National Council for Curriculum and Assessment
An Chomhairle Náisiúnta Curaclaim agus Measúnachta

Review of Mathematics in Post-Primary Education

Report on the Consultation

April 2006

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Introduction

The review of post-primary mathematics education arose in the context of concerns about the low level of mathematical skills displayed by students emerging from post-primary schools. Also, a number of studies had highlighted issues in relation to curriculum provision and uptake, the teaching and learning of mathematics in our schools, the appropriateness and effectiveness of the assessment arrangements, and the performance of candidates in the Junior Certificate and Leaving Certificate mathematics examinations. Mathematical knowledge and skills are held in high esteem in Ireland and are seen as having a significant role to play in the development of the knowledge society to which Ireland aspires.

In October 2005 the NCCA published a discussion paper which presented an overview of the issues surrounding mathematics education at post-primary level in Ireland. The paper presented data on the uptake of mathematics at the three different syllabus levels and considered the performance of candidates in the state examinations and in international tests of achievement. It identified particular areas of concern that needed to be addressed as part of the review, with a view to stimulating discussion on the nature and purpose of post-primary mathematics education as well as on the particular difficulties facing it at this point in time.

A companion paper on international trends in mathematics education was commissioned by the NCCA to further inform the debate on the issues identified in the discussion paper, and to give an insight into how other countries are addressing similar concerns about mathematics education.

This report describes the consultation process and the responses and submissions received. It revisits the issues raised in the discussion paper in light of the consultation responses, and summarises the findings that have emerged. Finally, it draws together a set of conclusions and makes recommendations on progressing the review so that post-primary mathematics education can be re-shaped to fulfil its many roles and functions in the lives of learners.

2. The consultation process

Consultation documents and feedback mechanisms

Following the publication of the NCCA discussion paper and its companion paper on international trends in mathematics education, a consultation process was engaged in during October and November to allow those with an interest in the issues raised to respond to these and to raise any other concerns which they considered should be addressed under the review.

To facilitate feedback, a consultation questionnaire was developed that was based on the main issues identified in the discussion paper. An online version of this questionnaire was made available on the NCCA website. Submissions by post and email were also invited and, to encourage as wide an audience as possible to participate in the consultation, a free text messaging service (SMS) was established. A 'flyer' was designed to draw attention to the response channels available.

Consultation documentation was circulated widely, including to all post-primary schools and to the education partners. In particular, third-level education departments and the science/engineering departments of Universities and Institutes of Technology were invited to respond. Consultation documents were also sent to individuals who had expressed an interest in the review. The initial period of consultation was extended to mid-December to facilitate those groups and organisations who wished to engage their members in discussing particular aspects of the review, and who needed time to facilitate the collation of their views.

Focus discussions

As part of the consultation, the NCCA held discussions with a number of groups to engage in focused consideration of the issues as they perceived or experienced them. These included two parents' representative groups, three branches and the Council of the Irish Mathematics Teachers' Association, and the Junior Certificate Mathematics Support Service team. Discussions also took place with a representative of the Union of Students

in Ireland and informally with individual mathematics teachers who were attending inservice courses in the period of the consultation. A combined meeting was convened of three NCCA course committees in the mathematics area (Junior Certificate and Leaving Certificate mathematics, and applied mathematics at Leaving Certificate). In collaboration with the School of Mathematical Sciences, University College Cork, a well-attended meeting was held in January 2006 at which teachers of mathematics and lecturers (from both University College Cork and the Cork Institute of Technology) engaged in wide-ranging discussion of aspects of mathematics education that were of common interest.

Collecting the data

Towards the end of the consultation period, questionnaires received in hard copy were uploaded to the online facility, and email and other submissions were collated under headings similar to those used in the questionnaire. This enabled the responses and submissions to be considered in a structured manner. According as further responses and submissions were received these were included in a similar fashion.

The structure of the questionnaire was such that those responding could be easily grouped into pre-determined categories (2nd or 3rd level students, teachers or lecturers, parents, school principals, employers, etc.). The survey facility that was used also enabled the identification of 'skipped' items and the tabulation of items in a multi-part section that required response on a three-part Likert scale.

The SMS provider facilitated the collection of the text messages received in the form of a spreadsheet, which allowed for easy handling and collation of the data. This also enabled the identification of repeat text messages, so that the number of individual SMS respondents could be gauged accurately.

3. Responses and submissions

The total number of responses to the consultation is indicated below in Table 1. Two-thirds of the responses were in the form of returned questionnaires (by post, as an email attachment or using the online facility). When repeat or trivial responses (mainly text messages) were taken into account this number was reduced to 340. Of these, 21 contained limited comment on just some of the issues, while about half of the questionnaires contained one or two skipped sections. It should be noted that about 10% of those who made a response did so on behalf of a group or organisation and that, in some cases, this was the result of focused internal discussion within the group or organisation. In addition to the formal submissions, notes were also taken at the discussion meetings mentioned previously.

Table 1. Number of responses or submissions by type

Response format	**No.**	**Comment**
Questionnaire (postal or email attachment)	150	15 of these had less than three of the twelve sections completed
Questionnaire (online)	114	7 of these were blank except for a name, while a further 6 had less than three of the twelve sections completed
Text message	98	36 of these were repeat or trivial messages
Other submissions	22	Including email comments and feedback obtained at group meetings
Total	384	

Table 1 shows the total number of submissions by type while Table 2 opposite provides a further breakdown of the returned questionnaires by category of respondent, where this was indicated in the response. Sixty-five text messages of a non-trivial nature were received from individuals. In a small number of cases written submissions were received that did not follow the format of the questionnaire; these tended to focus on specific

aspects of mathematics education, such as syllabus content, syllabus levels, examinations, or the low level of mathematical knowledge and skills of students emerging from post-primary schools. Just over half of the non-questionnaire/SMS submissions dealt in great detail with all of the issues raised in the discussion paper.

Table 2. Categories of questionnaire respondents

Category	Number	% of total
2nd level student	24	9.3
Teacher (mathematics)	117	45.5
Teacher (other subject)	7	2.7
3rd level student	22	8.6
Lecturer (mathematics)	26	10.1
Lecturer (other subject)	12	4.7
Parent	11	4.3
School Principal	14	5.4
Employer	0	0
Other	16	6.2
Not indicated	8	3.1
Total	257	

As can be seen, almost half of all questionnaires came from teachers, the majority of whom were teachers of mathematics. No individual business or employer completed a questionnaire, despite the concerns that this sector has expressed in recent years about standards and achievement in mathematics. The 'other' category includes a small number of professional individuals or groups, including ICT Ireland which represents the high technology/ICT sector within IBEC. A submission was also made on behalf of the Office of the Financial Regulator.

The remainder of this section of the report describes and summarises the submissions received using the section headings provided in the consultation questionnaire, even where this structure was not used by the respondents. These are not evaluated or responded to in this section; comment is provided in sections four and five in relation to what has emerged in the consultation submissions.

Where it was deemed relevant in the case of particular points of view expressed in a submission, indication is given in the commentary as to whether these were from an individual or from a group. In a small number of cases, the type of group is identified (teachers, representative organisation, parents, etc.).

3.1 Role and purpose of mathematics education

All respondents commented on the importance of mathematics education to the individual and to society. They pointed to its significance in the development of logical thinking and problem-solving skills, as well as its importance as a foundation for other subjects, especially the science and technology subjects. At senior cycle, mathematics was noted as having an added importance, given its requirement for admission to third-level courses.

Many respondents commented on the need for post-primary mathematics to be more in tune with real-life contexts so that students can see its relevance to themselves and to their current and future lives. Some also commented on the need for students to appreciate the beauty of mathematics, as well as the need to overcome the sometimes negative attitudes towards the subject. In one group discussion, a parallel was drawn with English, which students also use in other subjects. Indeed, the learning of mathematics as a series of formulae to be learnt, or procedures and techniques to be practised, was likened to learning English through the study of grammar, but with no exposure to literature or poetry.

Some respondents also commented on the need to refer to applications of mathematics, including its use in what could be termed non-mathematical fields or situations. They see it as important that students at post-primary level should know how and where they will use mathematics in their lives and work, and how it underpins many of the developments that affect our daily lives.

(Mathematics) provides an essential foundation and language for science and technological subjects.

Numeracy is an essential life skill…poor mathematical skills impede progress.

Learners … (should) acquire a certain amount of competence in applying the mathematics they learn to meaningful problems drawn from both the personal and scientific domains.

3.2 Concerns about mathematics education

The discussion paper identified specific areas of concern in relation to post-primary mathematics education:

- the emphasis on procedural skills rather than on understanding
- poor application of mathematics in real-world contexts
- low uptake of Higher level mathematics, especially in the Leaving Certificate
- low grades achieved at Ordinary level, especially in the Leaving Certificate
- gender differences in uptake and achievement
- difficulties in mathematics experienced by some students in third-level courses.

A common theme across many submissions was the emphasis that is placed in classrooms on 'rote learning' of mathematical knowledge and on 'procedural skills' which students can then demonstrate in examinations. This approach means that teaching for understanding does not receive much consideration. The revised syllabus in Junior Certificate mathematics is seen as going some way towards bringing about a change, but more needs to be done.

There is a real need to explain the ideas behind … procedures.

The emphasis in maths education needs to change to one where understanding and an ability to apply knowledge are the main goals.

More emphasis on experimentation, investigations and project work in the classroom would have a beneficial effect.

However, some respondents pointed to the benefits of developing a reasonable level of procedural skills, as distinct from rote learning, so that students can call on these skills when solving problems. The application of mathematics in real-world contexts entails a measure of practised skills in solving problems.

Many respondents commented on the fact that mathematics is perceived as a difficult subject, in which only those with a high level of mathematical ability can expect to achieve success.

I feel (at least from my own school days, which aren't so far away) that a lot if emphasis is put on 'strange' mathematical questions... I never really understood these types of questions...and still fail to understand their full importance in after school life (3rd level student)

This perception is seen as contributing to the fall-off in uptake of Higher-level mathematics at Leaving Certificate where, currently, the uptake levels are 11% at Foundation level, 71% at Ordinary level, and 18% at Higher level. The lack of recognition of Foundation level mathematics for entry to many courses at third level is viewed as contributing to the 'problem' of the excessive numbers taking Ordinary level mathematics at Leaving Certificate.

Many teachers commented on the lack of sufficient class time for mathematics, especially in the junior cycle, and the need to 'cover the course' so that students can do well in the Junior Certificate examinations. The reduction of time for mathematics is attributed to an overloaded curriculum as a result of the introduction of additional subjects or modules.

As a result, they argue, it is difficult to find time to explain the concepts involved. In order to prepare students for the examinations, the focus is mainly on student practice of the technical application of procedures.

> *A lot of the time with maths I just follow the method and do not understand why I should do so. After a while I get some understanding of the reasoning.* (2nd level student)

Some teachers commented on the declining commitment on the part of students to make the effort that is required to achieve a good level of understanding, but acknowledge that this is not confined to mathematics. The growing tendency for post-primary students, particularly in the senior cycle, to engage in part-time work is also noted; this is seen as contributing to lower attendance rates and less time being devoted to homework. Other factors identified as contributing to difficulty in mathematics at Leaving Certificate level are the 'gap' in standards between the junior cycle and senior cycle mathematics syllabuses (especially since the revision at junior cycle), a lack of basic numeracy skills, which some attribute to over-reliance on—or inappropriate use of—calculators.

> *The fact that Junior Cert maths was reduced in content and the topics dropped are still on the LC means that the gap is too wide for many LC students.* (mathematics teacher)

> *The recent changes at both primary level and Junior Cert. level are steps in the right direction. The main emphasis is on teaching and learning for understanding. These ideas must be carried on to Leaving Cert. The problem arises at second level in that these changes are not continuous. There is a huge void between Junior Cert. and Leaving Cert. at present.* (mathematics teacher)

> *The intuition associated with mental arithmetic (tables) is gone as a result of the introduction of calculators. Revision of maths in primary and junior cert with the associated revision in senior cycle (is required).* (group of mathematics teachers)

While some respondents see the lack of choice in the Junior Certificate mathematics examination as problematic, leading to lower achievement levels, others see this as of benefit in ensuring that students have fully covered the course.

Only a small number of respondents commented on gender differences, most of these making the point that the number of girls taking Higher level mathematics needs to increase. It was noted that girls generally perform better than boys in the examinations.

Mathematics lecturers also cited evidence of low levels of understanding on the part of students, even those who had studied Higher level mathematics at Leaving Certificate. They claimed that many students lack a basic understanding of concepts in some areas of mathematics, including arithmetic, algebra and geometry. These difficulties may go back to junior cycle, or even to primary school. Some commented on the narrow interpretation of the syllabus in textbooks and in the examinations.

> *At present one might describe the teaching of mathematics as a preoccupation with the 'how' of the subject to the almost total neglect of the 'why' of the subject. It is vital that the mathematical material presented in future texts and examinations cater more to applications so that students can acquire a greater understanding and appreciation of mathematics as a problem-solving discipline.* (mathematics lecturer)

Concern was also expressed that teachers of mathematics, especially in the junior cycle, may not themselves have an in-depth understanding of the subject, or have received pre-service training in the teaching of mathematics, thus leading to a narrow focus or 'comfort zone' in their teaching. Some respondents noted that not all teachers of mathematics attended the inservice courses that were provided to support the implementation of the revised Junior Certificate mathematics syllabuses. In

discussions with the Junior Certificate Mathematics Support Service, it was pointed out that longer-term support for teachers is needed if they are to make the kinds of changes in practice that are called for in the move towards teaching for and learning with understanding, particularly if further syllabus and assessment changes are introduced.

The Irish Mathematics Teachers' Association and some mathematics lecturers pointed out that a move to greater emphasis on applying or relating mathematics to the world in which we live is a complex task, one which is likely to prove challenging for teachers, particularly for those who are not specialist mathematicians. The continuing professional development of teachers of mathematics will need to be supported through the preparation of appropriate resources to facilitate this change of approach in the teaching and learning of mathematics.

3.3 Recent developments in mathematics education

Many respondents indicated that the 'teaching for understanding' approach in Junior Certificate mathematics has not been fully embraced by teachers. Some see this being due to the 'traditional' style of the revised syllabuses and examnations, while others point to the lack of time required to do this well. There is a general view that teachers need further support in changing their classroom methodology.

Individual teachers have commented that students find it difficult to make the transition from primary school mathematics to the kind of mathematics they meet at second level. Teachers at second level may not be aware of the changes that have taken place in primary school mathematics and vice versa.

> *How much communication is there between Primary and Post-Primary teachers with regard to the knowledge / implementation of these curricula? Virtually none, I would say.* (mathematics teacher)

Some respondents (parents, teachers) raised concerns about the early use of the calculator and its effect on numeracy skills, some arguing for a separate, calculator-free assessment of these skills. The lack of connectedness to real-life contexts in the way mathematics is taught was also commented on. There is a need to ensure a seamless progression in the kind of mathematics being taught from primary school to Leaving Certificate. There was no follow-through at Leaving Certificate following the changes in Junior Certificate mathematics and 'gaps' exist as a result.

While a small number of respondents commented on the lack of choice in the Junior Certificate mathematics examination papers, there were differing views on this issue. One second-level student expressed the view that the lack of choice ensures full course coverage and avoids situations where students can omit some sections of the (Junior Certificate) course:

> *For years, topics such as trigonometry were left out by candidates – showing their lack of understanding, only to realise they needed to understand it more fully for Leaving Cert.*

Mathematics lecturers were generally very much in favour of the 'no choice' situation in the Junior Certificate mathematics examination, some even suggesting that this could also be beneficial at Leaving Certificate. On the other hand, some teachers expressed the view that the lack of choice served only to reinforce the perception that mathematics is difficult. It was suggested that, in making curriculum changes, the views of a wide-ranging group of experts should be sought; this should include primary school teachers and mathematics lecturers. One lecturer suggested that third-level scientists could provide suitable insights and examples for topics that might be included in mathematics syllabuses at second level.

Respondents also referred to the influence of the examinations on what is taught and how it is taught in mathematics classrooms. Teaching and learning continue to be exam driven.

> *There is strong consensus that while the examination remains the same, classroom practice will be to resist change.* (lecturer in mathematics education)

In expressing concern about the competence levels of students, especially in the junior cycle but also at senior cycle, some respondents suggested that there should be a separate assessment of basic mathematical skills, which all students should be required to achieve. If a high 'pass' mark was required, this would give assurances to the system regarding the ability of students in basic mathematics. Students should be allowed to undertake this assessment as many times as required to achieve a 'pass'.

Some respondents (students and teachers) identified algebra as an area of particular concern, which needs to be addressed by different teaching methods. Geometry is another area seen to pose difficulties for students.

Finally, a number of respondents commented on the reluctance of (second-level) teachers to change their methods, since their students are very successful in the examinations. One school principal expressed the view that mathematics teachers would feel very insecure in an environment that promotes and assesses an explorative, innovative and investigative assignment type of mathematics. Another school principal noted that

> *project work and/or continual assessment through different methodologies and skills should give time and space to enjoy maths and its essence, thereby breaking the emphasis on procedural skills, etc and relying more on understanding of maths and its principles.*

3.4 Current trends in mathematics education

This section had the highest level of non-response in the questionnaire. Where it was addressed in detail, respondents tended in the main to be lecturers in mathematics. Some second-level teachers indicated a lack of familiarity with current trends.

The comment of a mathematics lecturer captures the points expressed in a number of the responses about 'modern mathematics':

> *… its nature and emphases rendered it susceptible to the type of reductionism to which it fell victim: where abstract techniques were taught and examined while the applications that should have accompanied them were neglected because of the greater demands they place on both teacher and learners.*

This respondent went on to say that adopting a genuine 'modern mathematics' approach would require careful work to see it implemented successfully, including a change in textbook and examination treatment of many syllabus topics and a change in the mode of teaching to address student motivation and to include the application or evident purpose of these mathematics topics.

Some respondents commented on the need for a balance between the various approaches to mathematics education, noting that, while students need to learn mathematics in context, skills of logical thinking, rigorous argument and abstraction must not be neglected. The various approaches to mathematics education are not independent or mutually exclusive, they argue; the focus should not be on one to the virtual exclusion of the other.

A group of teachers pointed out that opportunities arise, especially in the Transition Year, to take alternative approaches to topics in mathematics, allowing students to look at these topics differently, but that time constraints are a definite hindrance at other levels.

One group (a professional body), summarised this need for balance as follows:

> *It is likely that the RME approach will do more to address the problems we encounter. However, a balance must be found here; students should also appreciate the elegance of mathematical proof and the rigour involved. Beyond that, there is no need for us to 'prescribe' minutiae – if students enter third level with an appropriate*

disposition towards mathematics, we should be capable of providing them with the rest, whether it be in guiding them into the world of 'pure' mathematics, or facilitating their acquisition of the additional 'knowledge' required for data analysis, 'modelling' etc.

There seemed to be general agreement that more concrete experiences of mathematics in real-world contexts are needed for primary and junior cycle students, with gradual movement towards abstraction as they become more comfortable with the underlying concepts and structures of mathematics. It should be possible to integrate some applications of mathematics as well as problem-solving approaches within the current syllabuses, but the examinations would need to change in order for this to be taken seriously.

Many respondents referred to the fact that students will need mathematics in many aspects of their daily lives, so these should be used as the contexts when they learn mathematics: interest calculations, discounts, tax, statistics, currency conversion, calculation of area, volume, etc. While some of these contexts may not have immediate relevance to junior cycle students, reference to them at this level is important so that the students can appreciate how and where mathematics affects their lives.

3.5 Mathematics in relation to other subjects

This section focused on the dual nature of mathematics—geared to applications but also worthy of study in its own right—and on its relationship to other subjects, and was responded to in sixty percent of submissions. Many respondents pointed to the importance of a good foundation in mathematics so that students could engage productively in a wide range of subjects, not just the science and technology areas (specific mention was made of business subjects, music, geography, P.E., home economics, history, CSPE, ICT). These subjects can also contribute to mathematics. While commenting on this overlap, it was noted (mainly by teachers, but also by some students) that the manner in which topics are treated in the different subjects can vary significantly.

> *Teachers teach in isolation and they are happy to do so. How does the business teacher teach percentages? How does the science teacher (unless they are one and the same) teach measure, scientific notation, area and volume? How does the geography teacher teach scale, data handling and data presentation?* (mathematics teacher)

Some of this variation is attributed to the lack of communication or discussion between teachers of these subjects (and, perhaps, a lack of opportunities for this) and/or the lack of in-depth mathematical knowledge on the part of teachers of subjects other than mathematics. The establishment of good in-school planning structures can help this situation. Linkages with other subjects will also enable students to see the connectedness of mathematics and its applications in other disciplines. This could be facilitated if there was coursework in mathematics.

Some respondents pointed out that it is important not to treat mathematics solely as a 'service' subject; it has a beauty in itself. As with many other subjects, 'usefulness' is not the only criterion by which mathematics should be judged or justified.

> *Mathematics as a universal language and as a subject to enrich our students in their problem solving and creative abilities rather than as a subject of routine procedures to service other disciplines is a must for the education of our future generations.* (mathematics teacher)

Third-level students commented on the need for second-level students to be aware of the importance of mathematics for many courses at third level and how understanding of mathematics is critical for success in such courses.

3.6 Provision and uptake of mathematics

In this section respondents were asked to comment on the adequacy of the current mathematics courses in meeting the needs of all students at both junior cycle and senior cycle. Views were also invited on the relatively low uptake in Higher level mathematics. Just over forty percent of respondents commented on these points.

The responses were very wide-ranging, reflecting the diversity of opinion that exists regarding the current courses and their assessment. Many of the points made in responding to this section were repeated elsewhere in submissions.

A small number of respondents made initial remarks concerning the lack of continuity between mathematics in the primary school curriculum and mathematics syllabuses at junior cycle and senior cycle. This is seen as creating difficulties for students, who may not make the connections themselves. A lack of awareness and understanding among teachers of the mathematics curriculum/syllabuses at each level means that there is not a coherent treatment of topics. When added to the different methodologies that are used to address the same concept, students are confused; it is little wonder that they consider mathematics difficult.

> *In Ireland there is a serious discontinuity between the mathematics curriculum for senior primary school and that for junior post-primary school. … This discontinuity has been highlighted in past reports but little has been done to address the situation. By the time students reach senior post-primary level, many of them have become alienated from mathematics and will find the easiest option available in terms of further study of mathematics.* (lecturers in mathematic education)

On the other hand, teachers generally (and some mathematics lecturers) were of the view that the revised mathematics syllabuses at Junior Certificate did not require much alteration, but that the approaches used in

the teaching and learning of the topics were critical. Many of the teachers pointed out that there is potential to introduce topics in context and to adopt a problem-based approach to these, while ensuring that students develop the requisite skills that will enable them to progress to more advanced topics and problems. However, as noted already, this is not a simple step and will require

> *the development of a different mindset and much greater pedagogical content knowledge among teachers – at least among many of those who teach mathematics as a second subject – … many good teachers already model the approaches one would like to see; the current in-service initiative taps the resource that they can provide.* (lecturer in mathematics education)

A common theme among respondents was the need to alter the style of the examination paper if assessment is to reflect a problem-solving emphasis and a context-based approach in teaching and learning.

Most respondents recognised the need to encourage more students to study Higher level mathematics in the Junior Certificate; some reference was made to timetabling and in-school structures that could facilitate this. It was recognised that a concerted effort is required to change the attitude that mathematics is difficult, and this message must be communicated to students, teachers, parents, and the public generally.

Some respondents recommended a focus on numeracy and algebra in the junior cycle, so that students grasp the basics concepts and skills. Teachers, especially those for whom mathematics may not be their main subject, should be given support to ensure that they cover these aspects well.

Reference was made once more to the gap between Junior Certificate and Leaving Certificate mathematics courses. A number of mathematics lecturers favoured a reduction in the breadth of the Leaving Certificate courses so that a deeper understanding of some topics could be achieved

by students at all levels. As one lecturer pointed out, the current choices mean that students can opt out of some topics:

> *Many students can completely disregard whole sections of the course, e.g. trigonometry, and still enter a third level college where this can be a major component of their required knowledge.*

The issue of non-recognition of Leaving Certificate Foundation level mathematics for third level courses continues to give rise to difficult choices for students, some of whom find themselves in classes with others who are capable of talking Higher level mathematics, but who choose to take the Ordinary level course and focus their efforts to gain points in other subjects. Some students opt out of Higher level through fear of failure in the examination; they need to 'pass' mathematics to get into college.

In a discussion with a parents' group, one parent suggested that the timetable for the exams should be changed so that students following the Higher level course could sit the Ordinary level examination in order to secure the required 'pass', and then also take the Higher level examination without the fear of 'failing'.

As an alternative to this, it was suggested, where a school offers a three-year senior cycle along the lines of the NCCA senior cycle proposals, students could take the Ordinary level examination in mathematics at the end of their fifth year and then proceed to study the Higher level course for sixth year. This would require some re-alignment across the courses, with perhaps a common course for one year followed by a measure of specialisation in the final year. As noted by some respondents elsewhere in the questionnaire, a common examination paper, focusing on basic mathematical knowledge and skills (perhaps a calculator-free one), could form one element of assessment. The second paper could offer a range of choices, depending on what courses or careers students were aiming for.

A different approach was suggested by some respondents. This advocates the merging of Foundation and Ordinary level mathematics syllabuses into

a common course, accepted by third level, which should be aimed at those who do not intend to study pure mathematics at third level. The common course should be designed to ensure that students will acquire a good grasp of fundamental mathematics concepts and will develop an adequate level of mathematical skill to enable them to use mathematics in their chosen courses at third level.

A re-structuring of Higher level mathematics was also suggested, with a reduction on the 'options' and giving relevance to the topics retained. By way of illustrating how real-life context could be used in the treatment of a topic (difference equations) one lecturer in mathematics offered the following suggestion.

> *…the recurrence relation governing the amortisation of loans should be treated in depth as this is perhaps one of the most useful applications of mathematics lying outside of the scientific situation that bears on a student's (future) personal life and which can be undertaken at the level of senior cycle.*

The same respondent, having commented on the 'technical exercise' nature of many examination questions went on to state:

> *The manner by which a difference equation is arrived at is more often than not a greater practical skill than the routine by which the solution of the difference equation is obtained*.

Many of the respondents, but particularly students, commented on the fall-off in the numbers taking Higher level mathematics at Leaving Certificate. Students pointed out that, while they may begin studying mathematics at Higher level, they drop back to Ordinary level when they find the time demand too great and believe that they can get points in a much easier fashion in other (Higher level) subjects.

Although it is not possible to determine the categories of respondents who submitted text messages, a large proportion of comments related to maths

as being a 'hard' subject to study/learn, with difficult topics to be covered especially at Higher level, making success (as measured in points) beyond the reach of all but the most able students.

3.7 Influence of the examination papers – 'teaching to the test'

In this section, respondents expressed a variety of views on assessment of mathematics. The influence of the examinations on student choice was mentioned frequently in responses. Teachers acknowledged that they tend to focus on 'getting their students through' the exams successfully. The exam focus was also noted by students, parents and lecturers.

> *Maths is taught almost exclusively for the purpose of getting students the best possible results in the exams. Our present syllabus can be ruined by bad exam papers. Our present exam tests predominately a student's ability to memorise – and to substitute into a formula*. (2nd level student)

> *As long as teachers are judged on their results teachers will `teach to the test'. Note the interest in recent times on who goes to college.* (mathematics teacher)

> *Assessment is the driving force for change and the current system will not change unless exam questions demand thinking skills from our students. The repetitive nature of examination questions forces rote learning rather than understanding.* (mathematics teacher)

Once again, there was extensive comment on the difficulty that students experience with Level Certificate Higher level mathematics, and the wide gap in standard between this and the Ordinary level course. While recognising that sole reliance on a terminal examination paper puts pressure on students, caution is advised by some respondents as regards the introduction of coursework. Students of average or below average ability in mathematics, it is argued, might not be well served by coursework assessment, depending on how it is conducted.

> *In the UK there seems to be evidence that investigations are becoming procedural. Assessment of coursework also raises social issues as students from privileged backgrounds often have advantages.* (mathematics teacher)

However, in general, respondents seem to be favourably disposed to the introduction of coursework, 'practical work', or some other form of second assessment component, in mathematics as an aid to deepening students' understanding of topics. This would also facilitate closer alignment between the syllabus aims and objectives and its assessment.

A number of respondents stressed the need for extensive professional development for teachers in relation to any second assessment component that might be developed, so that they are well prepared and the system can be assured that this is not a 'dumbing down' of the mathematics course.

Following on from earlier views on the need to move away from rote learning, many respondents in this section stressed the need for the examinations to adopt a different style of question, which is designed to assess the candidate's mathematical understanding as well as problem solving skills rather than practised procedural skills in predictable question types. This could also include

> *unseen applied questions … which test the ability of students to apply familiar techniques in unfamiliar situations.* (mathematics lecturer)

Some respondents referred to the need for continuous assessment, both as a formative tool and to enable students to obtain some credit/marks in advance of the final examination.

> *I think continuous assessment would be a brilliant idea in all subjects - particularly maths! You spend two years studying a subject and then it all comes down to what you can remember or write on a page in the space of 5 hours….* (2nd level student)

> *Sílim gur cheart measúnú leanunach a dhéanamh le linn na mbliana agus go mbeadh bfhéidir 20% nó 30% den pháipéar déanta acu roimh an Teastas Sóisearach agus Ard Teist – ní bheadh an méid céanna brú orthu… sílim go mbeadh torthaí níos airde.* (mathematics teacher)

The potential for using ICT in assessment was mentioned by a number of respondents, with some commenting on the need for greater use of ICT by teachers in the mathematics classroom before students could be expected to use it as both a formative and a summative diagnostic/assessment tool. Steps would need to be taken to ensure that its use is monitored and that external and/or moderation procedures are established. A number of submissions advocated a piloting of possible alternative assessment arrangements, perhaps in Transition Year.

If there is to be a specific assessment of basic mathematical knowledge and skills (especially in arithmetic and algebra), then ICT has a role to play in this. Students could take (and re-take, if necessary) such a test to obtain a 'satisfactory' grade, which could then be incorporated into their final result in mathematics.

Many respondents pointed to the 'fairness' of the current system and the trust that it enjoys. Any changes would need to be well researched and also well resourced, and their introduction preceded by extensive information and education of all concerned, students, parents, teachers, examiners, third level institutions, employers, etc.

3.8 Syllabus levels and range of courses

Less than half of the questionnaire responses commented on the issues raised in this section. Some referred back to earlier comments, especially in relation to provision and uptake, while others focused almost exclusively on the issue of non-recognition of Leaving Certificate Foundation level mathematics for course entry at third level. A few submissions made the point that students who are capable of studying Foundation level

mathematics only are probably not suited to third level courses where there is any mathematical content.

As already noted, the lack of follow-on changes at Leaving Certificate after the revisions in mathematics syllabuses at Junior Certificate is seen as a major difficulty for both teachers and students. This needs to be addressed as a matter of urgency. Other changes can be developed and introduced more gradually. While some respondents advocated a reduction to two course levels, others saw benefit in retaining the three levels, provided a re-balancing was undertaken to bring about a more evenly spread progression across the levels. If there was to be a re-structuring of mathematics syllabuses, this could accommodate both general and specialist mathematics, with the former taking a more context-based and applications approach while the latter could have greater emphasis on 'pure' mathematics.

In some responses and group discussions, the status of applied mathematics at Leaving Certificate came in for consideration. The present course is seen as offering students with an interest in mathematics the opportunity to solve problems based on realistic applications (albeit mostly restricted to mechanics). If appropriate aspects of this course were incorporated into the main mathematics syllabuses, this could provide an opportunity to extend the coverage of (advanced) applications to other interesting and more recently developed areas of mathematics. ICT is seen as offering great potential in this regard.

As an alternative, it is suggested, if Higher level mathematics were made accessible to a greater number of students, following an amalgamation of the Foundation and Ordinary level syllabuses, an additional, more advanced mathematics course might be considered that would provide a challenge to more able students who were interested in further study of mathematics.

3.9 Student achievement in mathematics

This section sought respondents' views on the effectiveness of identified measures for improving student performance in mathematics examinations. Approximately two-thirds of respondents completed this section of the questionnaire. Table 3 shows the levels of agreement with each of the three effectiveness ratings for the suggested measures (respondents were invited to suggest other measures).

Table 3. Effectiveness of measures for improving the performance of students in mathematics examinations (% of responses; rounded).

Suggested measure	Very effective	Effective	Not effective
allocation of more class time to mathematics	51	35	14
better pre-service and inservice education for teachers of mathematics	64	27	9
improved mathematics textbooks and other learning resources	51	42	7
provision of learning support for students who are experiencing difficulties with the subject	73	26	1
provision of 'general' as well as 'specialist' mathematics courses	52	34	14
increased emphasis in examination questions on the application of mathematics to real-world problems	53	37	10
the introduction of additional forms of assessment, such as coursework	46	29	25
improving the perception of mathematics among parents and the general public	43	37	20

Over sixty percent of respondents completed this section. As shown in the table above, the provision of learning support for students who experience difficulty is almost universally seen as very effective or effective in improving their performance in examinations, whereas the introduction of additional forms of assessment is judged to be an ineffective measure by over a quarter of respondents.

Over ninety percent of those who responded consider that better pre-service and inservice for teachers, as well as improved mathematics textbooks and other resources, are likely to be effective or very effective in this regard, although comments elsewhere in responses point out that the 'performance' should not be equated with the technical execution of practised routines in answering questions.

The allocation of additional class time for mathematics was seen as a very effective measure by most of the respondents who had previously drawn attention to the reduction of class time for mathematics as a result of an expanded junior cycle curriculum. In some cases, teachers had advocated a minimum of one class period per day for mathematics, which they felt would allow them to develop teaching strategies that did not rely almost exclusively on rote learning of techniques and procedures.

Among the other measures suggested by respondents were

- use of ICT and other innovative teaching methods
- continuous assessment linked to overall results
- topics on the history, evolution and applications of mathematics
- greater emphasis on (and reward for) accuracy in mathematics
- reduction in class size to allow for group work and discussion
- better information on the mathematics required in third level courses
- teachers of mathematics to be qualified in mathematics
- increase in the points allocated to maths for some courses
- improving the connectivity between mathematics and other subjects
- mathematics courses for parents.

Finally, a number of respondents (in questionnaires and text messages) commented on the need to make mathematics more 'real' and to develop additional resources for students, such as web-based tutorials.

3.10 Teaching and learning in mathematics

Almost sixty percent of respondents expressed views in this section. Many focused on the need to change the way mathematics is taught; they see both syllabus revision and changes in assessment as the drivers for this. There was some repetition of points made earlier in responses regarding the need for teachers of mathematics to have a qualification in mathematics and for the availability of resources to facilitate and encourage changes in classroom practice. However, respondents did comment on the good work being done by inspiring teachers and recommended that best practice needs to be disseminated widely so that all teachers can gain insights into the varied approaches that could be taken in the mathematics classroom.

Reference was made to developments in our understanding of how students learn and the need for teachers to adapt their approach so that all students are catered for. Active teaching methodologies help to engage students more and to encourage enquiry-based, constructivist learning. A move to teach for skills development was advocated, allied to changes in assessment so that the emphasis is on students applying their mathematical knowledge and skills in answering examination questions.

> *A more hands on approach is needed involving active methodologies leading to a greater understanding of topics and an ability to apply knowledge to realistic problems. This would require a reduction in content of the courses with topics taught to a greater depth… (and) a commitment to CPD for maths teachers involving both content and pedagogy.* (mathematics teacher).

The issue of continuous assessment was also raised in the context of changing teaching and learning practice. Student confidence can be improved and learning can be reinforced if students are required to demonstrate skills in problem solving over a period of time. This can lead to the assessment of their application of such skills to unfamiliar contexts in a terminal examination paper.

> *There is definitely a place for the teaching of procedural skills and teaching for answering exam-type questions as repetition of certain mathematical skills does have merit. However, I feel there would need to be a dramatic cultural change among teachers and parents to accommodate the highly desirable shift of emphasis towards different, unfamiliar contexts. (school principal)*

The valuable support provided to Junior Certificate mathematics teachers in recent years is acknowledged by teacher respondents, who want to see this support continued. Teachers should have available to them expert tutors or guides who can assist them in changing their teaching methods, but this will take time.

There is generally good support for establishing a better balance between the 'traditional' approach and emphasis on understanding of concepts. Many students come to understand the concepts when they have achieved a measure of confidence in their own ability to follow the procedure for answering examination-type questions.

3.11 Attitudes to and beliefs about mathematics

A little over half of the respondents offered opinions on the issues surrounding perceptions, attitudes and beliefs about mathematics. There was general acknowledgment in the submission that mathematics is perceived as a difficult subject, with many students and adults lacking confidence in dealing with mathematical issues and processes. The symbolic language that is used in mathematics is off-putting for some and the fact that there is little understanding of the concepts means that, for many, any mathematical skills they have are the result of rote learning.

Respondents see a need to make school mathematics more interesting and engaging for students, through activities and learning approaches that relate the mathematics being learned more closely with students' everyday lives and experiences. Teachers should make connections between and across the often isolated topics as they are presented in textbooks. This

requires that teachers be confident and have a 'big picture' understanding of the subject.

> *… students need their best maths teachers at a young age. Teachers who really know what they are doing and really understand the simplicity of what they are doing. Once confidence is in place at a young age, I think the other issues… will right themselves*. (mathematics lecturer)

Many third-level respondents commented on the need to give second-level students insights into the beauty of mathematics and to provide them with appropriate challenges so that they can experience the satisfaction that comes from solving a difficult problem or discovering an interesting aspect of a problem or solution. The quality of teaching, especially in the early years, is very significant. Some students also commented on 'good' and 'bad' mathematics teachers that they have had in school.

> *A teacher who loves mathematics will communicate this to students. Many successful mathematicians will attribute their success to the enthusiasm of teachers who inspired them to work at the subject.* (mathematics lecturer)

In discussions with parents, the point was made that many parents do not themselves have a sufficient grasp of mathematics (or the methodologies now being used in class) to enable them to help their children. Mathematics is one of two subjects (Irish is the other) where parents feel inadequate and regularly seek outside help for their children when they are struggling with homework. This begins as early as first year at second level, and reinforces the perception of mathematics as a difficult subject.

> *… provide support (to be defined) to parents who can ultimately influence student attitudes to maths and who can supervise homework assignments.* (text message)

A number of students (and some second-level mathematics teachers) commented that it is not worth the effort, in terms of getting points, to spend a lot of time at mathematics when it is easier to get high grades in other subjects. There is a large discrepancy between the effort required to achieve a grade C at Higher level and a grade A at Ordinary level in mathematics. Students who do persist with Higher level mathematics tend to be those who are very good at it or who are targeting courses or careers that rely heavily on mathematics. This contributes to the 'elitist' perception of the subject. A text message stated that large class size means teachers do not have the chance to give individual attention to students who are struggling with mathematics, and so the students just 'give up'.

Some respondents suggested that greater use of ICT (computers and calculators) in mathematics classes could help students to overcome the drudgery or boredom of long computational problems or repetitive routines and allow them to gain insights into patterns and concepts so that mathematics is made more interesting and enjoyable. For the majority of students, it is argued, a focus on real-life applications will help them to relate mathematics to aspects of their own lives in ways that will hold their attention. The media have a role to play also, particularly in not perpetuating negative attitudes towards the subject.

The view was expressed that the 'fear' of mathematics is associated with getting the 'wrong' answer, and this may begin at primary school. It was advocated that students need to focus more on the processes involved in solving problems of a mathematical nature and realise that their accuracy will improve once they have confidence in the methods used. Teachers can help by pointing out that a variety of approaches may often be possible, all of which are valid and will give the correct result. This requires teachers themselves not to have an absolutist view of the subject. In this regard, one respondent quoted the Russian mathematician Kolmogorov:

> *"Mathematics can be taught really well only by a person who himself is fascinated by it and perceives it as a living, evolving science."*

One third-level college introduced a 'primer' mathematics course for its Higher Certificate Electronic Engineering course as an intervention aimed at bringing students back into contact with mathematics in a manner that will boost their confidence, as well as being a foundation for the main course. A lecturer in the college noted that, in particular, many mature students returning to education have a negative 'mathematical self-image'. Material for the primer course is chosen carefully so that students are engaged initially in concrete concepts related to number manipulation, then progressing to simple symbol manipulation and finally to realistic applications of the concepts and processes. The course is structured in such a way that students can learn at their own pace and in an individually supported manner.

Not all students need to undertake this primer course; a diagnostic test, which has no bearing on student grades, has been developed to determine students' needs. Other third-level colleges also report successful operation of mathematics learning centres and/or tutorial programmes in mathematics for students who find it difficult to make the transition from the rote-learned mathematics that they experienced at second level. In some of these colleges diagnostic testing is also carried out on the first-year intake.

Respondents pointed to the need for students at second level to be given opportunities in class to build confidence in their own ability at mathematics, building on what they know rather than presenting them with a set of procedures to practise when answering questions. It was suggested that second-level mathematics teachers might benefit from training in diagnostic testing.

3.12 Other influences affecting mathematics

Many of the responses under this heading referred back to previous comments they had made regarding attitudes and beliefs, the pressure to get 'points' at Leaving Certificate, and the need to make greater use of ICT in the teaching and learning of mathematics. Some respondents favoured the return of 'bonus points' so that more students would be encouraged to

study mathematics at Higher level. The status of Foundation level mathematics at leaving Certificate also surfaced again in responses.

The lack of coherence and appropriate progression between primary and post-primary mathematics, and between the different syllabus levels in post-primary mathematics were again pointed to as being problematic. There was a repeated call for immediate action to be taken at Leaving Certificate in light of the changes that have been introduced at Junior Certificate, and for professional development and support for teachers to be maintained. It was recommended that opportunities for teachers to support their peers should be facilitated and encouraged. Mention was made of the wealth of web-based material that is suitable for use in mathematics classes, but teachers need to be made aware of it and be supported in using it appropriately. Some applications can ease the workload of the teacher by providing students with opportunities to practise (and assess) their technical skills in problem solving and to be facilitated in some measure of self-directed learning.

There was very little comment on some other issues that had been identified in this section of the Discussion Paper: cultural, equality, gender (uptake and achievement), socio-economic factors, educational disadvantage, and students with disabilities or special educational needs.

Additional comments

Almost one-third of questionnaire respondents provided additional comments, many of these using this section to summarise or re-state their main concerns and recommendations. A number of respondents used this opportunity to look forward to what they would like to see happening in mathematics education in relation to syllabus revision or development, assessment arrangements, accreditation, and teacher professional development.

> *If we wish to see the vast majority of our 15-18 year olds continuing to study mathematics then understanding and an ability to apply*

knowledge rather than an ability to follow procedures should be the benchmark. Syllabus review by itself will not achieve this. The culture in the maths classroom needs to be changed. This needs to be supported by time for interaction and reflection for maths teachers and a proper support service to foster and inform this change. (mathematics teacher)

What is needed is a slow but steady change in the way mathematics is taught in Ireland today. Most of the relevant issues are adequately covered in the discussion paper, however now the real challenge lies in addressing these issues. This will require a long look at best practices in other countries, supply of adequate resources to teachers especially in the form of training and materials to aid education. Making maths relevant to the students is one area that needs urgent attention at all levels from primary to third level. (mathematics lecturer)

By way of contrast with their earlier comments, which stressed the urgency of addressing the issues in mathematics education, in this section respondents acknowledged that it will take time and will require a range of actions to be undertaken. All respondents were in agreement on one point – change is required; there was not the same measure of agreement on the extent or nature of that change.

Responses in this section also contained a number of specific suggestions for addressing identified issues. These included

- combining Foundation level and Ordinary level syllabuses, with a greater emphasis on 'practical' mathematics and reflect this in the assessment arrangements
- introducing an element of coursework that would involve students researching topics outside the narrow confines of the syllabus, thus enabling them to see how/when mathematics is applied in real-world contexts

- making examination questions less predictable; including 'unseen' questions/problems so that students are required to apply familiar techniques in unfamiliar situations
- introducing some form of continuous assessment (perhaps piloting this in schools)
- providing the resources and training for teachers to make greater use of ICT in mathematics; computer-based assessment for some aspects of mathematics should be investigated
- supporting mathematics teachers with short courses, developed by the relevant authorities and interest groups and delivered by personnel with in-depth knowledge and experience of teaching the subject, so that they can renew their enthusiasm and reflect on their understanding of mathematics and how they teach it in their classes
- re-visiting the issue of the mathematics requirement for entry to courses at third level that do not have particular mathematics content; if necessary, including at OL and FL topics that students need to have covered before going to third level
- researching best practice elsewhere and adopting whatever would improve mathematics education here
- commissioning the development of resources (CDs, web, handbooks, etc.) for teachers so that they can adopt different teaching methods
- encouraging more mathematics graduates to take up teaching
- finding out the real-life contexts for mathematics that students are interested in
- making more use of an assessment for learning approach in class
- considering the re-introduction of bonus points for Higher level mathematics in respect of some third-level courses.

4. Reflecting on the issues

The consultation set out to stimulate discussion of a range of issues affecting post-primary mathematics education; in this it has certainly succeeded. The responses received reflect a wide variety of perspectives on mathematics and mathematics education, ranging from mathematics in the primary school, through second and third level, encompassing its past and present developments as an area of human interest, and touching on its current and potential applications in the world in which we live.

Many individuals and groups have given serious thought to the issues raised in the consultation, and communicated their views in a manner which is both an honest recognition of the challenge and an expression of hope that serious consideration will be given to this important area of learning. The concerns surrounding mathematics education arise not only because of its importance in the development of human knowledge and skills, but also because of the high esteem in which it is held within and outside of education in Ireland and the potential contribution that it can make as Ireland aspires to become a knowledge-based society.

The review has provided the first opportunity for almost forty years to take an in-depth look at mathematics education and there is broad welcome for this. Many respondents point to the discontinuity that now exists between mathematics education in the primary school and that at second level. The philosophy underpinning post-primary mathematics, which has its origins in the 'modern mathematics movement' of the sixties is at odds with the realistic mathematics education (RME) philosophy which underpins the revised curriculum for mathematics in primary schools. The review affords the opportunity to take a 'bigger picture' look at mathematics education so that that students will experience a continuum of mathematics learning over their years in formal education.

The remainder of this section presents a summary and analysis of the issues in light of the responses received during the consultation.

There is a general recognition – and expectation – that change is required in the mathematics education which students experience in post-primary school. The discussion documents and the consultation responses present the rationale for change in respect of four broad aspects of mathematics education which are inter-related: the mathematics curriculum, teaching for and learning with understanding, assessment, and the culture that surrounds mathematics. Other issues identified in the NCCA discussion paper which elicited a range of responses as presented in the previous section of this report, can be considered under these four headings.

4.1 Looking at curriculum

Although not a compulsory subject to Leaving Certificate, mathematics is studied by the vast majority of post-primary students, most of whom remain in school until the end of the senior cycle. Internationally, the proportion of students who study mathematics in upper-secondary education is comparatively lower than is the case in Ireland. Students' performance in Leaving Certificate mathematics plays a significant role in progression to third-level education as well as in preparing them for their future personal, social and working lives. This has given added weight to the concerns that have arisen over the past number of years in relation to their mathematical knowledge and competence.

A traditional mathematics curriculum allied to a relatively narrow emphasis on the development of de-contextualised procedural skills and the backwash effect of examinations on teaching and learning have meant that many students leave school with only a superficial understanding of the subject and little or no conceptual knowledge. While this may serve them well in the short term, that is in terms of examination results, its longer-term effect is one of limited value in respect of their future mathematical needs.

The Junior Certificate mathematics syllabuses were revised in 2000, but this was more a minor adjustment of content than a genuine revision and was followed (rather than informed) by a move to focus on teaching for understanding. Although arising just after the introduction of the revised

primary school curriculum in 1999, the revised syllabuses for Junior Certificate mathematics did not adopt the emphasis placed in primary school mathematics on developing mathematical ideas in real-life contexts and their application to real-life problems. They were still very much a content-based presentation of mathematics that was almost totally devoid of any meaningful context. Leaving Certificate mathematics syllabuses are of a similar style, not having been revised since the early nineties.

Many of the submissions point to the need for a complete overhaul of the post-primary mathematics syllabuses due to a number of factors:

- discontinuity with the mathematics curriculum in the primary school and the 'gaps' that have arisen at Leaving Certificate as a result of the syllabus changes at Junior Certificate
- over-emphasis on technical, procedural skills at the expense of conceptual understanding
- the lack of context for much of the syllabus content and the virtual absence of reference to applications of mathematics in real life
- the perception that mathematics is a difficult subject in which only the intellectually talented students can expect to succeed
- questions arising with regard to the appropriateness of the three syllabus levels and the mismatch between the aims and objectives of the current syllabuses and their assessment
- the identified mathematical deficiencies of students emerging from post-primary school.

A large number of submissions indicate topics in mathematics that should be considered for inclusion in revised syllabuses, and some that could be considered for reduced treatment or removal altogether. Some of the respondents have suggested that two different types of mathematics courses are required to accommodate the needs and abilities of all students, including their future mathematical needs in terms of their further studies and careers.

4.2 Looking at teaching and learning

The teacher is the critical agent of change. Syllabus changes can indicate a change in content, emphasis and approach to mathematics education and can set out appropriate strategies and structures for its assessment, but the context and conduct of the teaching and learning situation is critical to effecting a change in the way students develop their knowledge, understanding, skills and attitudes in mathematics.

International trends suggest that an approach which develops from the concrete to the abstract and which presents mathematics in a context that relates it to real-life situations is likely to engage students' interest and enable them to develop their knowledge and skills to an appropriate level. Many of the respondents to the consultation pointed to the need to make mathematics more related to the lives of students, to let them see how it applies in real-world contexts and how it enables them to develop their thinking and problem-solving skills. They need to realise that mathematics is not an unconnected series of procedures whose meaning and logic are impenetrable to all but a small minority, but that it has an integrity and beauty in itself as well as myriad applications in their daily lives.

There is an acknowledgement that this requires a fundamental change in the way in which many teachers teach mathematics, but such a major change is required if students are to learn with understanding. Some respondents (teachers and lecturers), referring to the findings from research on the impact of change on teaching practice, argue that this change will take time, that it needs to be well resourced, and that sustained teacher professional development and support will be required for it to become embedded in practice.

A major concern in the responses is the level of pedagogical content knowledge that some mathematics teachers have, particularly those for whom mathematics is not their principal subject and who may not have any third-level qualification in mathematics. This concern also extends to teachers in primary school, where some of the students' perceptions of and

attitudes to mathematics are formed. It has been suggested that serious consideration needs to be given to both pre-service and inservice aspects of mathematics teaching. Lecturers in mathematics education who responded to the consultation commented on the difficulty that teachers experience in teaching mathematics in a manner that is qualitatively different from the way they themselves learned mathematics.

The use of ICT in mathematics can dramatically change the students' engagement with the subject. The use of computers and calculators acts as a motivator and increases productivity, overcoming the tedium and boredom of repetitive procedures and computation and allowing for a level of intuitive exploration that was impossible heretofore. The potential of this technology needs to be recognised when syllabuses are being developed.

The backwash effect of examinations on teaching and learning was noted in many of the comments from students, parents, teachers and lecturers. In recognising this, there needs to be a closer alignment between learning and assessment. Assessment for learning has a significant role to play in this regard but, once again, the need for teacher professional development was highlighted.

The following were among the suggestions made by respondents in relation to the teaching and learning of mathematics.

- Undertake an audit of the mathematics qualifications of teachers of mathematics in post-primary schools; identify the needs of teachers in terms of professional support and put measures in place that will address these comprehensively.
- Develop teaching resources that will facilitate teachers in changing their teaching methods where necessary and support teachers in making this change.
- Prepare examples of well-structured contexts and applications that relate mathematics to the lives of students and to real-world situations (these need to be genuine, not pseudo-applied) and that will stimulate discussion and exploration by the students.

4.3 Looking at assessment

Throughout the various sections of the questionnaire responses and in the many discursive submissions that were received, the centrality of the examinations and their dominant influence on teaching and learning was evident. This was also recognised in the discussion groups, where some suggestions were made for steps that could be taken to alleviate the situation. However, there was no agreement on whether such suggestions would gain the necessary support. In the absence of alternative forms of assessment, students and teachers view preparation for the terminal examination as the focus of much of the learning in mathematics. The predictable nature of the examination papers, in many cases at the level of the individual question, reinforces—some would argue rewards—this focus.

In meetings with parents and, to a lesser extent, with teacher groups, the possibility was discussed of a common examination that would be aimed at assessing basic mathematical competency. This examination, which students could undertake as often as necessary to obtain a satisfactory rating, could be operated independently of the final examination (Junior or Leaving Certificate). It could provide a student with a certificate of competency, a sort of 'mathematics driver's licence', which would give assurances to the system that those who obtained this certificate had demonstrated a minimum level of knowledge, skills and application in particular aspects of mathematics. Much of this learning and assessment could be technology based, allowing students to progress at their own pace and to build up their competency as well as confidence in their ability to 'do' the mathematics. Students who experience difficulty in making progress could be identified easily and given individual support to overcome the particular difficulty. In this way, individual barriers to progress could be surmounted.

The focus on examinations that prevailed in most of the responses is in itself indicative of the need to broaden the debate about assessment and the different roles that it can and should play in teaching and learning. The benefits of some form of continuous assessment were mentioned in many submissions and responses, although it was not always clear what the

respondent meant as 'continuous assessment'. In some cases, this appeared to refer to the undertaking of coursework over the two or three years of the course, which would be assessed towards the end of the cycle and the result combined with that obtained on the examination paper(s). In other cases, it seemed to mean a number of assessment events spread over the period of the course, each contributing a part mark towards the final grade. Some respondents expressed reservations about the former, indicating disadvantages that accompanied it, while others were not in favour of the latter due to its lack of flexibility for accommodating different rates or stages of intellectual development.

Very few teachers indicated that they used forms of assessment other than traditional tests to provide their students with formative feedback on their knowledge and understanding of mathematical concepts. This is an area which needs to be addressed. Assessment for learning initiatives currently being undertaken by the NCCA may provide insights into ways in which teachers can be assisted in making greater use of this approach in mathematics.

As mentioned above, and also in conjunction with teaching and learning, there is great potential for the use of ICT in assessing specific aspects of mathematical knowledge, understanding and skills. At present, initiatives of this kind are mainly confined to calculator and/or computer-based tests that focus on routine computational and procedural 'drill and practice' learning in mathematics. However, computer software is available that assesses logical reasoning and problem-solving skills in a mathematics context, which might easily be adapted to encompass a range of syllabus topics.

4.4 Looking at the culture of mathematics education in Ireland

Despite the high regard in which mathematics is held in this country—or perhaps because of it—the perception of mathematics as a difficult subject persists. The majority of respondents acknowledge this and agree that this perception needs to be challenged and changed if mathematics education is to achieve its potential in contributing significantly to the development of both the individual and society.

A few respondents noted that there are some indications that the changed approach to more active methodologies in mathematics classrooms in primary schools is having an effect on pupil's attitudes to the subject. This is supported by the findings of the Primary Curriculum Review, which reported very positive attitudes to mathematics among children who participated in the case study schools. As primary school pupils progress to the post-primary junior cycle, it will be important that these attitudes are reinforced. More needs to be done to ensure greater coherence between the approaches in mathematics education at both of these levels. As mentioned elsewhere in this report, the mismatch in the underpinning philosophies as well as in the classroom experiences are barriers that must be overcome.

Parents indicated that their own perceptions of mathematics as being difficult are picked up on by their children, especially as they progress through second level. There is a role here for the wider community to adopt a more positive attitude and to be conscious of how negative messages regarding mathematics can reinforce already poor attitudes.

Teachers need to be encouraged to adopt more imaginative approaches in their teaching and to encourage their students to 'make sense' of the mathematics they learn. This in turn will require them to have confidence in their own mathematical ability as well as in their teaching of the subject. Once the student's confidence is established at a young age, they will not be daunted by the more challenging aspects of the subject as they progress through school. A number of respondents recommended that the 'best' mathematics teachers in a school should teach at first year so that a solid foundation in mathematical knowledge and skills is laid at that stage. Some lecturers point to the success of mathematics tutorial or assistive initiatives in overcoming the negative attitudes of third-level students to the subject, and cite examples of students who emerge from second level with relatively low grades in Leaving Certificate mathematics, yet who manage to graduate with the highest honours in courses that have a significant mathematical content.

Mathematics needs to be seen as a subject which, although it may present challenges, provides the means for overcoming these challenges. Many respondents cautioned against falling into the trap of 'dumbing down' the subject through the removal of essential mathematical concepts from syllabuses or by a lowering of the standards required in the examinations (particularly the Leaving Certificate). Rather, they suggest, efforts should be made to support students in their learning of mathematics by identifying the problems they experience and developing methods of overcoming them. In this way, their confidence and self-esteem will be enhanced.

5. Towards effecting change

The most significant finding in the consultation is the broad agreement that teaching and learning practices have the greatest influence on students' understanding of mathematics. Teachers, and their own attitudes to mathematics, have a major role to play in student attitudes to and perceptions of the subject.

Syllabus change will not of itself bring about a transformation of student attitudes and perceptions or in their mathematical knowledge, skills and understanding, or result in improved examination results. While it is recognised that syllabus change is needed, as is a change in the assessment arrangements—including the examinations—it is in teaching and learning the 'why' and 'what if' of mathematics as well as the 'how' that deeper understanding of mathematical concepts can be achieved. This will, in turn, bring about a change in the student perception of, and attitude towards, mathematics.

A dominant theme in the consultation feedback is the need to make mathematics more related to the lives of students. Two main approaches are advocated for this:

(i) introducing mathematics concepts in real-life contexts so that it leads from the concrete to an appropriate level of abstraction

(ii) highlighting the many ways in which mathematics is applied in the real world.

It has been pointed out that this is not a straight-forward or simple process, not can it be expected to succeed in a short time-frame. Appropriate contexts and applications need to be selected carefully; their ability to underpin the mathematical ideas and processes being taught and learned must be assured. Teachers must be confident in their own ability to use contexts and applications in the class situation and in the effectiveness of such an approach to facilitate student learning and understanding.

The development of a range of teaching resources that can be drawn on in planning classroom activities is critical to its success. A collaborative effort between experienced teachers (primary and post-primary) and third-level lecturers (of mathematics and mathematics education) may provide a means of selecting or developing appropriate exemplars. A representative sample of students should be consulted to ensure that the contexts used are indeed of interest or relevance to them.

ICT is seen as having a significant role in enabling greater exploration and investigation in mathematics, both within and beyond the classroom. Its potential as a tool for teaching and learning remains to be exploited fully, although some progress is being made in this area as a result of the teacher support programmes put in place in recent years. Teachers are gradually availing of this technology (advanced calculators and computers) to enhance students' learning of mathematics. The ICT framework being developed by the NCCA can provide a means for teachers to both explore and develop exemplars of ICT use in mathematics classes.

Most of the responses in the consultation identified the rote learning of mathematics, particularly of procedural techniques specifically aimed at answering predictable types of examination questions, as one of the major problems that must be overcome. Textbooks were mentioned as narrowing down mathematics to this feature. Some respondents, mainly mathematics lecturers and lecturers in mathematics education, stressed the need to shift the emphasis in procedures from drill-and-practice routines to a deeper understanding of the processes involved in solving problems and the varied strategies and approaches that can be adopted to achieve a solution.

Students need to be given insights into the reasoning behind the kinds of decisions that are taken in devising solutions to problems, and to see how 'real' and 'live' mathematics can be. Teachers can also facilitate this by encouraging class discussion, by brainstorming ideas, and by considering alternative approaches to the solution of similar problems.

As noted by many of the respondents in the consultation, change is also required in the manner in which learning in mathematics is assessed. There needs to be debate on how the development of basic mathematical knowledge and skills can be supported and assessed. The potential of alternative forms of assessment, and assessment events, should be explored and the role that ICT might play in these should be investigated.

Potential avenues for change need to be explored, which base change on the findings of research. They should take an inclusive approach so that all involved, students, teachers, parents, and the wider community and education interests can work together to bring about much-needed improvement in how students engage with mathematics and in their understanding of it.

The remainder of this section sets out a number of short-term and longer term steps that can be taken to re-shape the teaching, learning and assessment of mathematics. These are grouped under the four main headings identified previously.

5.1 Changing curriculum

The opportunity presented by the current re-balancing exercise for Junior Certificate syllabuses (in which mathematics is included) should be availed of to reflect, especially in the learning outcomes, the change in approach to one of teaching for and learning with understanding. This emphasis had not been a driver of the syllabus change in mathematics that took place in 2000, but was subsequently the focus in much of the implementation support for teachers that accompanied the introduction of the revised syllabuses. Added value can be obtained if this is combined with the context and applications approach suggested earlier, and accompanied by focused support for teachers.

The types and range of mathematical skills that students are expected to develop through their study of mathematics need to be identified. These could be categorised (computational, graphical, communication, procedural, analytical, logical reasoning, problem-solving, decision-making,

etc.) and syllabus topics and learning outcomes could be selected that will support and emphasise the development of these skills. Resources should be developed that teachers can use in the classroom and teachers should be supported in adopting a changed approach, where necessary. On an ongoing basis, syllabus revision that is deemed necessary can be informed by developments along these lines, and can move the focus away from content, facilitating greater emphasis on contexts, applications and skills.

Another aspect of mathematics that featured prominently in submissions was the need for students to realise and appreciate the many ways in mathematics is applied in the real word. Although applications of mathematics are myriad, it is important that those chosen for reference in class should be appropriate to the students' level of knowledge and understanding and should underpin the particular topic(s) being considered. As an initial step, links should be identified between mathematical concepts and processes as they arise in the mathematics class and the same topics (or their applications) as they arise in other subjects.

The integration of applications of mathematics into the mainstream subject will mean that the position and nature of the syllabus for applied mathematics at Leaving Certificate must be considered. This should take into consideration the potential for short courses in one or more specialised areas of mathematics and its applications. The current Leaving Certificate applied mathematics syllabus, which has not changed for over thirty years and is studied by a small proportion of Leaving Certificate students, is almost exclusively focused on mechanics topics from mathematical physics. Many other applications of mathematics have emerged in recent times, facilitated in no small way by computer technology which itself is based on mathematical principles.

Post-primary syllabus change in mathematics, especially in the junior cycle, must build on the changes that have taken place in the primary school curriculum so that there is greater coherence between the two levels. Students and teachers must be able to see a progression in the

mathematics that is taught and learned. The 'gap' between junior cycle and senior cycle syllabuses also needs to be addressed. Of course, syllabus change *per se* will not transform the mathematics experiences of students. Their engagement with mathematics in the classroom is much more than learning what is set down in the syllabus.

Much attention in responses to the consultation was focused on the impact that lack of recognition for Foundation level Leaving Certificate mathematics has on student uptake at Ordinary level and, consequently, on examination results. If the suggestions outlined in this section of the report are implemented and evaluated, any subsequent revision of syllabuses can take into account whether the continued existence of three syllabus levels is either desirable or necessary.

5.2 Changing teaching and learning

If change is to occur in the mathematical experiences of students, then teachers will need to consider ways in which the approaches they use in class can become more effective in providing the kinds of experiences that will engage students. Bringing about a change in teaching practice is a challenging process that needs to be managed carefully. Over a period of time, teachers have built up and honed their repertoire of resources and teaching strategies, which have proven successful with successive cohorts of students. It is important to appreciate that changing practices will involve extending the teachers' comfort zones to embrace new approaches, and that they will need to be supported in doing this.

Teaching involves building on students' existing knowledge, understanding and skill. A problem that has constantly faced teachers in first year at second level is that of knowing what it is that students have learned at primary school and how well they have done so. Using standardised tests, all primary schoolchildren will be tested twice on their mathematical achievement. The NCCA is working on the development of report card templates and on supporting the process of transfer of information on student progress from primary to post-primary schools. Post-primary

teachers will thus know more about the mathematical achievement of primary schoolchildren as they enter second level. However, they will need support in using this information effectively. Opportunities for primary and post-primary teachers to engage in discussions about mathematics and the teaching methodology employed at each level should be both encouraged and facilitated.

To bring about a change in the teaching of post-primary mathematics, two specific approaches have already been identified: a move to more context-based mathematics (including its applications) and an emphasis on skills development. To gauge the suitability and effectiveness of a context-based approach to teaching and learning, a number of mathematics topics in the current syllabuses could be selected and some exemplar lessons developed for teachers to use in class. The support services, the subject association (IMTA) and the emerging teacher professional networks operating through Education Centres could be involved in providing support for teachers in adapting and using these resources in their teaching of mathematics. This initiative could focus on junior cycle mathematics for a defined initial period.

In their review of the literature on international trends in mathematics education, which the NCCA commissioned, Conway and Sloane described in detail a lesson study initiative which is proving very successful among teachers in Japan. This involves groups of teachers collaborating in the development of specific lessons which are then implemented in class. The group reviews the lessons, refines them in light of practice, and puts them through a repeat cycle of implementation and evaluation. Further iterations are undertaken until the group is satisfied with the quality of the lessons. This method of professional collaboration could be tried on a pilot basis in conjunction with the proposals outlined in 5.1 above. The development of appropriate assessment procedures and mechanisms for classroom use would enable teachers to gauge the extent to which students have developed their mathematical understanding and skills in these topics, and

at the same time allow them to evaluate the effectiveness of their own teaching using this approach.

By limiting the number and variety of topics involved, teachers could gain confidence in the new approach and, perhaps, begin to devise ways of using it in other mathematics topics. Such an initiative could also inform, and be informed by, a process of ongoing syllabus review, in an alternative model for syllabus revision that involves teachers as action researchers, contributing as professionals to subject development. Teachers should be encouraged to see themselves as learners, adopting and adapting a variety of strategies in their teaching. Where students and teachers engage in open discussion of mathematics, this can be to the benefit of both. Students can gain insights that they might not otherwise obtain, and teachers can gain a better understanding of the students' thinking.

Students will not necessarily make the connections within and between subjects, so it is important that the teacher does this with them. As mentioned already in connection with a move to include relevant applications of mathematics, this will require collaboration between teachers at the school level. Some respondents suggested the development of support materials and/or short courses for teachers so that they can improve their familiarity with such applications of mathematics, thus building their confidence in referencing these in class. Here again, the Japanese lesson study approach referred to earlier comes to mind.

5.3 Changing assessment

Much has already been said regarding the need to reform the assessment of mathematics, including the development of complementary assessment components, not least because of the backwash effect of the examinations on teaching and learning. The benefits of assessment for learning (AfL) in building students' confidence in their own learning and in the early detection and solution of difficulties which students experience must become more widely appreciated and accepted. The current NCCA initiative in AfL includes mathematics. If this is extended in conjunction with

a context and applications based approach at Junior Certificate, as suggested above, all teachers can engage in formative assessment as part of a developmental process. Feedback provided through AfL can also help to reassure students that appropriate standards are being attained or to identify areas of knowledge or skills that require attention if the student is to achieve a particular target.

A greater focus on skills and applications of mathematics in both the curriculum and classroom practice must be supported and reinforced in the examinations. Many respondents pointed to the backwash effect of the examinations, particularly due to the predictable nature of the questions, resulting in classroom emphasis on practising routine procedures that can be reproduced in the examination. They advocate the inclusion of questions that require students to apply their knowledge and skills in unfamiliar contexts (through 'unseen' questions). However, this should not be the only way in which a changed teaching and learning approach is recognised in the examination papers. There needs to be a move away from a focus on demonstration of routine procedures in other questions also.

Syllabus changes will need to be reflected in changes to the assessment procedures, including the examination papers. Closer liaison with further and higher education in syllabus revision and in teaching and learning initiatives would ensure confidence in the revised arrangements from the point of view of progression to courses at third level.

5.4 Changing the culture surrounding mathematics

The review of mathematics undertaken by the NCCA has generated much discussion and debate, but this has mainly been within the education community. There is a need to widen the debate and to challenge the traditional, negative views of mathematics. Students need to be given both the opportunity and the means to achieve their potential in mathematics, so that they can have a better self-image and confidence that they can 'do' mathematics. They need to see the subject as a set of challenges that they can meet and overcome rather than ones they should avoid.

The rationale for and nature of the changes to be implemented will need to be publicised. Parents, especially, will need to be informed of the changes and assisted in understanding their implications. They will need to be facilitated in helping their children to develop their mathematical ability. Changing attitudes and perceptions will involve a long-term campaign. It took time for the present situation to develop; it will take time to reverse that development.

Children in primary school are experiencing a different kind of mathematical education from that experienced by their parents. The Primary Curriculum Review indicates that children are enjoying the active engagement with mathematics and the methodologies being employed in class. This must be continued and reinforced at second level and teachers of mathematics at both levels must be supported in adopting practices that will ensure its continuation. The suggested approaches outlined above, if implemented fully, will go a long way to ensuring that for students their years in post-primary education and, more specifically, their experiences in mathematics will also prove to be enjoyable.

In the past, significant numbers of students have emerged from post-primary education with negative memories of mathematics as a result of 'failure'. These can be overcome, as evidenced by successful interventions at third level, but the problem must be tackled where it arises – in primary and post-primary mathematics classrooms.

Looking ahead

The next phase of work under the review is about effecting change. This includes not only curriculum and assessment change but also engaging with students, teachers and schools, and those in further and higher education, in developing and implementing new approaches in mathematics education that will impact positively on the experiences of post-primary students.

Ireland has a long tradition of student engagement in mathematics education throughout the post-primary years, unlike in many other countries where a much smaller proportion of the student cohort in upper secondary education studies mathematics. The concerns that exist about the level of knowledge and skills among school leavers can be harnessed into support for the steps needed to bring about improvement, for the benefit of the individual learner, the learning community and society generally.

Appendix 1: Mailing list for the consultation

Groups or Organisations

All Post-primary Schools

An Chomhairle um Oideachas Gaeltachta agus Gaelscolaíochta

Association of Community and Comprehensive Schools

Association of Primary Teaching Sisters

Association of Secondary Teachers, Ireland

Association of Teachers' Centres in Ireland

CDVEC Curriculum Development Unit

Centre for Early Childhood Development and Education

Chambers of Commerce of Ireland

Chief Executive Officers of Vocational Education Committees

Church of Ireland Board of Education

Co-operation of Minority Religions and Protestant Parents' Association (COMPASS)

Curriculum Councils for England, Scotland, Wales and Northern Ireland

Department of Education and Science

Directors of Education Centres

Economic and Social Research Institute

Educate Together

Education Departments in Third-Level Colleges

Educational Research Centre

Engineers Ireland

Federation of Catholic Brothers and other Catholic Schools Parent Councils (FED CBS)

Forfás

Heads of Mathematics, Science and Engineering Departments in Universities and Institutes of Technology

Heads of Universities and Institutes of Technology

Industrial Development Authority

Institute of Public Administration

Irish Business and Employers Confederation

Irish Mathematics Teachers' Association

Irish National Teachers' Organisation

Irish Primary Principals' Network

Irish School Heads' Association

Irish Small and Medium Enterprises Association

Irish Universities Association

Joint Managerial Body

Junior Certificate Mathematics Support Service

Mater Dei Institute of Education

National Association of Principals and Deputy Principals

National College of Art and Design

National Education Office for Travellers

National Educational Psychological Service

National Parents Association of Vocational Schools and Community Colleges (NPAVSCC)

National Parents' Council (Post-primary)

National Qualifications Authority of Ireland

NCCA Council Members

NCCA Senior Cycle Committee

NCCA Junior Cycle Committee

NCCA Committee for Junior Certificate mathematics

NCCA Committee for Leaving Certificate mathematics

NCCA Committee for Leaving Certificate applied mathematics

Office of the Chief Science Advisor

Parents Association of Community and Comprehensive Schools (PACCS)

Primary Curriculum Support Programme

Queen's University Belfast

Royal Irish Academy

Second Level Support Service

Shannon Curriculum Development centre

St. Mary's University College, Belfast

State Examinations Commission

Teachers' Union of Ireland

Union of Students in Ireland

University Libraries

University of Ulster

Individuals

Aidan Savage	National Co-ordinator, School Completion Programme
Aideen Cassidy	National Co-ordinator, Junior Certificate School Programme
Alan Gilbert	COMPASS
Alan Mulligan	COMPASS
Anna Walshe	National Co-ordinator, Junior Certificate Science Support Service
Anne Marie Ryan	National Co-ordinator, Leaving Certificate Home Economics
Anne O'Sullivan	Castlebar, Co. Mayo
Brendan Duane	National Co-ordinator, Leaving Certificate Chemistry
Christy Tyrrell	NCCA Education Officer, Accounting
Ciarán O'Sullivan	Lecturer, Institute of Technology, Tallaght
Connie Carolan	PACCS
Daithí Mac Sithig	Union of Students in Ireland
Diane Birnie	Lucan, Co. Dublin
Eamonn Sheppard	Thurles, Co. Tipperary
Eileen Flynn	National Co-ordinator, School Development Planning
Eleanor Petrie	COMPASS
Elizabeth Oldham	Trinity College, Dublin
Frances Holohan	National Co-ordinator, Leaving Certificate Vocational Programme
Geraldine Horgan	NCCA Education Officer, Junior Certificate Science
Geraldine Mooney Simmie	University of Limerick
Gerard O'Reilly	FED CBS
Geraldine Perkins	FED CBS
Gerry Nolan	Intel Ireland
Humphrey O'Riordan	PACCS
Jerry Shiel	Educational Research Centre
Jim Jackman	PACCS
Joan Crowley-	National Co-ordinator, Special Education Support

O'Sullivan	Service
Joe Kennedy	NPAVSCC
John Mulcahy	NCCA Education Officer, Geography
John O'Donoghue	University of Limerick
Joyce Ryder	COMPASS
Louise Holden	The Irish Times
Margaret Donohue	PACCS
Marion Palmer	Institute of Art, Design and Technology, Dún Laoghaire
Maureen Connolly	NPAVSCC
Michael O'Leary	National Co-ordinator, Transition Year Support Service
Paddy Flood	National Co-ordinator, Leadership Development for Schools
Pádraig Ó Siochrú	Daingean Uí Chúis, Co. Chiarraí
Pat Murphy	PACCS
Pat O'Connor	Coachford, Co. Cork
Pat Younger	NCCA Education Officer, Economics
Patricia Forde-Brennan	NPAVSCC
Patricia O'Malley	COMPASS
Paul McElwee	St. Catherine's College, Sion Hill
PJ Garvan	NPAVSCC
Rose Tully	NPAVSCC
Sean Close	St. Patrick's College, Drumcondra
Sheila O'Driscoll	National Co-ordinator, Leaving Certificate Applied
Sinéad Breathnach	National Co-ordinator, School Development Planning Initiative
Tim Regan	National Co-ordinator, Leaving Certificate Physics
Tom Geary	University of Limerick
Tom Mullins	University College, Cork
Valerie O'Dowd	Assistant National Co-ordinator, PCSP
Vincent Brett	FED CBS

Appendix 2: List of respondents

Aileen Clancy	Teacher (other subject)
Alex Hogarty	3rd level student
Alfie O'Doherty	Lecturer (mathematics)
Amanda Fennell	Teacher (mathematics)
Andrew Wood	Teacher (mathematics)
Ann Allen	Teacher (mathematics)
Ann Vereker	Lecturer (mathematics) WIT
Anne Brosnan	Teacher/Researcher NUIM
Anne McNamara	Teacher (mathematics)
Aoife Maher	2nd level student
Association of Community and Comprehensive Schools	
Association of Secondary Teachers, Ireland	
Ayla Tuohy	3rd level student
B McDonnell	Teacher (mathematics)
Bernadette McLean	Teacher (mathematics)
Bernie McMahon	Teacher (mathematics)
Bernie O'Callaghan	Teacher (mathematics)
Bertie Keely	Teacher (mathematics)
Br Thomas Hickey	Teacher (mathematics)
Breda Collins	Teacher (mathematics)
Breda Doherty	Teacher (mathematics)
Breda Fallon	Teacher (mathematics)
Breda Morrissey	Teacher (mathematics)
Brendan Kelly	Teacher (mathematics)
Brendan McGill	Teacher (mathematics)
Brendan O Sullivan	Teacher (mathematics)
Bríd Galligan	Teacher (mathematics)
Brien Nolan	Lecturer (mathematics)
Cammie Gallagher	Teacher (mathematics)

Carmel McGee	Teacher (other subject)
Cathal Jordon	Teacher (mathematics)
Catherine O Donnell	Parent
Catherine Roddy	Teacher (mathematics)
Catherine Walsh	Teacher (mathematics)
Christine Dunne	Teacher (mathematics)
Christy Maginn	Teacher (mathematics)
Ciaran O'Sulllivan	Lecturer (mathematics) IT Tallaght
Claire Thomas	Teacher (mathematics)
Claire O Neill	2nd level student
Coláiste Íde	School Principal
Colm Doyle	3rd level student
Colm McGuinness	Lecturer (mathematics)
Colm O Connor	School Principal
Conall Kelly	Other
Diarmuid Lalor	Teacher (mathematics)
Daithí Ó Máirtín	Múinteoir (matamaitic)
Damian Cooke	SEC
David Doyle	Lecturer (mathematics)
David Flannery	Lecturer (mathematics)
David Hobson	Teacher (mathematics)
Declan Casey	Teacher (mathematics)
Declan Dunne	Teacher (mathematics)
Declan McConnell	Other
Deirdre Barry	Teacher (mathematics)
Deirdre Gardiner	Teacher (mathematics)
Deirdre O Halloran	2nd level student
Denis Dunne	Parent
Dominic Guinan	Teacher (mathematics)
Donal Hurley	Lecturer (mathematics) UCC

Donna McGowan	Teacher (mathematics)
Donncha Ó hÉallaithe	Lecturer (mathematics) GMIT
Dorothy Hughes	
Dr. Aidan Seery	Lecturer (other subject) TCD
Dr Ann O'Shea, Dr David Wraith	Lecturers (mathematics) NUIM
Dr Brien Nolan + others	Lecturers DCU
Dr. Diarmuid O Sé	Lecturer (mathematics) IT Carlow
Dr James Grannell	Dept. of Mathematical Sciences, UCC
Dr. John Corr	Lecturer (mathematics) IT Tralee
Dr Joseph Manning	Lecturer (other subject)
Dr Leo Creedon	Lecturer (mathematics) IT Sligo
Dr. Michael Brennan	Lecturer (mathematics)
Dr Noel Colleran	Tipperary North VEC
Dr Paul Robinson	Lecturer (mathematics) IT Tallaght
Dr S M McMurray	Lecturer (mathematics) TCD
E Kernan	Teacher (mathematics)
Eamonn Grennan	Lecturer (mathematics) IT Sligo
Eamon McNulty	Teacher (mathematics)
Eileen Gildea Moran	Teacher (mathematics)
Eileen McCrory	Teacher (mathematics)
Elizabeth Oldham	Lecturer (maths; maths ed.) TCD
Emer O Neill	3rd level student
Emma Keenan	2nd level student
Ena Boyle	Teacher (mathematics)
Engineers Ireland	
Eugene Hickey	Lecturer (other subject)
Eugene Kernan	Lecturer (mathematics)
Fiona Desmond	Teacher (mathematics)
Flora-Louise Carey	3rd level student
Frances Weymes	Teacher (mathematics)

Francis Kavanagh	Other (school psychologist)
Frank Ryan	Teacher (mathematics)
Gary Hammond	Other
Gemma O'Dwyer	Teacher (mathematics)
Geraldine Browne	Teacher (mathematics)
Gerard Brennan	Teacher (mathematics)
Gerry Kelly	Lecturer (other subject) IT Letterkenny
Gina Hyland	Teacher (mathematics)
Grace Poole	3rd level student
Grainne Ni Mhuiri	Teacher (mathematics)
Helen O'Mahony	Teacher (mathematics)
Helena McLoughlin, Mary Waters-Wynne	Teachers (mathematics)
Higher Education and Training Awards Council	
Ian McCulloch	Teacher (mathematics)
Ian O Donnell	3rd level student
ICT Ireland	High-tech sector within IBEC
Imelda Butler	2nd level student
Irish Mathematics Teachers' Association	
Irish National Teachers' Organisation	
Jacqui Lehane	Teacher (mathematics)
James Reilly	Lecturer (mathematics) IT Tallaght
Jessica Stack	2nd level student
Joan Cleary	Lecturer (mathematics) IT Tralee
Joe Bridges	Teacher (mathematics)
John	Teacher (other subject)
John A Desmond	Teacher (mathematics)
John Colleran	Teacher (mathematics)
John Fanning	Teacher (mathematics)
John King	Teacher (mathematics)
John Mac Sweeny	Lecturer (mathematics)

John McGuinness	Teacher (mathematics)
John Moore	Teacher (mathematics)
John S Davin	School Principal
John Scannell	Teacher (mathematics)
Joseph McCarthy	Lecturer (mathematics)
Karan Murphy	Teacher (mathematics)
Karina Plunkett	Parent
Kathleen Cotter	Teacher (mathematics)
Kevin J Kelly	Dept. of Mathematics, CIT
Kevin Lynch	Lecturer (other subject) IT Tralee
Kevin Swords	Teacher (mathematics)
Lauranne Kelly	Teacher (mathematics)
Leo Hogan	School Principal
Liam Ó Callanáin	Teacher (mathematics)
Lorraine Doherty	Teacher (mathematics)
Lynda	3rd level student
Mairéad Hourigan	Other
Mairéad Ó Shaughnessy	Teacher (mathematics)
Majella Healy	Teacher (mathematics)
Marc van Dongen	Lecturer (other subject)
Margaret Convey	Teacher (mathematics)
Margaret McKeon	
Maria O'Brien	Parent
Marie Griffen	School Principal
Marie Reilly	Teacher (mathematics)
Marion Palmer	Lecturer (other subject) DLIADT
Martha Burton	
Martina Plunkett	Teacher (mathematics)
Mary Clancy	Teacher (mathematics)
Mary Clayton	Teacher (other subject)

Mary Cronin	Teacher (mathematics)
Mary Doohan	Teacher (mathematics)
Mary Fahey	Teacher (mathematics)
Mary Friel	School Principal
Mary Gallagher	Teachers (mathematics)
Mary Geary	Teacher (mathematics)
Mary Irving	Parent
Mary Lafferty	Teacher
Mary McHugh + Maths teachers	Teachers (mathematics)
Mary Morrissey	3rd level student
Mary O Donnell	Teacher (mathematics)
Mary O'Malley	Other
Mary Smith	3rd level student
Mary Storey	2nd level student
Mary T Reany	Teacher (mathematics)
Maths Dept, Bishopstown	Teachers (mathematics)
Maths Dept, Boherbue	Teacher (mathematics)
Maura Carroll	Teacher (mathematics)
Maura Farrell	Teacher (mathematics)
Maureen Spain	Teacher (mathematics)
Melissa Finn	3rd level student
Michael Cannon	Teacher (mathematics)
Michael FitzGerald	Lecturer (mathematics) GMIT
Michael Kelly	Teacher (mathematics)
Michael McCann	School Principal
Michael Stacey	School Principal
Michael White	Teacher (mathematics)
Mícheál D.O Muineacháin	Teacher (mathematics)
Michel Schellekens	Lecturer (other subject)
Michelle Conroy	Teacher (mathematics)

Monica McKenna	3rd level student
Nadia Duffy	Parent
Neil Hallinan	Teacher (mathematics)
Nicholas Sweetman	School Principal
No Name	Teacher (mathematics)
No Name	3rd level student
No Name x 11	2nd level student
Noreen Noonan	Teacher (mathematics)
Nuala Ní Cheallabhui	Teacher (mathematics)
Office of The Financial Regulator	
Oliver McCormack	Teacher (mathematics)
Oliver Murphy	Teacher (other subject)
Orna Lucey	Parent
Owen McConway	Teacher (Mathematics)
Owen O'Mahony	2nd level student
Paddy Ward	Teacher
Pamela Reidy	Teacher (mathematics)
Patricia Bridges	Parent
Patricia Mc Hugh and Geraldine O Shea	Teacher (mathematics)
Patricia Nunan	Teacher (mathematics)
Patrick Flood	Teacher (mathematics)
Patrick McVicar	School Principal
Patrick Plunkett	Teacher (mathematics)
Paul Allen	3rd level student
Paul Brady	Parent
Paul Curran	Lecturer (other subject) UCD
Paul Doyle	Teacher (mathematics)
Pauline Ryan	Parent
Peadar Hanratty	Teacher (mathematics)
Peter McLoughlin, Angela Donoghue	Teachers (mathematics)

Rachel Maguire	2nd level student
Rasmus Noeske	3rd level student
Rebecca Quirke and Yvonne Hannafin	2nd level students
Regina Casey	
RIA Mathematical Sciences Committee	Royal Irish Academy
Richard O'Donnell	3rd level student
Rita Murphy	Teacher (mathematics)
Rita O Donoghue	Teacher (mathematics)
Roísín Scally	Teacher (mathematics)
Roisin O'Sullivan	Teacher (mathematics)
Rynagh McNally	3rd level student
S Moloney	Teacher (mathematics)
Seamus O Neill	School Principal
Seán Close, Dolores Corcoran, Therese Dooley, St. Patrick's College Drumcondra	
Seán Dowling	Teacher (mathematics)
Sean Kealy	Teacher (mathematics)
Shane Dowdall	Lecturer (mathematics)
Sheila Carroll	Teacher (mathematics)
Sinéad Fitzsimons	Teacher (mathematics)
Sr Berchmans Whelan	School Principal
Students' Union IT Carlow	3rd level students
Stuart Barry	
Syed Ahmed	3rd level student
Teresa Cushen	Teacher (mathematics)
Teresa Mc Namara	Teacher (mathematics)
Teresa Mulhall	3rd level student
Teresa Nolan	Teacher (mathematics)
Thomas Doyle	2nd level student
Thomas K Murphy	Teacher (mathematics)
Tim Brophy	Teacher (mathematics)

Tom O Connor	Teacher (mathematics)
Tomás Mac Eochagáin	
Una Healy	2nd level student
Union of Students in Ireland	USI
Union of Secondary Students in Ireland	USSI
Valerie O'Keeffe	Teacher (mathematics)
Veronica Kerin	Teacher (mathematics)
Victoria Clarke	3rd level student
Winifred O'Toole	Teacher (mathematics)
Yvonne Hassett	3rd level student
Yvonne O Neil	Teacher (mathematics)