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National Council for Curriculum and Assessment
An Chomhairle Náisiúnta Curaclaim agus Measúnachta

Review of Mathematics in Post-Primary Education

a discussion paper

October 2005

REVIEW OF MATHEMATICS IN POST-PRIMARY EDUCATION

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1 Introduction

2005 marks the bi-centenary of the birth of William Rowan Hamilton, one of Ireland's leading mathematicians and scientists. One of his most important discoveries was general methods in dynamics, which virtually predicted modern wave mechanics (which has many and varied applications). Computer users might appreciate his contribution to developments in graphics technology through his discovery of quaternions (while walking along the banks of the Royal Canal!). It is appropriate, therefore, that mathematics in post-primary education should come in for close scrutiny in the course of this year. Coincidentally, the United Nations declared 2005 as the International Year of Physics, noting that it is also the centenary of important scientific discoveries by Albert Einstein. In the UK, the year is being celebrated as 'Einstein Year'.

Mathematics matters. And it matters for different reasons. On the one hand, in its manifestations in terms of counting, measurement, pattern and geometry it permeates the natural and constructed world about us, providing basic language and techniques for handling many aspects of everyday and scientific life. On the other hand, it deals with abstractions, logical arguments, and fundamental ideas of truth and beauty—an intellectual discipline and a source of aesthetic satisfaction. Its role in education reflects this dual nature: it is both practical and theoretical—geared to applications and of intrinsic interest—with the two elements firmly interlinked.

Mathematics has traditionally formed a substantial part of the education of young people in Ireland throughout their schooldays. Its value as a component of general education, for employment, and for further and higher education is recognised by the community at large. The development of mathematical skills impinges on the individual's opportunities for development, with consequent economic implications in a society increasingly reliant on and influenced by advances in science and technology, which have a high dependency on mathematical principles. Accordingly, it is of particular importance that the mathematical education offered to and experienced by students should be appropriate to their abilities, needs and interests, and should fully and appositely reflect the broad nature of the subject and its potential for enhancing the students' development.

A recent UK report (Hoyles et al., 2002) concluded that mathematical literacy[1] can contribute to business success in an increasingly competitive and technologically based world-wide economy and that there is an inter-dependency of mathematical literacy and the use of information technology in the workplace. Of significance in this study is the fact that mathematical skills cannot be considered in isolation, but rather in the context of the work. The use of information (and communications) technology has 'changed the nature of the mathematical skills required, while not reducing the need for mathematics' itself (ibid, p.10).

In undertaking this review of mathematics education, the NCCA seeks to address a range of issues surrounding mathematics at post-primary level in Ireland. This paper presents an overview of these issues, outlines current trends in mathematics education, and provides data on uptake in mathematics at post-primary level. The paper also considers the performance of candidates in the state examinations and in international tests of achievement. Finally, it identifies particular areas of concern that will need to be addressed in any plan for revision of mathematics syllabuses that may arise as a result of this review. These are intended to act as a stimulus for discussion on the nature and role of mathematics education in Irish schools, while keeping in mind developments that are currently under consideration at both junior and senior cycle.

1 Mathematical literacy is defined in the OECD Programme for International Student Assessment (PISA) as "an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgements and to engage in mathematics, in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen" (OECD, 1999).

2 Context of the review

The review is not simply an exercise in syllabus revision—although this may be an outcome of the review—but rather a more fundamental evaluation of the appropriateness of the mathematics that students engage with in school and its relevance to their needs. It must take into consideration broader reviews that are currently taking place (the implementation of the primary school curriculum; junior cycle) and the proposals being developed for senior cycle education.

2.1 Concerns regarding mathematics

Internationally, there is concern about the low level of mathematical skills of students emerging from second-level education and, in particular, of those proceeding to third-level education (Tickly and Wolf, 2000). These uneven and inadequate mathematical skills affect not only the individual's development and career prospects, but also have more general implications for society.

Issues in relation to mathematics education in Ireland have been highlighted in a number of studies in recent years (Smyth et al., 2004; Lyons et al. 2003; Elwood and Carlisle, 2003; Smyth and Hannan, 2002). These include the provision and take-up of Higher level mathematics and the gender differences that exist in this take-up; the performance of students in state mathematics examinations and in international tests; and the teaching and learning practices that prevail in mathematics classrooms in Ireland.

Over recent years, growing concern has been expressed regarding mathematics in the senior cycle of post-primary education, especially in relation to the numbers of candidates achieving low grades in the Leaving Certificate Ordinary level mathematics examination papers. However, there has also been concern at the low level of mathematical knowledge and skills shown by some students proceeding to further and higher education, and their inability to cope with basic concepts and skill requirements in the mathematical aspects of their courses. O'Donoghue (2002), in particular, noted observations by university lecturers regarding the lack of fluency in fundamental arithmetic and algebraic skills, gaps in basic knowledge in important areas such as trigonometry and complex numbers, and an inability to use or apply mathematics except in the simplest or most practised way.

2.2 Recent developments in mathematics curriculum and assessment in Ireland

Mathematics in the primary school

A revised primary school curriculum was introduced in 1999 and is being implemented on a phased basis; mathematics was among the first group of subjects to be implemented. The 2003-2004 academic year was designated a year of consolidation and review of the Primary School Curriculum. It was also the first year of the NCCA's Primary Curriculum Review, which focused on teachers' and children's experiences with the English curriculum, the Visual Arts Curriculum and the Mathematics Curriculum.

In case studies conducted as part of that review, teachers reported a perceived improvement in motivation for mathematics learning among children, particularly where everyday, real-world materials and contexts were used. Children found mathematical games, puzzles and interesting problems a good motivational influence in their mathematics learning. However, data are not yet available to indicate whether increased engagement has resulted in improved performance.

Teachers reported doing practical (hands-on) work as their greatest success with the mathematics curriculum. There was also an awareness among teachers themselves of the need to integrate mathematics with other areas of the curriculum. Among the challenges in implementing the mathematics curriculum, almost half of the teachers identified catering for the range of children's mathematical abilities as the greatest challenge. In their ongoing implementation of the mathematics curriculum, teachers prioritised focusing more on specific curriculum content, increasing their use of practical work and giving more attention to the use of mathematical language.

Mathematics in the junior cycle

When the Junior Certificate was introduced in 1989, the syllabuses in mathematics were not revised, having been introduced (as syllabuses A, B and C) in 1987, but were renamed Higher, Ordinary and Foundation level syllabuses and first examined under their new titles in 1992. A revised syllabus covering all three levels was introduced in 2000 and first examined in 2003. This was accompanied by in-career development for teachers of mathematics through a dedicated support service. A particular focus of this support was the type of teaching methodology that might best facilitate the aims and objectives of the revised syllabus, thereby leading to improved mathematical understanding on the part of students (rather than learning mathematics by rote, which had tended to predominate previously).

A review of the curriculum at junior cycle is currently under way and part of the present focus involves the re-balancing of syllabuses and their presentation in a common format. Consideration is also being given to the role that *assessment for learning* can play in improving teaching and learning across a range of subjects. Mathematics is one of the subjects to be included in the second phase of both of these review elements.

Mathematics in the senior cycle

The current Leaving Certificate mathematics syllabuses at Ordinary and Higher level were introduced in 1992 and first examined in 1994. The Ordinary Alternative syllabus, introduced in 1990 for first examination in 1992, was re-designated as Foundation level in 1995. The proportion of the student cohort taking each of the three syllabus levels in mathematics (approximately 11% at Foundation level, 72% at Ordinary level, and 17% at Higher level) does not match the expected pattern of uptake when these syllabuses were being developed (20-25%, 50-60% and 20-25% respectively).

Mathematics is one of only two subjects at Leaving Certificate (Irish is the other) which is offered at three syllabus levels. Significantly, when the English syllabus was being revised, the course committee discussed the desirability of providing a third (Foundation level) English course. However, following consideration of the issues involved, it was decided to continue with just two syllabus levels. The aims and objectives of the English syllabus are the same for all learners, and the skills being fostered are the same. Unlike mathematics, differentiation in Leaving Certificate English is not achieved by reference to content, nor is there a specific intention to target the Higher level English syllabus at 'specialists'.

Proposals are being developed for a major re-structuring of the senior cycle of post-primary education. These developments are aimed at improving the rate and quality of participation, at sustaining excellence, at creating greater flexibility and choice for learners, and at meeting educational, social and economic needs. The proposals involve the restructuring of the curriculum to include subjects, short courses and transition units, balancing content and skills, a greater variety and frequency of assessment methods, and a focus on independent learning.

Mathematics is seen as a significant subject for all students in the senior cycle, both as a subject in its own right and as a support for the teaching and learning of other subjects and courses where mathematical competence is a pre-requisite. It is one of only two subjects (English was the other) which the vast majority (84%) of all respondents to the NCCA online questionnaire survey considered should be compulsory for all students. Significantly, 88% of employers who responded to the survey were of this view, as were 79% of students.

2.3 Current trends in mathematics education

Internationally, current trends in mathematics education include emphasis on *problem-solving, modelling* and so-called *'realistic mathematics education'*. It is worthwhile exploring each of these a little and considering how mathematics education here in Ireland compares with the developing international scene.

Problem-solving has always been an important mathematical activity, but it has been given special emphasis in some national curricula (or guidelines, or their equivalents) in the last twenty years. In the USA, the 1980s were designated as the decade of problem-solving. At the end of that decade, the highly influential *Standards* document produced by the National Council of Teachers of Mathematics (NCTM) in the USA placed problem-solving as the first of its list of 'standards' (NCTM, 1989). The newer version, *Principles and Standards for School Mathematics* (NCTM, 2000), may be a little more realistic in its assessment of what can be achieved and what works in the classroom, but fundamentally advocates a similar approach.

By solving mathematical problems, students acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that serve them well outside the mathematics classroom. In England & Wales and Northern Ireland, emphasis was placed in the 1990s on *'investigation'* more than on problem-solving. The two concepts are related, but investigation is perhaps more geared towards exploratory work rather than the solution of clearly-defined problems.

In Ireland, Leaving Certificate mathematics courses were revised in the early 1990s. Higher level mathematics was aimed at the more able students, including those who might not proceed to further study of mathematics or related subjects, and it placed particular emphasis on syllabus aims concerned with problem-solving, abstracting, generalising and proving. Ordinary level mathematics (and to a lesser extent the Ordinary Alternative course, which later became Foundation level) on the other hand, was designed to move gradually from the relatively concrete and practical to more abstract and general concepts, with particular emphasis on syllabus aims concerned with the use of mathematics. Thus, it was designed essentially as a service subject, providing knowledge and techniques needed for students' future study of science, business and technical subjects.

The exploratory, open-ended style associated with investigations does not seem to fit Irish teachers' and students' views of mathematics. Possible reasons for this may lie in the culture of mathematics teaching in this country (see Section 6), in the demands that this approach would make on teacher knowledge, skills and attitudes, and in the fact that such work is not currently subject to assessment in the examination. Elwood and Carlisle (2003) suggest that there is a very narrow view of achievement in mathematics promoted by the examinations, ' … one that does not sit comfortably with the aims and objectives outlined in the syllabuses on which the courses of mathematics … are based' (p.111).

Modelling is an approach traditionally associated with applied mathematics or applications of mathematics. It involves analysing a problem, translating it into mathematical form, solving it in that form, and translating back to the original (real-life or other) situation—and checking that the solution is plausible. It is time-consuming, and typically requires a very different approach from the explain-and-drill one associated with emphasis on basic skills and routine procedures.

Realistic mathematics education (RME) stems from the Netherlands. Developing from a reaction against the 'modern mathematics' movement, it emphasises the solution of problems set in contexts which engage students' interest. It thus combines elements of the problem-solving and modelling approaches. It is probably the most 'fashionable' approach among mathematics educators at present, and underpins the OECD Programme for International Student Assessment (PISA)[2] (Shiel et al., 2001).

While there are dangers in following fashion unquestioningly, our lack of opportunity to engage seriously with the issues—in the context of a radical critique of our junior cycle syllabus—has been an unfortunate accident of history. Post-primary mathematics syllabuses in Ireland do not currently make reference to the modelling or RME approaches.

The adoption of an underpinning philosophy along the lines of RME is not a step to be taken lightly, nor could it be expected that such a change would be successful, or have measurable effects, within a short period of time. A change of culture is required, together with a change in practice. Past experience, nationally and internationally, tells us that a longer-term strategy of implementation and support is required. There is also a need to consider the pre-service education of mathematics teachers, whose own experience of mathematics education (particularly at post-primary level) has been very much along the traditional lines identified by Lyons et al (2003) and whose 'comfort zone' may not extend to encompass more modern approaches in the teaching and learning of mathematics.

Increasingly, more students with special educational needs are being included in mainstream education. In common with other teachers, mathematics teachers will need to be able to adapt their teaching methodology so that these students can develop their mathematical knowledge and skill appropriately. Ireland is also seeing an increase in the numbers of students from other countries whose early mathematics education differs significantly from that of Irish students. Teachers will need to be able to make the connections for such students, and this requires some degree of familiarity with alternative approaches and methodologies.

The revised Primary School Curriculum is more in line with the RME philosophy and, in particular, with the problem-solving approaches to mathematics education. In time, it may eventually permeate second level education 'from the bottom up' according as students transferring to post-primary schools have had longer experience of such approaches at primary school and teaching and learning in mathematics at junior cycle adopt the changed approach advocated by the syllabus revisions implemented in 2000.

Although it is not compulsory in the senior cycle, almost all students in Ireland study mathematics to Leaving Certificate. Elsewhere, national requirements or cultural pressures to take mathematics lead to different patterns in uptake. In the context of the Second International Mathematics Study (SIMS) in the early 1980s, education systems were identified in which mathematics in the senior cycle is

- compulsory
- effectively compulsory (i.e. needed for further study/job purposes, so taken by almost all students)
- taken only by those in certain tracks
- genuinely optional [as for A-level GCE] (Travers and Westbury, 1990).

2 In PISA 2000, reading literacy formed the major domain of assessment, with mathematical literacy and scientific literacy as minor domains. Mathematical literacy formed the major domain in PISA 2003 (problem-solving was introduced as an additional minor domain) and scientific literacy will be the major domain of assessment in PISA 2006.

Other factors to be considered in relation to subject uptake include the number of subjects that students take, the amount of time given to each subject, and how students are allocated to classes/levels. It should be borne in mind that decisions on the time given to individual subjects and the allocation of students to classes are taken at school level. Thus, for example, it is perfectly acceptable to give reduced weight to mathematics in order to allocate time to other subjects, if this truly reflects the goals that are set for the education provided in the school. Where this occurs in a large number of schools, it effectively becomes the national norm. Under these circumstances, however, Irish students cannot be expected to reach the same standards in mathematics as do students in countries giving appreciably more time to the subject.

2.4 Mathematics in relation to other subjects

While mathematics is a discipline in its own right, it also plays an important role in a variety of other subjects, such as business, geography and, most notably, the science and technology subjects (mathematics has sometimes been called the language of science).

> 'Serious concern about the mathematical competence of students in schools and in higher education permeates the debate on the declining uptake in the (physical) sciences.' (Report of the Task Force on the Physical Sciences, 2001)

In his report on the inquiry into post-14 mathematics in the United Kingdom, Smith (2004) pointed to the need for teachers to be aware of the links between mathematics and other subjects, as well as the links within mathematics itself. He also drew attention to the need for continuing professional development in respect of mathematics for teachers of other subjects, something seen as important for integrating the teaching and learning of mathematical skills in other subjects and areas of the curriculum.

The concern that third-level institutions have expressed regarding the standard of mathematical knowledge and skills among their first year intake is not solely related to mathematics courses, but extends to other courses where mathematics provides an important basis for progression (O'Donoghue, 2002). Inadequate mathematical skills were also noted as an issue affecting science, social science, and technology courses in British universities, threatening the quality of degrees in a wide range of key disciplines (Tickly and Wolf, 2000). In this regard, the ability to apply mathematics in what, at first glance, might appear to be non-mathematical contexts is a significant consideration. Part of the problem may lie with the perception that students at second level have of individual subjects being self-contained areas of study, unconnected to other subjects or curriculum areas (and unrelated to real life).

Given the relatedness of mathematics to a variety of subjects, it may seem obvious that, in other subjects involving some level of mathematics, teachers should be able to cross-reference their work with what happens in the mathematics class. The findings of the primary curriculum review have indicated that integration is proving difficult for teachers to achieve. If this is the case where teachers have the same class for a range of areas of the curriculum, then it is likely to be more problematic at second level where, given the subject-specific nature of staffing and timetabling, teachers are likely to have even less opportunity (or need) to look beyond the boundaries of their individual subject to consider its relatedness to other subjects or programmes.

3 Provision and uptake of mathematics

3.1 Mathematics in the primary school

In the primary school, all pupils study mathematics, which is concerned with the acquisition, understanding and application of mathematical knowledge and skills. The curriculum emphasises that mathematics is both a creative activity and a process of managing and communicating information. In the Teacher Guidelines for mathematics, mathematical literacy is noted as being of central importance in providing the child with the necessary skills to live a full life as a child, and later as an adult. It is seen as necessary to make sense of data encountered in the media, to be competent in terms of vocational mathematical literacy and to use appropriate technology to support such applications. Mathematics is used in everyday life: in science, in industry, in business and in leisure activities. Society needs people who can think and communicate quantitatively and who can recognise situations where mathematics can be applied to solve problems. [Teacher Guidelines (1999), p. 2]

The areas of content in the primary school mathematics curriculum are presented as strands that form a network of related and interdependent units: *number, algebra, shape and space, measures, and data.* These are further developed as strand units, which range across four groupings of classes from infants up to sixth class. These strands do not form a hierarchy, but rather are seen as interrelated units in which understanding in one area is dependent on, and supportive of, ideas and concepts in other strands; integration opportunities are indicated in some strand units. *Number* is an integral component of all of the strands.

3.2 Mathematics in the junior cycle

In common with other subjects, mathematics education is seen as contributing to the personal development of the students, helping to provide them with the mathematical knowledge, skills and understanding needed for continuing their education, and eventually for life and work. Thus, students should be able to recall basic facts and demonstrate instrumental understanding; they should acquire relational understanding (appropriate to the syllabus being followed), be able to apply their knowledge and skills in analysing and communicating mathematical information, and develop an appreciation of mathematics—including its history—and its role in their lives.

Almost all students study mathematics, which is one of only two subjects in the junior cycle that are provided at three syllabus levels: Foundation, Ordinary and Higher. However, unlike most other subjects where Higher level is intended for the majority of students, the Higher level mathematics syllabus states that it is targeted at students of above average mathematical ability. Thus, the cohort of students who study the Higher level course is much smaller than is the case for many other subjects.

Table1.1 gives the numbers of Junior Certificate mathematics candidates taking the examination at different levels in the period 2002-2004. These figures show that over 41% of the candidates took the Higher level paper in 2004 (a slight increase on previous years), 47% took the Ordinary level paper (a slight decrease), and less than 12% took the Foundation level paper (also a slight decrease). This is in contrast with other subjects (except Irish), where considerably more than half of the examination candidates take the Higher level paper.

Table 1.1 JC Examination candidates taking mathematics at each level

Year	Total number of examination candidates	Maths (FL)	Maths (OL)	Maths (HL)	Total Maths
2002	60,439	7,886	29,588	21,821	59,295
2003	59,633	7,324	27,383	23,734	58,441
2004	57,074	6,584	26,345	23,006	55,935

Detailed results show that 44.5% of the candidates at Higher level achieved an A or a B grade, while 6.4% obtained less than a D grade at this level. At Ordinary level, a similar proportion achieved the top two grades, with 7.3% failing to get at least a D grade. At Foundation level, 56.8% of the candidates achieved an A or B grade, with just 2.1 % obtaining less than a D. Overall, 6.3% of the candidates who sat the mathematics examination in 2004 obtained less than a grade D at any level.

As mentioned previously, consideration needs to be given to how students choose, or are allocated to mathematics classes, especially in their first year of post-primary education. Class allocation and timetabling processes should facilitate as many students as possible having the opportunity to study the higher-level course and, particularly, that they are not locked into a level due to either late development of their mathematical knowledge and skills or initial under-achievement.

3.3 Mathematics at Leaving Certificate

Leaving Certificate mathematics forms part of a broad educational experience for students in the senior cycle as they complete their post-primary education, preparing them for further education, for the world of work and for citizenship. In addition, Leaving Certificate mathematics plays a significant role in terms of entry to courses at third level, something which is not always understood by second-level students.

Three levels of mathematics course—perhaps sufficiently different to be considered as three distinct courses—are currently provided in the established Leaving Certificate: Foundation, Ordinary and Higher. This has been the case since 1992, as indicated above, and is in contrast to most subjects where there are just two levels (Ordinary and Higher), but similar to the provision in mathematics in the Junior Certificate. A course in applied mathematics is also provided (at two levels; uptake of the Ordinary level is very low).

In other countries, the provision of a range of courses for the senior cycle cohort is not unusual (however, the provision of a course in 'Applied Mathematics' is unusual, and may be restricted to countries that were influenced by practice in England). There appears to be a strong link between having different strata of educational provision (e.g. secondary, vocational) and having different kinds of mathematics courses. Also, internationally, the proportion of students who study mathematics in upper second-level education is comparatively lower than is the case in Ireland, where almost all students study mathematics to Leaving Certificate level.

The need to provide a range of syllabus levels, and in particular to provide what might be called 'general' as well as 'specialist' courses, is likely to be greater when a high proportion of the age cohort is retained in school and is required to take—or opts to take—mathematics as a subject in the senior cycle. However, it should be noted that the revised syllabus for Leaving Certificate English is a common one, with differentiation between Higher level and Ordinary level being achieved by specifying different texts as well as having separate examination papers for the two levels.

Table 1.2 gives the numbers of Leaving Certificate mathematics candidates taking the examinations at different levels in the period 2002-2004. These figures show that just under 18% of mathematics candidates took the Higher level paper in 2004 (similar to previous years), over 71% took the Ordinary level paper (a slight decrease), and 11% took the Foundation level paper (a slight increase). By contrast, in English (also taken by the vast majority of Leaving Certificate students, but with only two syllabus levels) over 60% of candidates take the Higher level examination paper. Students seem willing to take the Higher level paper in other subjects, but drop to Ordinary level in mathematics. This can be attributed in part to the perceived difficulty of mathematics, but also to attitudes and beliefs about mathematics (see section 6.3) and the 'elitist' status that Higher level mathematics can sometimes have in schools among students and teachers.

Table 1.2 LC Examination candidates taking mathematics at each level

Year	Total number of examination candidates	Maths (FL)	Maths (OL)	Maths (HL)	Total Maths
2002	55,496	5,296	38,932	9,430	53,658
2003	56,237	5,702	39,101	9,453	54,256
2004	55,254	5,832	37,796	9,429	53,057

The proportion of candidates taking mathematics at Leaving Certificate Higher level is less than half of those achieving an A or B grade on the Junior Certificate Higher level mathematics examination (and less than a quarter of those who achieved a grade C or higher). A compounding factor here is the comparatively smaller number base of Junior Certificate students in Higher level mathematics, as previously mentioned. The relatively poor take-up of Higher level mathematics rightly gives cause for concern, since it has implications for the follow-on study of mathematics to degree level.

As indicated already, mathematics is effectively compulsory in Ireland; it must also be noted that Irish students take a larger number of subjects in the Leaving Certificate than their counterparts in other countries. A consequence of this lack of specialisation is that the total time available has to be shared among many subjects, so the time allocated to any one subject at senior cycle is low in international terms. Evidence from international studies indicates that the proportion of time allocated in Ireland to mathematics in the junior cycle is also low by international comparison, and that the actual amount of time, taking into account the length of the school day and year, is likewise low (Travers and Westbury, 1990; Lapointe et al., 1992).

Anecdotally, comments from teachers during the in-career development programme that supported the implementation of the revised Junior Certificate mathematics syllabus over the period 2000-2004 point to a further erosion of the time allocated to mathematics in some schools.

In the Leaving Certificate Applied, students have an opportunity to consolidate and improve their conceptual understanding, knowledge and skills in mathematics through the practical, analytical, problem-solving approaches of the Mathematical Applications modules, as well as though integration of mathematics in other modules. The four Mathematical Applications modules, which reflect the applied nature of the Leaving Certificate Applied programme are:

- mathematics for living
- enterprise mathematics
- mathematics for leisure and civic affairs
- mathematics for working life.

4 Syllabus style and standard of examination papers

4.1 Syllabus style

The style of the present Leaving Certificate syllabus was set in the 1960s at the time of the 'modern mathematics revolution'. This emphasised abstraction, rigorous argument and use of precise terminology. The 'modern' emphasis has been diluted in subsequent revisions, and a more eclectic philosophy has taken its place. There have been minor 'trouble-shooting' revisions but no genuinely radical critique of the aims of mathematics education in the junior cycle or of the style of content, pedagogy and assessment that is appropriate for the cohort served by the programme. [This is documented in the *Guidelines for Teachers* that accompany the present Junior Certificate mathematics syllabus (Department of Education and Science/National Council for Curriculum and Assessment, 2002).] Perhaps because of the absence of such a root-and-branch revision, more recent trends in mathematics education did not permeate discussions in Ireland.

4.2 Examination papers

Examination papers still reflect the formal language and rigorous specification of questions that typify the 'modern mathematics' era. Most questions are presented as mathematical tasks (for example, 'solve the equation…') without being set in a context. Contextualised questions tend to involve a great deal of reading and/or some imprecision in specifying aspects of the problems. Also, individual contexts may appeal to some students while failing to engage others. The Irish examination papers in mathematics have aspired to fairness with regard to students' ability to read the questions and to answer them without the need for prior knowledge of a non-mathematical nature: hence, to test mathematical rather than other skills. However, the de-contextualised nature of questions has contributed to increased emphasis on recall and on the application of routine procedures.

Mathematics examination papers from some other countries, at least for lower second-level students, appear less technical than do the Higher and Ordinary level papers in Ireland and may not cover such advanced or formal mathematics. However, there is a greater emphasis in some countries on solving problems set in everyday contexts.

The Higher level Leaving Certificate mathematics examination papers up to 1993 probably over-emphasised problem-solving, in that candidates were not given adequate opportunities to display the more routine skills they possessed. By contrast, examination papers at Ordinary level were very routine.

From 1994 onwards, Leaving Certificate questions have displayed a 'gradient of difficulty' with a problem-solving section at the end. This reflects a more balanced emphasis on a fuller range of objectives as listed in the current syllabus. However, by placing the problem-solving material at the end of each question and allocating it approximately 40% of the marks, it does allow teachers or students who are targeting a safe 'C' grade to focus on the lower-order objectives at the expense of the problem-solving ones.

A further difference between mathematics examinations in Ireland and elsewhere is the absence of any form of coursework as part of the final assessment for certification (this is also true for many other subjects). As a consequence, the likelihood of achieving some of the syllabus aims and objectives, which do not lend themselves to being assessed by externally set terminal examinations, is diminished. Experience has shown that, where objectives are not assessed, they tend not to be emphasised in teaching and learning. Furthermore, in the absence of coursework, there is little opportunity or encouragement for students or teachers to engage in a more extended investigation of any one area of mathematics. However, there has been no pressure from the teaching body for coursework assessment of mathematics. Genuinely complementary, rather than supplementary, forms of assessment are probably outside the experience of almost all mathematics teachers.

Additionally, as noted in the Junior Certificate mathematics syllabus and the *Guidelines for Teachers*, while the syllabus aims and general objectives together provide a framework for all three syllabus levels, *level-specific* aims are identified for Foundation, Ordinary and Higher levels. That the general syllabus objectives are not all assessed (or assessable) by the terminal examination is acknowledged by the separate identification of assessment objectives which, although the same for all three syllabus levels, are meant to be interpreted in the context of the level-specific aims. The examination-focused teaching and the rote learning that appear to characterise mathematics classrooms in Ireland (Lyons et al, 2003) could mean that objectives which are not assessed are not likely to be addressed in class. While this is also true of many other subjects, the absence of a second mode of assessment, which could address additional objectives, means that the problem is more acute for mathematics.

It is noted in the guidelines that,

> 'Given the exclusion of some of the objectives from the summative assessment process, it is all the more important to ensure that these objectives are addressed during the students' mathematical education.' (page 91)

Difficulty level

It remains to comment on the general level of difficulty of the examination papers. Before the first examination of the revised Leaving Certificate syllabus in 1994, the NCCA course committee provided specimen questions that duly informed the production of sample papers. However, in subsequent years, anecdotal evidence from meetings where the mathematics examination papers were reviewed indicates that teachers believe there has been escalation in difficulty level of examination papers, by comparison with the sample papers. This perception may have contributed to the lower than expected increase in the proportion of candidates taking the Higher level course (an increase from 10% to almost 18% over the period since 1994, although still not achieving the 20-25% target aspired to).

While it must be borne in mind that a significant function of the examinations is to differentiate between candidates' levels of achievement, this could be managed through more rigid application of marking schemes rather than through more difficult questions.

5 Student achievement in mathematics

5.1 Leaving Certificate examination results

The level of low grades obtained in Leaving Certificate mathematics has given cause for concern. In particular, media attention has focused on the mathematics performance of Ordinary level candidates, with an average failure rate[3] of slightly more than 13% in recent years (see table of results below). When combined with the number of candidates who take the Foundation level mathematics examination (which, as already mentioned, is not accepted for entry to a range of third-level courses), this represents a sizeable proportion of Leaving Certificate candidates who 'fail' to get places at third level institutions. (Of course, such comment does not take into account the possibility that some of these students may not have had aspirations to progress to further or higher education in the first place.)

The tables below show the performance of candidates in the Leaving Certificate mathematics examinations since 2000. These are the overall grades obtained by candidates on the two examination papers at each level. Apart from 2001 when a greater percentage of candidates at Higher level achieved the top grades, the distribution of grades is fairly consistent over the five years shown.

LC HL Mathematics, percentage of candidates achieving the various grades

	A	B	C	D	E	F	NG
2000	14.0	28.3	31.9	20.7	3.8	0.9	0.2
2001	21.2	32.6	26.9	15.4	3.1	0.8	0.1
2002	13.2	28.3	33.5	20.7	3.4	0.9	0.1
2003	13.2	30.1	32.7	19.6	3.7	0.6	0.1
2004	16.1	30.0	31.2	18.4	3.3	0.9	0.1

LC OL Mathematics, percentage of candidates achieving the various grades

	A	B	C	D	E	F	NG
2000	14.4	26.0	25.5	21.4	8.2	4.0	0.5
2001	14.1	24.9	23.1	21.3	10.2	5.6	0.8
2002	13.6	24.4	24.6	23.0	9.4	4.4	0.6
2003	10.9	26.6	26.6	24.2	8.3	3.1	0.3
2004	15.7	28.6	24.9	19.3	7.7	3.3	0.5

3 'Failure' is used for convenience; it is more correct to speak of grades lower than a 'D'.

LC FL Mathematics, percentage of candidates achieving the various grades

	A	B	C	D	E	F	NG
2000	8.0	33.1	33.4	18.8	4.8	1.6	0.3
2001	7.8	31.8	33.3	19.8	5.2	1.8	0.2
2002	9.0	31.7	32.7	19.4	4.9	1.9	0.3
2003	12.2	34.9	30.5	16.6	4.3	1.2	0.2
2004	10.0	33.4	32.0	18.2	4.9	1.4	0.2

Unacceptable though the 'failure' rate is, it has been worse in the past. The figures in the late 1980s—prior to the introduction of a third Leaving Certificate course in mathematics—show that more than one-fifth of the Ordinary level mathematics examination candidates 'failed' . Moreover, at that time, the Higher level examination was taken by only 13% of the examination cohort (some 7000 candidates; the figure subsequently fell below 6000, or around 10% of the examination cohort).

The acute problems were somewhat alleviated by the introduction of the Ordinary Alternative syllabus in 1990 (for first examination in 1992) and the revision of the Higher and Ordinary courses in 1992 (for first examination in 1994):

- Percentages of candidates taking the Higher level examination rose considerably over the following years, but levelled out before reaching the aspirational range of 20-25%; at present around 17.5% of the cohort take the Higher level examination.
- Numbers taking the Ordinary Alternative/ Foundation level examination remained much lower than the Course Committee had expected—under 10%, rather than the 20-25% for whom the syllabus was designed.
- The Leaving Certificate Applied programme accounts for approximately 5% of the student cohort at this level. (The Ordinary Alternative course was not originally targeted at students who now take the Leaving Certificate Applied; they were to be served by a 'Senior Certificate' course.)

5.2 Junior Certificate examination results

In the Junior Certificate, mathematics is also assessed at three syllabus levels. As already indicated, a revised syllabus was introduced in 2000 and first examined in 2003. Thus there are only two years of examinations results relevant to the current syllabus. Also, a style was adopted for the examination papers that indicated to candidates where their working of solutions was required to be shown (at the risk of losing marks where this was not complied with).

The table opposite shows the number and performance of candidates taking the three syllabus levels in Junior Certificate mathematics examinations for 2003 and 2004. In the case of Higher level and Ordinary level, these are the overall grades obtained by candidates on the two examination papers (there is only one examination paper at Foundation level).

JC Mathematics, percentage of candidates achieving the various grades at each level

		A	B	C	D	E	F	NG
HL	2003	17.2	33.6	28.6	17.0	3.1	0.5	0.0
	2004	16.1	28.4	28.9	20.3	5.2	1.1	0.1
OL	2003	9.2	31.0	31.3	20.8	5.8	1.8	0.1
	2004	10.1	34.3	30.8	17.7	5.2	1.9	0.2
FL	2003	15.4	37.8	29.5	13.6	3.2	0.4	0.0
	2004	16.4	40.4	29.1	12.0	1.8	0.3	0.0

5.3 Evidence from cross-national studies

International studies of achievement have to be interpreted with great care because, all too often, they do not compare like with like. Nonetheless, when due account is taken of the context, they can provide helpful pointers to strengths and weaknesses in student achievement.

Ireland has not participated in studies of mathematics achievement at Leaving Certificate level, but those for younger students provide interesting information.

- In the first (1988) and second (1991) International Assessments of Educational Progress (IAEP I and II), considering 13-year-old students, Irish performance was decidedly moderate. In IAEP II, in particular, the test content was well matched to the Irish curriculum; average Irish performance was similar to that of Scotland, but the Irish results showed a worrying 'tail' (Lapointe et al., 1989; Lapointe et al., 1992).
- The mathematics tests for the Third International Mathematics and Science Study (TIMSS) (1994) were also well matched to the Irish syllabus content; on this occasion, performance of Irish second-year students was better than that of the comparable cohort in a number of countries with similar cultural and developmental level (Beaton et al., 1996).
- By contrast, students in Ireland achieved a score in mathematical literacy not significantly different from the OECD average on both the first (2000) and second (2003) cycles of the Programme for International Student Assessment (PISA). Mathematics was the major domain of assessment in 2003, and the average performance of Irish students was below that of several countries that might be deemed 'comparable'. While the mathematical concepts underlying the majority of PISA items in 2003 would be generally familiar to Irish students (although somewhat less familiar for Foundation level students), the situating of mathematics problems in a context (e.g. embedded in a real-life setting) was recognised as unfamiliar for the majority of items at all three syllabus levels (Shiel et al., 2001; Cosgrove et al., 2004)

Altogether, therefore, the message from the studies is somewhat mixed; but they provide evidence that the performance of some Irish students at junior cycle gives cause for concern. This suggests that the seeds of a least part of the problem at senior cycle may be sown during the junior cycle, or even earlier.

5.4 Evidence from Chief Examiners' reports

Chief Examiners' Reports for Leaving Certificate mathematics were produced in 2000 and again (in response to the poor Ordinary level results) the following year. These very valuable documents highlight specific areas of strength and weakness in students' answering and relate them to the objectives of the syllabus.

- For Ordinary level students, weaknesses include poor execution of basic skills in some areas and an apparent lack of *relational understanding* (understanding of 'why' rather than just 'how'—hence, the basis for applying knowledge in even slightly unfamiliar circumstances). The implications of this are considered below.
- The strengths of Ordinary level candidates are seen to lie in the area of competent execution of routine procedures in familiar contexts.
- The report points out that it 'is clear, both from the continuing relatively high failure rate and from the type of work presented by the candidates who are failing, that there are significant numbers of candidates who are wholly unsuited to taking this examination'.

The Examiners' Report for the Junior Certificate in 1996 also highlights basic weaknesses especially among students taking the Ordinary and Foundation level examination papers. (It helped to counteract any undue optimism from the comparatively 'good' results from TIMSS.) Thus, again, there is evidence that the problems observed at Leaving Certificate level start further down.

5.5 Other evidence

One measure (not, of course, the only measure) of the effectiveness of the Leaving Certificate mathematics course is the extent to which it prepares students for study at third level. Of particular interest here is the role of the Ordinary level course in equipping students for further and higher education courses in science, technology, other technical subjects, and other subjects requiring a good grasp of mathematics.

Evidence is accumulating that the incoming level of mathematical expertise—hence, the expertise of students who achieved a D grade or higher in the Leaving Certificate Ordinary level examination—is insufficient, and does not match expectations created by the objectives and content of the syllabus and by the standard of the examination papers.

The report by Morgan (2001) points to difficulties leading to failure and dropout from Institutes of Technology (ITs); this quantifies anecdotal evidence from lecturers in ITs that poor mathematical competency contributes to non-completion of courses. Some third-level institutions have identified problematic areas in students' mathematical knowledge and skills and have put in place successful interventions to address these (O'Donoghue, 2002).

6 Teaching and learning

6.1 Focus of teaching

The results of TIMSS provided insights into the approaches that Irish teachers feel are best for ensuring success in school mathematics.

- In international terms, memorising and routine performance were given exceptionally high emphasis in Ireland, while logic, creativity and applications were given very low emphasis (Ireland came last as regards applications) (Beaton et al., 1996).
- This could be attributed to the style of the examination papers. However, any move to make the papers less routine and more 'applied' are met with considerable opposition, for example through feedback via the Irish Mathematics Teachers' Association (IMTA). This may indicate that teachers are philosophically comfortable with the current style: that they see mathematics (or mathematics for school students) as being about fairly routine performance in no particular contexts. Alternatively, it may be that this style fits the classroom methodologies that they know and with which they feel secure.

IMTA meetings devoted to 'post-mortems' on the examination papers—these are regular events and are often among the better attended meetings, especially with regard to the Higher level Leaving Certificate papers—are at times obsessively devoted to 'what will get marks' rather than 'what may improve students' learning' or 'what might be good mathematics education'. This is understandable in the immediate lead-up to examinations, but increasingly seems to start a long way before the examinations.

The Chief Examiners' Reports, as mentioned earlier, emphasised students' lack of *relational understanding*. Research suggests that relational understanding, appropriately complemented by *instrumental understanding* (knowing 'what to do'), is important for successful learning that can be retained and applied (National Council of Teachers of Mathematics, 2000). The examination-focused teaching described above is not conducive to the development of relational understanding, which tends to require an emphasis variously described in the literature as 'sense-making' or 'meaning making' (Hiebert et al., 1997). It is facilitated by somewhat 'progressive' teaching, allowing for constructivist approaches in which concepts are explored, individuals' imperfect concepts and procedures are reflected on and 'de-bugged', and expository teaching is appropriately complemented by activities such as discussion and journal-writing.

Evidence from the international studies suggests that Irish classrooms are largely 'traditional,' involving teacher exposition and (probably, followed by) individual pupil work (Lapointe et al., 1989; Lapointe et al., 1992; Beaton et al., 1996). Of course this too can be used to facilitate relational understanding, but is not such a natural format for its development. The study by Lyons et al. (2003), involving collection of videotape evidence in a small number of classrooms, tends to support the idea that mathematics teaching tends to be unduly instrumental (see 6.2 below).

In considering the 'short-cuts' that some teachers are taking, the shortened and decreasing time allocated to mathematics should be borne in mind. Not all teachers *want* to teach in that way… and of course some, despite the shortage of time, do not. A further problem may be teachers' own knowledge base. This is considered in more detail below.

6.2 Focus of learning

The findings of research (Lyons et al., 2003) into the teaching and learning of mathematics in second-level schools in Ireland suggest a high level of uniformity in terms of how mathematics lessons are organised and presented. There is a concentration of class time on the two interrelated activities of teacher demonstration of mathematical procedures and skills, and student practice of these. The practice exercises were typically set by the teacher (in the majority of cases from the textbook) and undertaken by the students during class time or as homework. A procedural rather than a conceptual or problem-solving approach to mathematics prevails in the predominantly 'traditional' mathematics classroom. However, observations of English classes indicate that the use of traditional approaches to teaching is not confined to mathematics.

Research (carried out in England, but there is evidence of a similar phenomenon here) with students in teacher education courses indicates that some have gone through their undergraduate career, even perhaps in mathematics or related degree courses, without gaining a truly relational understanding of the subject (Suggate et al., 1999).

Teachers, of course, cannot do all the work themselves. Students' approaches may be unhelpful in this respect. Some students may be too inclined to sit back and expect the teachers to do the work so that they (the students) learn painlessly. Other students may be prepared to work very hard, but may put their hard work into inappropriate learning strategies: ones that do not promote meaningful learning. (Examples would include learning the proof of a geometrical theorem by 'learning it off by heart' without reference to a diagram, and therefore being entirely unable to carry out the proof if the diagram is labelled differently.) In fact, students may have a *learnt helplessness* that suggests to them that they cannot tackle even slightly unfamiliar work. Students who have suffered from a 'tell and drill' or 'busywork' approach (bereft of meaning) may already have learnt this helplessness before they enter second level school. However, the revised mathematics curriculum introduced in 1999 places increased emphasis on a practical, hands-on approach to the learning of mathematics, which is reported as promoting greater engagement in, and enjoyment of, mathematics learning on the part of the children (see 2.2 above).

Of relevance here is the culture of the classroom and especially the *didactical contract* implicitly made between students and their teacher (Nickson, 2000). This may be of the form: 'I am here to get my exams., you are here to teach me to do it.' Evidence of such a pragmatic approach is found in students' comments on being faced with a more meaning-related or discursive approach at third level.

6.3 Attitudes to and beliefs about mathematics

Implicit in much of what has been said above is the issue of *attitudes to mathematics* and the related issues of *beliefs, perceptions or conceptions* about mathematics.

Consideration can be given first to teachers. Research suggests that there is a connection between teachers' views of mathematics and their approach to teaching it (Thompson, 1992). A teacher who believes that mathematics is a bag of useful but unconnected tricks is likely to emphasise different things than will a teacher who believes that mathematics is a body of knowledge as near to absolute truth as we can get, a web of beautiful relationships, or an activity involving the formulation and solution of problems. Standard research on the characteristics of a good teacher indicates that one such characteristic is enthusiasm for the subject being taught.

For students, several issues arise. Research indicates that attitudes and achievement are correlated, albeit not particularly strongly. Notably, not all successful students like the subject (McLeod, 1992). Moreover, the within-country associations between attitudes and achievement do not necessarily hold across countries. In international studies, some of the highest-scoring countries had the most negative attitudes to mathematics and *vice versa* (Robitaille and Garden, 1989; Lapointe et al., 1989; Lapointe et al., 1992). However, these may be a reflection of cultural tendencies (with regard to it being 'OK to say you like schoolwork') rather than being related to approaches specifically to mathematics.

In PISA 2003, students were asked about four aspects of their approaches to learning in mathematics: motivation, self-related beliefs, anxiety, and learning strategies. The study found that interest in and enjoyment of mathematics is closely associated with performance in all OECD countries. Students who believe in their own abilities and efficacy, and who are not anxious about mathematics, are particularly likely to do well in the subject (OECD, 2004).

More generally, some findings noted in the context of the review of research for the *Cockcroft Report* (Committee of Inquiry into the Teaching of Mathematics in Schools, 1982) suggest that students like the simple, routine aspects of mathematics that are of limited educational value and have limited application to industrial prosperity (they can be mechanised), and that they dislike the aspects which highlight problem-solving—a rather depressing situation for mathematics educators.

Students see mathematics as effortful—'hard work' and 'natural ability' are required to do well in the subject. The issue of 'mathophobia', or fear of mathematics, is important (and this is not confined to students; many adults are uncomfortable when faced with numerical data and even relatively straight-forward number operations). Students for whom mathematics does not make sense might be expected to experience failure and to be scared of the subject. However, qualitative research dealing with rather gentle problem-solving approaches with weaker students suggest ways forward which might combine *appropriate courses* (from an educational and social point of view) with *appropriate pedagogy*.

In their study of Irish mathematics classrooms, Smyth et al (2004) found that students typically saw mathematics at second level as the same or 'harder' than in the primary school, and more than was the case for either Irish or English. This was particularly so in respect of students in the higher stream classes. Almost all of the students in the case study schools considered mathematics useful. Of those who had not received extra help or learning support in school, approximately one-third indicated that they would have liked to receive help with mathematics. Mathematics was the second least popular subject (after Irish) identified by the students. However, over 70% of the students considered that the time spent doing mathematics was about right, whereas about half of the students thought this was not the case for Irish and other languages (too much time), or the 'practical' subjects such as P.E., information technology, art, or materials technology wood (too little time).

An interesting aspect of the study undertaken by Lyons et al. (2003) was the decision to include interviews with parents of students observed in the mathematics classes, and their classification into three types: 'insiders', 'outsiders', and 'intermediaries' in terms of their knowledge of the education system, education level, and levels of intervention with their child's school. This approach offered a unique insight into the connections between school and home, with a particular focus on mathematics.

The insider parents had extensive experience of the education system, with most having obtained a third-level qualification. These parents held positive views of mathematics and monitored their children's progress in the subject. While believing that good teaching and hard work was needed for success in mathematics, 'most (of these parents) also thought that success at mathematics was dependant on having 'natural ability' in the first instance' (Lyons et al. 2003; p.342).

Outsider parents, while having reasonably good levels of education, had much less knowledge than insiders about how the education system works, or what was required to succeed in formal education. They had more negative attitudes to schooling and mathematics, based on their own experiences. However, like insider parents, they also had a strong belief that innate ability was crucial for successful learning in mathematics.

Parents were classified as intermediaries on the basis of being somewhere between the insiders and the outsiders. They had some knowledge of what they should do to ensure their children's educational success, but were concerned about the adequacy of this knowledge or their capacity to act in supporting their children's education. While they had concerns about their children's performance in, and attitudes to, mathematics, they regarded these as in some way linked to their own negative experiences in the subject.

6.4 Teacher competencies

Mention has already been made of problems with teachers' knowledge base. This is likely to be true of some primary teachers and some second-level teachers for whom mathematics is their second or third teaching subject—teachers who may have (at best) limited mathematics in their degrees.

A considerable amount of research points to limitations in student-teachers' content knowledge in mathematics (Brown and Borko, 1992). In particular, their knowledge of concepts may be poor (hence, they may have weak relational understanding—much the same notion).

The situation may differ in different countries. A small-scale piece of work emphasises the difference in 'profound understanding of fundamental mathematics' (PUFM) between a group of Chinese and a group of American primary level teachers (Ma, 1999). The Chinese displayed both relational and instrumental understanding; they could do elementary computations in different ways, giving reasons, and get them right. The Americans were less likely to understand the method they used and some made errors. It would be wrong to read too much into such a small study, but it has highlighted important issues relating to the teacher's own knowledge base in mathematics and their approach to teaching the subject.

It is likely (though perhaps not as well established by research) that some teachers and prospective teachers of mathematics may not have adequate pedagogical content knowledge, i.e. knowledge of (*inter alia*) how to develop relational understanding in their students. Much of the discussion of post-primary mathematics education was dominated by consideration of syllabus content and assessment issues (English and Oldham, 2004). Thus, even if their general knowledge and skills in teaching are good, teachers may not be able to use it suitably in teaching mathematics.

The recent in-career development programme for mathematics teachers at junior cycle addressed pedagogical content knowledge and other issues that may enhance teachers' and students' enjoyment of mathematics—and may even lead to the establishment of a different didactical contract (see 6.2 above). There is a need for teachers to recognise the emotional dimension to learning that, in light of the comments above regarding student attitudes, has particular relevance to mathematics. The sense of failure (and, possibly, of frustration) that some students feel at an early stage in relation to mathematics must be acknowledged and addressed if these students are to engage successfully with later learning in this subject.

There is no formal provision that facilitates teachers in routinely updating their skills, other than when new or revised courses are being implemented. Thus, for example, where developments emerge through particular computer applications or in our understanding of the different ways in which students learn, there are no established structures whereby the general body of teachers can become familiar with these in the context of mathematics teaching. This absence of a culture or provision of ongoing professional development impacts on all teachers; that in-career support is provided only when there is syllabus change communicates a message of change as event rather than process and suggests a role for the teacher as the recipient of change rather than its agent. The work of the subject associations—the Irish Mathematics Teachers Association in this case—is extremely important in challenging this prevailing culture.

6.5 The culture of the classroom

Typical classrooms have not facilitated the 'concrete' approach to mathematics education recommended in the in-career development programme for the revised Junior Certificate mathematics syllabus. They are often set up in such a way as to reinforce the 'expository plus seatwork' style referred to earlier.

The fact that many schools do not have designated 'mathematics classrooms' not only adds to difficulties in providing concrete materials; it means, for example, that classrooms are not decorated with posters that create a lively, interesting environment for the learning of mathematics.

The main classroom 'aid' is the textbook. Again, Irish textbooks are somewhat functional by comparison to those in some other countries (the small population base militating against large, glossy texts with many discussion points and suggested activities). As evidenced by inspection visits, teaching is highly dependent on the class textbook (which tends to reinforce the 'drill and practice' style) and the examinations, and there is frequently a very close relationship between these two. Lyons et al (2003) found that students were generally not given insights into the applications of mathematics in everyday life; learning was a matter of memorising mathematical procedures and facts:

> 'Mathematics was presented to students generally as a subject a) that had a fixed body of knowledge; b) that was abstract in character; c) that required demonstration of procedures rather than explanation; and d) that comprised discrete elements.' (Lyons et al., 2003; p. 143).

Elwood and Carlisle (2003) suggest that the better performance of girls in both the Junior Certificate and Leaving Certificate mathematics examinations (particularly at Ordinary level) may well be explained by them being better prepared and organised, more familiar with the conventions and requirements of the topics covered, and better able to recall the learnt rules and formulae as required by the questions asked. The 'traditional' mathematics classroom, as exemplified above by Lyons et al., facilitates such learning.

The image of mathematics as linked to 'real life' may not have been enhanced by the late and slight adoption of information and communications technology (ICT) in mathematics teaching. The use of calculators has been introduced for fifth and sixth classes in the primary school, and their use in the junior cycle has increased following the introduction of the revised syllabus in 2000 (recent research suggests that only limited use was made of them prior to their being allowed in the examinations). Computers are not regularly used in mathematics classrooms, although the Schools IT2000 initiative has resulted in more teachers making use of ICT in a range of subjects.

7 System and cultural influences

7.1 Backwash effect from institutions beyond school

Third level colleges have played a considerable part in the uptake or non-uptake of the different Leaving Certificate syllabuses. This has been the case particularly for Ordinary Alternative and Foundation level.

Initially, the universities accepted Ordinary Alternative as a course for matriculation purposes for entry to courses that would not require mathematics (if 'mathematics' was a requirement at all for such courses; for some, it is not). Naturally, they did not accept Ordinary Alternative for entry to subjects requiring a substantial level of mathematics.

However, the (then) Regional Technical Colleges, while naturally requiring at least the Ordinary level for entry to technological and scientific courses, also required this level for entry to a number of courses for which its content (including co-ordinate geometry and calculus) seemed irrelevant—in fact, for which a *good* knowledge of the Ordinary Alternative material would have been highly preferable to a rote-learnt and very imperfect knowledge of the Ordinary level material.

High passing grades represent worthwhile knowledge and skill. Acceptance of such grades might be conducive to more meaningful learning than is the case at Ordinary level at present for weaker students.

Outside the third level sector, other training bodies have also been unwilling to accept the 'third course' (with the exception of the Gardai where a grade B at Foundation level is accepted as an alternative to a grade D at Ordinary level). Again, it would probably be appropriate for such bodies to accept a high passing grade on that course.

7.2 CAO points

It is worth noting that the growth in numbers taking the Higher level examination has occurred despite the discontinuation of 'double points' for Higher level mathematics. The re-scaling of points that equated a C3 on the Higher level examination with an A1 on the Ordinary level examination has probably had a significant, but negative influence on the uptake at Higher level. While a very high mark at Ordinary level is not, perhaps, as easy to obtain as it was before the syllabus was revised, the current standard of the Higher level examination papers has meant that 'good, but not very good' candidates may not feel sufficiently confident of getting the C grade, and so take the 'easier' option of Ordinary level. These candidates feel that they can obtain the top Ordinary level grade with much less work than is required for the points-equivalent grade at Higher level. This reinforces the perception that, for mathematics, the points mapping between the grades is mismatched.

Worse effects emerged following the decision to award no points at all to the Ordinary Alternative, and subsequently to the Foundation level examination. The result was extremely damaging to the perceived status of the 'third course' and hence to the self-esteem of students taking it. The non-acceptance of Foundation level grades discourages students from taking this course, with a consequent increase in the number of low grades at Ordinary level.

7.3 Education as a passport to a career

Recalling research done in the early 1970s, and corroborated in TIMSS, it is perhaps reasonable to conjecture that students—even more now than then—perceive education in terms of access to third level or to careers rather than 'a love of the learning' (Beaton et al., 1996). In fact the very high level of study (by at least some students) that was noted—in newspaper articles on 'pressures of the points race' and so forth—in the comparatively hungry early 1990s was produced by extrinsic motivation, not by love of the subject.

A very pragmatic approach, in which students target the points they need *now* rather than the knowledge and skills they may need *later*, seems to be prevalent. This could mean that some low passing grades were fairly intentionally targeted (as being sufficient for, say, matriculation purposes)—and perhaps even some failing grades were student misjudgments. Second level students appear not to recognise that successful engagement with many third level courses is dependent upon mathematics knowledge and skills and that, without these competencies, progress may be severely hampered—irrespective of the grades achieved or the number of points obtained in the Leaving Certificate.

8 Equality issues

Issues of equality in mathematics education have tended to focus almost exclusively on perceived gender differences in relation to provision, uptake and achievement. This focus has persisted, despite research evidence that socio-economic status and educational disadvantage also represent significant factors in explaining differences in uptake and achievement between boys and girls across a range of subjects. In PISA 2003, for example, the difference in mean scores between boys and girls was about one-sixth of a standard deviation, whereas the difference in mean scores of students in Ireland with low and high economic, social and cultural status is around nine-tenths of a standard deviation (Cosgrove et al., 2004).

In relation to gender, girls' uptake of mathematics courses has improved in recent years, and so has achievement. Smith and Hannan (2002), while noting that the pattern of take-up for mathematics at senior cycle in Ireland contrasts with that in many other countries where it is an optional subject, found significant differences in the take-up of Higher-level mathematics. A greater proportion of girls than boys take Higher level mathematics in the Junior Certificate, but this is reversed for Leaving Certificate. Moreover, in schools which focus on preparation for higher education, a greater proportion of Leaving Certificate students study the Higher level mathematics course.

The age at which gaps in achievement appear to exist has been rising, but there are differences in the achievement of higher grades between boys and girls at the different course levels (Elwood and Carlisle, 2003). While Lyons et al (2003) found that girls tend to achieve overall better results than boys across all levels in Junior Certificate mathematics, a greater proportion of boys than girls obtain A grades at Higher level, with boys in single-sex and comprehensive schools obtaining the highest proportion of A grades. This disparity in top grades has decreased in recent years and more recent studies 'have suggested that gender may not be as important a variable in explaining performance differentials in schools as it once was, particularly in the field of mathematics' (Lyons et al. 2003, p.12). However, differences have persisted with regard to how total scores on the examinations are obtained. Stereotypically, girls were better at routine work and boys at problem-solving.

Another possible area of differentiation is the use or non-use of contexts. The received view in the past has appeared to be that boys could deal with 'real, hard' mathematics, whereas girls were better when work was embedded in 'soft' contexts … or that girls were able to deal with mathematics embedded in contexts whereas boys were not so well able.

Classroom style may also have a role to play. Again, conventional wisdom has been that the individual, competitive approach—easily associated with the 'typical' classrooms described earlier—would suit boys better than girls; girls might be better suited by co-operative group work, which does not appear as a regular feature in studies of Irish classrooms.

Our examinations have not greatly emphasised good communication (stereotypically a female skill); on the other hand, little use is made of multiple choice questions, which have been seen as favouring boys. The projected increased emphasis on good communication in the Junior Certificate papers may in fact favour girls. Research by Elwood and Carlisle on gender and achievement in mathematics examinations has supported the opinion that

> *'a narrow view of achievement in mathematics is promoted by the Junior Certificate and Leaving Certificate examinations, and it is one that does not sit comfortably with the aims and objectives outlined in the syllabuses on which the courses of mathematics in schools in Ireland are based.'* (Elwood and Carlisle, 2003).

9 Conclusion

This paper sets out the background and context for a review of mathematics in the post-primary curriculum. It identifies a range of issues that surround mathematics education in Ireland and, in conducting the review, a number of distinct areas that need to be considered:

- the purposes of mathematics education, including societal expectations
- the mathematics curriculum/syllabuses and changes that may be needed
- the assessment and certification of mathematics
- the requirements of third level institutions and how these may (or should) be accommodated
- the perceived problems with regard to mathematics education in schools
- the role of information and communications technology in mathematics education
- the teaching and learning 'culture' of mathematics
- the inservice education and training of mathematics teachers.

In addition, there is a need to consider how the pre-service education of teachers can take into account curriculum (and assessment) changes that may have taken place since their own time in school, and with which they may not be familiar. In the case of those whose main teaching subjects do not include mathematics, but in which mathematics has a significant role, there is a need to develop strategies by which a coherent approach can be taken to the teaching and learning of mathematical concepts and processes.

Historical accidents which have militated against a root-and-branch consideration of junior cycle mathematics in the past have also affected revisions at senior cycle because the latter built on foundations that had not been critiqued for some time. A major 'root-and-branch' review of mathematics education in Ireland has not taken place since the 1960s. Such a review would afford the opportunity of considering the purposes of mathematics education and could extend also to the syllabus for Leaving Certificate applied mathematics, which has not undergone revision for a considerable time. The various models that were examined at the time of the 1992 revision of Leaving Certificate mathematics, but which were rejected for various reasons, should be re-visited, given the changes in the intervening period.

The appropriateness of the examination papers across the three levels needs to be considered, particularly since the uptake aspired to at the three Leaving Certificate levels has not been realised. A comparison needs to be made with the intended levels of difficulty as exemplified in the initial sample papers developed when the syllabuses were introduced. In particular, this would involve ensuring that Higher level Leaving Certificate examination papers are pitched at a level that provides for the needs of a greater number of students.

The non-acceptance of A and B grades at Foundation level by institutions of higher education and by the CAO (for points) has been a significant factor in the uptake at Foundation and Ordinary levels. Discussion around the acceptability of grades/levels in the Leaving Certificate, and the standards of achievement associated with them, should encompass the issue of whether, when a root-and-branch review of mathematics is complete, two levels may meet the needs of the student cohort and the education system as a whole.

The current problems at Ordinary level in the Leaving Certificate have deep-seated roots in cultural expectations about schooling and beliefs about mathematics. They start much further down the school than the Leaving Certificate and are about much more than the content of the courses and the associated pedagogy and assessment mechanisms. However, some of the issues are being addressed by the introduction of the revised Primary School Curriculum and the current in-career development being undertaken at junior cycle level. It is important that these moves are reinforced rather than disrupted by any 'quick fix'. (It may be worth noting here that the reforms in the Netherlands took some 25 years, starting in infant classes, and involved a concerted approach by teacher education institutions.) In the coming years, students coming into post-primary school will have experienced the full impact of the revised primary school mathematics curriculum. It will be important to have established an appropriate and cohesive programme of mathematics education at second level that builds on their previous learning so that they can maximise their potential in mathematics and related fields of study.

There is a need to consider both pre-service and inservice education and training for teachers of mathematics, and the extent to which real change in teaching and learning can take place. If a genuine re-appraisal of mathematics education is to lead to significant change, attention must be paid to the need for teachers to move away from the traditional approach, which may have been their own experience as students and/or which may have served them well as teachers up to now, and to embrace a new philosophy and associated methodology that will best serve future generations of students. Furthermore, teachers of other subjects in which concepts and processes of mathematics arise will similarly need to embrace change in practice.

The 'problem' with mathematics in our schools is not solely related to the issue of improving numeracy skills, although these undoubtedly need to be addressed, but goes far beyond this. The learning of mathematics also transforms our ability to conceptualise and structure relationships, to model our world and thus be able to both control and change it. Young people need to develop the ability to build connections across knowledge, to identify and explore patterns, to estimate and predict, to interpret and analyse numerical and statistical data, to communicate increasingly complex information, and to apply all of this in their daily lives and work.

As pointed out by Hoyles and Noss (2000), the argument is not that the world can only be understood through mathematics; it is that mathematics should be an essential tool for understanding it. This applies in particular to the digital technologies that rely for their development on the types of skills that mathematics education provides. Thus, mathematics education is a two-way street; it is inter-related with technology and can both support and be supported by modern developments in technology. An important feature of the review of post-primary mathematics must be the role that information and communications technology can play in facilitating greater engagement with mathematical concepts, in developing mathematical knowledge and skills and thereby, almost in cyclic fashion, contribute to future developments in this self-same technology.

There is a need to engage in discussion about the culture of mathematics in schools and in society, and to promote a 'can-do' approach to mathematics. In the words of Hoyles and Noss:

> 'We do need to find ways to connect mathematics with the broader culture. We need to find ways to break down the barriers between mathematics and art, music, humanities and the social sciences, as well as, of course, science and technology. We need to find entry points into the preoccupation and aspirations of children in ways that respect the integrity of their interests rather than patronizing and inevitably disappointing them. Most importantly, we need to introduce ways to make these connections by

ensuring that the solutions of problems given to our students need the use of mathematics, that children do have a chance to make choices of strategy for themselves and learn to reflect upon and debug them. We would like to encourage an appreciation that solving problems in and with mathematics is not a matter of routine or factual recall – although these might play a part.' (2000, p.155)

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