An Evaluation of the Impact of Project Maths on the Performance of Students in Junior Cycle Mathematics

Gerry Shiel and Cathy Kelleher

August 2017

Educational Research Centre

On behalf of the National Council for Curriculum and Assessment

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http://www.erc.ie

Cataloguing-in-publication data:

Shiel, Gerry.

An evaluation of the impact of Project Maths on the performance of students in Junior Cycle mathematics / Gerry Shiel, Cathy Kelleher

Dublin: Educational Research Centre

xxii, 190p., 30cm

ISBN: 978 0 900 440 55 7 (paperback)

978 0 900 440 56 4 (e-Pub)

1. Project Maths
2. Mathematical ability – Testing
3. Mathematics – Study and teaching (Secondary) – Ireland
4. Educational tests and measurement– Ireland
5. Academic achievement – Ireland

2017

I Title. II Kelleher, Cathy

510.71

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Preface

This evaluation of the impact of Project Maths on student performance in Junior Cycle mathematics occurs at an important time in the development of mathematics education in Ireland. On the one hand, new specifications for Junior Certificate mathematics will be developed over the next year or so, providing an opportunity to strengthen aspects of Project Maths and provide better linkages across strands and topics. On the other, there is a growing awareness of the importance of mathematics in driving interest in careers in science, technology, engineering and mathematics (e.g., STEM Education Review Group, 2016). The provision of bonus points to most students taking mathematics at Higher level in the Leaving Certificate examination is indicative of the importance attributed to strong performance in mathematics.

Yet, concerns about standards in mathematics and about uptake of Higher level mathematics are not new, and certainly pre-date the phased implementation of Project Maths in most post-primary schools from 2010 onwards. Concerns have been raised in the past about the effects of the transition from primary to post-primary schooling on students' mathematics performance (Smyth, McCoy & Darmody, 2004), while the performance of students in Ireland on international assessments of mathematics has lagged behind performance on assessments of reading literacy over many years (e.g., Shiel et al., 2016a). The impact of the examinations system in Ireland on teaching and learning of mathematics and other subjects has also come under scrutiny, and efforts to improve the assessment experience of students are being implemented on a phased basis, beginning with Junior Cycle (see DES, 2015a).

The implementation of new approaches to teaching and learning in the context of Project Maths is perhaps unusual in that the changes relate to a specific subject area, rather than to all curriculum areas, though the approaches advocated in Project Maths are clearly relevant beyond mathematics. The emphasis on numeracy in the new Junior Cycle Framework may also impact on mathematics performance by heightening students' awareness of the importance of mathematics across a wide range of contexts.

The current report provides a good opportunity to reflect on the effects of Project Math on the performance of students in Junior Cycle. While the evaluation looks at trends in examination performance and other indices of achievement such as scores on international assessments of mathematics, it also seeks to understand some of the factors associated with performance, including the practices of teachers in mathematics classrooms across a range of school types.

The report is divided into seven chapters. Chapter 1 provides background information on Project Maths and a summary of the literature on the implementation of Project Maths in schools. Chapter 2 looks at trends in performance on international studies of mathematics achievement, with a particular focus on performance across mathematics strands (content areas) and processes. Chapter 3 also draws on data from international studies as it examines students' attitudes to mathematics, including students' liking of mathematics, and their mathematics confidence and self-beliefs. In both chapters, the outcomes for students in Ireland are benchmarked against outcomes for students on average across OECD countries. Chapter 4 looks at the performance of students in Ireland on the Junior Certificate mathematics examination prior to Project Maths and again following its implementation. As in Chapter 3, students' strengths and weaknesses on different aspects of mathematics are considered. Chapter 5 once again draws on international studies to provide a description of mathematics instruction in Junior Cycle classrooms, addressing such issues as class size, instructional time, instructional practices and content coverage. Chapter 6 describes the outcomes of Focus Group interviews conducted as part of this evaluation. The interviews, which involved teachers of Junior Cycle mathematics, provide important context for some of the issues raised in earlier chapters.

Chapter 7 provides a summary of the findings and considers their implications for the development of a new specification for Junior Cycle mathematics.

As authors of the report, we wish to thank the NCCA for their support during the study. In particular, we acknowledge the assistance of Barry Slattery (Director), who provided advice and feedback at all stages, Brendan McGlynn (also of Coláiste Choilm, Tullamore), who assisted us at all seven Focus Group meetings, and John Hammond (Chief Executive), Rachel Linney and Bill Lynch, who gave us valuable feedback on an earlier draft of this report. We also want to thank colleagues at the Educational Research Centre for their support including Peter Archer (Chief Executive), Seán Close, Anne Comey, Imelda Pluck and Patricia Gaffney. And last, but not least, we wish to thank the 76 teachers of Junior Cycle mathematics who participated in our Focus Group interviews.

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Executive Summary

Project Maths is described as empowering students to 'develop essential problem-solving skills for higher education and the workplace by engaging teenagers with mathematics set in interesting and real-world contexts' (NCCA, 2016). Elsewhere, it is described as a 'root-and-branch' revision of the mathematics curriculum at both Junior and Leaving Certificate levels (NCCA, 2008, p. 1). The aims of the accompanying syllabus include: developing the mathematical knowledge, skills and understanding needed for continuing education, life and work; developing the skills of dealing with mathematical concepts in context and applications, as well as in solving problems; supporting the development of literacy and numeracy skills; and fostering a positive attitude to mathematics in the learner (DES/NCCA, 2013, p. 6). Initially, Project Maths was implemented on a phased basis in 24 post-primary schools (the 'initial' Project Maths schools). Beginning in 2010, it was rolled out, again on a phased basis, in all post-primary schools. Arising from the phased introduction of Project Maths, 2018 will be the first year in which all students taking the Leaving Certificate mathematics examination will have studied all strands of the syllabus under Project Maths from the beginning of First year.

In December 2016, the NCCA commissioned the Educational Research Centre to conduct an evaluation of the impact of Project Maths on the performance of students in Junior Cycle mathematics. A purpose of the study was to contribute to the development of new curriculum specifications for Junior Certificate mathematics, in line with the implementation of a new Junior Cycle Framework (DES, 2015a).

The current evaluation drew on existing sources, including the literature on the implementation of Project Maths and data based on national examinations and international assessments. The review also involved Focus Group interviews with 76 teachers of Junior Cycle mathematics across a range of school types. This summary is organised around a number of key themes that emerged from the evaluation. Readers are referred to Chapter 7 for a more detailed summary and specific implications for the development of revised curriculum specifications.

Overall Performance in Mathematics

An initial evaluation of Project Maths conducted by the National Foundation for Educational Research in England and Wales (NFER) showed limited effects on performance, with no significant differences on Statistics & Probability, or on Geometry & Trigonometry between students in the 24 initial Project Maths schools, and those in schools in which the initiative had been implemented from 2010 onwards (Jeffes et al., 2013).[1]

Data from the Programme for International Student Assessment (PISA) show that performance on overall mathematics among 15-year olds in Ireland has been relatively stable since 2003[2], though there was a drop in 2009 that was made up in 2012. Indeed, Ireland's mean score in 2003 (502.8) is almost identical to its mean score in 2015 (503.7). However, in contrast with the situation in many countries, including Australia and the Netherlands, the transition to computer-based assessment in

[1] These were the only strands on which all students in the study had been assessed. This arose due to the incremental introduction of Project Maths in non-initial schools, with two strands introduced for the 2010 First year cohort, an additional two strands for the 2011 First year cohort, and all five strands for the 2012 First year cohort.

[2] Ireland has participated in each cycle of PISA since 2000. PISA 2003 was the first year in which mathematics was a major assessment domain. PISA samples 15-year-olds in participating countries. In Ireland these students are drawn from the Second, Third, Transition and Fifth years, with just over 60% in Third year in PISA 2015.

2015 did not result in a drop in performance in Ireland. Data from another international study, the Trends in International Mathematics and Science Study (TIMSS)[3], show that, although students in Second year in Ireland achieved a significantly higher overall mean score in 2015 compared with 1995, the difference was not statistically significant. However, outcomes from the NFER, PISA and TIMSS studies should be viewed positively since the effects of new curricula may not become apparent for a number of years (Conway & Sloane, 2006). In the case of PISA, Project Maths may have offset a possible decline in performance related to the transition to computer-based testing.

Data on overall Junior Certificate mathematics performance must be interpreted with reference to an increase in the proportion of students taking Higher level in recent years (from 46.5% in 2011, the year before bonus points for taking Higher-level Leaving Certificate mathematics were re-introduced, to 55.3% in 2016). The proportion of students achieving grades A-C at Higher level has ranged from 79.2% in 2012 (for the last cohort that took the pre-Project Maths syllabus) to 76.3% in 2016 (for the first cohort that was examined on all five strands of the mathematics syllabus under Project Maths). Between these years, the proportion achieving Grade A dropped from 14.3% to 7.6%, while the proportion achieving Grade B dropped 33.9% to 30.8%. There was an increase in the proportion achieving Grade C, from 17.0% to 22.1%. The proportion of A-C grades at Junior Certificate Ordinary level declined from 76.2% in 2012 to 71.8% in 2016. There was also a small drop at Foundation level (81.4% in 2012 to 80.9% in 2016). However, for all examination levels, there are years prior to the introduction of Project Maths in which similar or lower proportions of students achieved Grades A-C, compared with 2016.

Low performance among higher-achieving students can also be observed in PISA and TIMSS. In PISA 2015, marginally fewer students in Ireland (9.8% in 2015) performed at Levels 5-6 (the highest levels of proficiency) than on average across OECD countries (10.7%), even though Ireland's overall mean score was significantly higher than the corresponding OECD average (503.7 vs. 490.2). In TIMSS 2015, 6.8% of students in Ireland performed at the Advanced benchmark, compared with 10.7% on average across OECD countries in the study, even though Ireland had an overall mean score that was significantly higher than the average of OECD countries in the study (523.5 vs. 513.2). The weak performance of higher-achieving students in Ireland can be contrasted with the relatively strong performance of lower-achieving students, with, for example, 15.0% performing below Level 2 in PISA 2015, compared with 20.8% on average across OECD countries.

Performance on Mathematics Strands

In addition to reporting on overall performance, most studies also report on performance by mathematics strand, although the names of strands and their scope may vary across studies and examinations.

Students in Ireland have performed relatively well on Uncertainty & Data in PISA mathematics, with mean scores that were significantly above the corresponding OECD averages in 2003 and 2012[4], though there was a significant drop of 8.5 score points between 2003 and 2012. Students in Ireland also performed well on the equivalent area, Data & Chance, in TIMSS 2015, with a mean score that was significantly higher than on average across OECD countries in the study. However, performance

[3] TIMSS has taken place every four years since 1995. Ireland participated in 1995 and 2015, but not in the intervening cycles. Unlike PISA, TIMSS uses a grade-based sample. In Ireland, students in TIMSS are in Second year, while, internationally, they are drawn from Grade 8.

[4] PISA reports performance on mathematics content areas (and processes) in those years in which mathematics is a major assessment domain (2003, 2012). In PISA 2012, 15% of 15-year olds in Ireland (those in Second and Fifth years) had studied under Project Maths.

among higher-achieving students in Ireland was not significantly different from the corresponding OECD-16 average, suggesting possible scope for improvement among higher achievers. In the 2015 Junior Certificate mathematics exam, Statistics & Probability represented the easiest content area for students taking Higher level (76.4% of available marks were achieved), along with Algebra (74%). Statistics & Probability was moderately difficult for students taking Ordinary level (73.3%, with higher percentages of available marks achieved on Number and on hybrid items – those drawing on a number of content areas). At Foundation level, Statistics was the easiest content area (89%).

Another area of relative strength for students in Ireland in international studies is Number. In PISA 2012, students in Ireland achieved a mean score on Quantity that was significantly above the corresponding OECD average (by 10.1 score points), though students in Ireland at the 90th percentile achieved a score that was not significantly different from the OECD average at that marker. In TIMSS 2015, students in Ireland achieved a significantly higher mean score on Number than students on average in participating OECD countries. Furthermore, performance on Number in Ireland was higher than performance in any other content area, and significantly higher than overall performance on the TIMSS test. Performance on Number in TIMSS increased significantly among students in Ireland between 1995 and 2005 (by some 33.2 score points), even though overall performance did not change significantly between those years. Performance on Number in the 2015 Junior Certificate examination was higher (in terms of the percentage of available points achieved) than on other content areas among students taking Ordinary level (77.6% of available score points), second highest (81.5%) behind Statistics & Probability among those taking Foundation level, and third highest (66.5%), behind Statistics & Probability and Algebra, among those taking Higher level.

Although national and international assessment data would suggest that performance on Number in Ireland is strong relative to other mathematics content areas, teachers in the Focus Group interviews expressed concern about performance in this area, especially among lower-achieving students. Concerns included a lack of proficiency in fractions and percentages among incoming First year students. Difficulty on fractions was viewed as impacting in a negative way on Algebra in particular. Several teachers also referred to students' difficulties in conceptualising irrational numbers.

A consistent finding in PISA mathematics has been the low performance of students in Ireland on Space & Shape items. In 2003 and 2012, students in Ireland achieved mean scores below the corresponding OECD average scores on Space & Shape, with female students, in particular, struggling on items in this content area. In PISA 2012, students in Ireland performing at the 10th and 90th percentiles achieved scores that were significantly lower than their counterparts on average across OECD countries.

Students in Ireland achieved a mean score on TIMSS 2015 Geometry that was not significantly different from the average of participating OECD countries, while higher-achieving students in Ireland (those performing at the 90th percentile) did significantly less well than on average across participating OECD countries. Moreover, this was an area of relative weakness for students in Ireland as they performed at a significantly lower level on Geometry than on the TIMSS test as a whole. In the 2015 Junior Certificate examination, Geometry & Trigonometry was identified as an area of relative weakness (SEC, 2015a). At Higher level, for example, students achieved proportionately fewer of the available marks for Geometry & Trigonometry than for Statistics & Probability, Algebra or Number.

In PISA 2012, students in Ireland achieved a mean score on Change & Relationships[5] (501.1) that was significantly higher than the corresponding OECD average (492.5). However, the score of students at the 90th percentile in Ireland was below the corresponding OECD average score, indicating lower relative performance among higher-achieving students in Ireland. In the corresponding TIMSS content area (Algebra), which is more clearly linked to school curricula than PISA and also includes Functions, students in Second year in Ireland achieved a mean score (501.0) that was not significantly different from the average of participating OECD countries (501.9). As in PISA Change & Relationships, the performance of students in Ireland at the 90th percentile on TIMSS Algebra was significantly below the corresponding average for participating OECD countries. Students in Ireland at the 10th percentile on TIMSS Algebra performed at a level that was not significantly different from the corresponding OECD average.

A somewhat different pattern of outcomes emerged on the Junior Certificate examination in 2015. Students taking the examination at Higher level achieved a greater percentage of the available marks on Algebra[6] (76%) than on any other content area. Students taking the Ordinary level exam achieved a smaller percentage of the available marks (40%) on Algebra than on any other content area, though they also performed relatively poorly on Functions (42%). Students taking Foundation level did better on Algebra (65%) than on Geometry & Trigonometry (51%) or on Hybrid questions (56.5%), but considerably less well than on Number (81.5%) or Statistics & Probability (89%). Focus Group teachers identified Algebra as the most challenging area for students, and noted that expressions and equations and inequalities were particularly challenging. However, a majority of teachers noted that the inclusion of patterns in the Algebra strand was a positive step, in that it allowed links to be established between Algebra and other aspects of mathematics.

As the Functions content area is integrated into Change & Relationships in the PISA assessment, and into Algebra in TIMSS, international assessments do not provide separate information on how students perform in this area. In the 2015 Junior Certificate mathematics exam, the weighting given to Functions (in terms of available marks) was lower than for any other content area, and no items addressed Functions at Foundation level. In general, Focus Group teachers were satisfied with the relative emphasis on Functions, with some noting that the content of this strand connected with and reinforced the concepts covered in Algebra.

A finding emerging from the Focus Group interviews was the high value that teachers placed on opportunities to support students in establishing links across mathematics strands. Several teachers cited how patterning in Number and Geometry impacted positively on students' subsequent understanding of Algebra. Others commented in a positive way on examination questions that allowed students to draw on content from different strands to arrive at their answers. Support materials provided by the Project Maths Development Team highlight how particular topics can be identified across a range of content areas, such as ratio/proportion in trigonometry, probability and geometry (scale, slope, ratio of circumference to diameter etc.). Other connections that might be established more explicitly include those between mathematics topics and other curricular areas and a variety of real-life situations. Such connections also capture the concept of mathematical literacy/numeracy, where students identify opportunities to apply and build on their mathematical knowledge outside of mathematics classes.

[5] Includes 'modelling . . . change and relationships with appropriate functions and equations, as well as creating, interpreting and translating among symbolic and graphical representations of relationships' (OECD, 2016a, p. 71).

[6] Some Algebra was also required to answer questions categorised as 'Hybrid'.

Performance on Mathematical Processes

International studies use a range of schemes to describe and report on students' mathematical processes. These essentially involve classifying questions based on the main thinking processes that students are believed to draw on. PISA 2012 based its categorisation on processes identified as underpinning problem solving – Formulating situations mathematically; Employing mathematical concepts, facts, procedures and reasoning; and interpreting, applying and evaluating mathematical outcomes (see Chapter 2 for definitions). In Ireland, students had a mean score on the Formulating subscale that was not significantly different from the corresponding OECD average, while mean scores on the Interpreting and Employing subscales were significantly higher. These outcomes suggest that, relative to other problem-solving processes, students in Ireland are less proficient on Formulating tasks such as translating real-world problems into a form amenable to mathematical treatment, and providing mathematical structures and representations that can then be used to solve problems.

TIMSS 2015 frames its processes (or cognitive domains) around the more traditional categories of Knowing, Applying and Reasoning (see Chapter 2). Relative to overall performance on the TIMSS mathematics test, students in Ireland had a mean score on Knowing items that was significantly higher than on the test as a whole, and a mean score on Applying that was significantly lower. Ireland's relatively strong performance on Knowing suggests that students in Ireland are most proficient on tasks in which they recall definitions, terms and properties, carry out basic algorithmic procedures, and retrieve information from graphs, tables, texts and other sources. Performance on Applying suggests that students are more challenged when confronted with tasks that require them to determine effective strategies for solving a problem, display data in tables or graphs, create equations and inequalities or geometric figures that model problem solving situations, and implement strategies to solve problems involving more familiar concepts or procedures. Surprisingly, perhaps, students in Ireland performed reasonably well on Reasoning tasks such as using intuitive or inductive reasoning based on patterns and regularities to arrive at solutions to problems set in unfamiliar situations, evaluating alternative problem-solving strategies and solutions, and providing mathematical arguments to support a strategy or solution. Worryingly, however, the performance of higher-achieving students in Ireland (those scoring at the 90th percentile) was significantly below the corresponding OECD-16 average score on Reasoning, while their scores at this marker on Knowing and Applying were not significantly different. In contrast, the scores of students in Ireland scoring at 10th percentile were significantly higher than the corresponding OECD-16 average on each process.

The stated objectives of the most recent Junior Certificate syllabus (DES/NCCA, 2013) are linked to cognitive processes in mathematics. These are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive dispositions. Despite the fact that these objectives are clearly stated at the start of all post-primary mathematics syllabi (albeit since 2013), teachers at the Focus Group interviews were generally unfamiliar with them, though they recognised the hierarchical nature of processes in mathematics.

Instructional Time for Mathematics

The amount of instructional time currently allocated to mathematics at Junior Cycle emerged as a key issue in the course of this review. Focus Group interviews revealed consensus among mathematics teachers on the view that it is not possible to implement the curriculum as intended within the instructional time currently available for Junior Cycle mathematics (a point also raised by teachers in the NCCA's 2012 review of Project Maths). The central issue described is not the volume of curriculum content per se (as teachers were broadly satisfied with its scope). Rather, the issue is with the time available to teach the content using the methodologies encouraged by the Project Maths initiative.

Teachers emphasised that the embedding of mathematical concepts requires time, and that both time and conceptual knowledge are needed to develop a deep understanding of mathematics.

In comparison with other OECD countries, Ireland already allocates considerably less time for mathematics instruction at Junior Cycle. TIMSS 2015 data revealed that the average weekly allocation of instructional time for mathematics in Second year is 193 minutes in Ireland, compared to 201 on average across the 16 OECD countries in the study. Annually, 109 hours are allocated to mathematics in Ireland, compared to 131 across the OECD-16 on average. In Ireland, mathematics instructional time is 11.3% of total instructional time, compared to 13.3% on average for the OECD-16, and 18% in Canada. Among the OECD-16, Sweden and Japan are the only countries which allocate fewer hours to mathematics instruction annually than Ireland (99 in Sweden and 106 in Japan), and are the only countries in which mathematics instruction constitutes a smaller proportion of total annual instructional time compared with Ireland (10.8% and 10.2% respectively).

Although there is no clear relationship between instructional time and performance in international studies, Focus Group teachers felt that an increase in instructional time for mathematics could create permitting circumstances for improvements in performance. According to teachers, the desired outcomes of increased instructional time are a deeper understanding of mathematics among students, and greater numbers of students meeting the learning outcomes specific to each strand and common across all strands.

The Curriculum and the Spectrum of Students' Abilities

Performance data from international assessments indicate that lower-achieving students perform relatively well on assessments and examinations, while higher-achieving student do not perform as well as expected. However, Focus Group teachers offered the view that students on both ends of the achievement the spectrum are being disadvantaged.

At present, students who have the most difficulty with mathematics have the option of taking the Foundation level examination at Junior Certificate. However, teachers reported that the teaching and learning of mathematics at this level is hampered by the absence of a specific syllabus, which means that teachers and students are unsure what is included. Teachers working with these students view the difficulty as being intensified by a perceived lack of structure on the examination paper, the unpredictability of the paper, and the linguistic complexity of the examination questions, though it might be argued that these challenges can be addressed through appropriate instruction. Notwithstanding these perceived difficulties, teachers working with lower-achieving students emphasised that a Foundation level is needed at Junior Cycle, even though the proportion of students taking Foundation level mathematics has dropped in recent years, and was just 5% in 2016.

According to teachers, weaker students taking Ordinary level mathematics at Junior Cycle are also likely to be challenged by the Ordinary level syllabus at Senior Cycle. They contend that the gap between the syllabi has increased since the introduction of revised syllabi under Project Maths and is now too large. The gap between Ordinary level at Junior and Senior Cycles may be especially pertinent now that higher-performing students who would have taken Ordinary level at Junior Cycle in the past have moved up to Higher level.

From the perspective of Focus Group teachers, students who are higher achievers in mathematics are not being adequately challenged across the three years of the Junior Cycle mathematics curriculum. Teachers attributed much of this to an increase in mixed-ability classes (and in some cases to large classes). Teachers pointed out that First year classes are not streamed for the Common Introductory Course (CIC), and that Second and Third year classes are more mixed-ability than before due to the increase in students taking Higher level mathematics. In their view, the pace of teaching is slower in mixed ability classes and the time available for methodologies that include reflection and evaluation is restricted, especially in larger classes.

Literacy and Mathematics Learning at Junior Cycle

A key theme emerging in the evaluation centres on the literacy levels required for mathematics at Junior Cycle and the related impact on students' problem-solving performance. Teachers expressed concern that students can be disadvantaged by the literacy levels required to access examination papers. They described an increase in the linguistic complexity of questions that is causing frustration for some students and can result in a loss of confidence. It was noted that students who have the most difficulty with mathematics may not have the linguistic skills to grasp what is required within a problem and hence may not persevere with it. Teachers reported that EAL students are also disadvantaged with regard to topic recognition.

Teachers' concerns are borne out to some extent in findings from a recent analysis of Junior Certificate examination papers, which revealed that between 2003 and 2015, the volume of material to be read increased substantially at all levels (Higher, Ordinary and Foundation) (Cunningham et al., 2016). The study also found that, while the difficulty of the embedded vocabulary, sentence length, and readability levels of the papers at Higher level and Ordinary level did not change, at Foundation level, there was an increase in the readability level from Grade 0 (pre-reading) to Grade 2. Hence, over time, greater demands have been put on all students in terms of the volume of material to be read on the examination papers. In addition, greater demands have been put on students who have the most difficulty with mathematics (who are likely to take the Foundation level paper) in terms of the linguistic complexity of the examination questions.

There is evidence to suggest that familiarity with mathematics vocabulary among students in Ireland is lower than among students in OECD countries on average. In PISA 2012, fewer students in Ireland than on average across OECD countries indicated that they were familiar (knew the concepts well or had heard of them often) with mathematical terms such as exponential functions, polygons, congruent figures, arithmetic means, divisors, complex numbers and probability. While this suggests that a stronger emphasis on mathematics vocabulary is warranted, the OECD notes that exposure to terminology alone is not enough; rather, it is argued that students need extensive exposure to problems that 'stimulate their reasoning abilities and promote conceptual understanding, creativity and problem-solving skills' (OECD, 2016b, p.3). It is positive to note that some teachers interviewed as part of this evaluation reported observing improvements in mathematics vocabulary among their students following the implementation of Project Maths.

This issue of literacy further raises the question of the extent to which cross-curricular linkages are supporting students' mathematics learning. Indeed, Focus Group teachers noted that students can be confused by differences or inconsistencies in wording (e.g., units of measurement) across subjects. An increased effort to integrate and ensure consistency of concepts and terminology across subjects such as geography, history and science, could be beneficial, especially as they relate to Shape & Space/Geometry, where performance on PISA in Ireland is weak.

A final point that needs to be highlighted relates to the view expressed in Focus Groups that many students do not have the skills needed to formulate answers to examination questions that require written explanations. This is of particular concern in the context of the common learning outcome across syllabus strands that expects that students will be able to explain findings, justify conclusions, and communicate mathematics verbally and in written form.

Establishing Greater Continuity between Primary and Junior Cycle Mathematics

The question of the adequacy of the CIC as a framework for bridging the gap between the primary mathematics curriculum and Junior Cycle mathematics arose in the course of this evaluation. The issue emerged primarily in Focus Group interviews with teachers, with teachers divided on their thoughts and feelings about the CIC. Comments uncovered both very positive *('excellent')* and very negative *('a waste of time')* opinions among teachers.

The CIC sets forth the minimum mathematics content to be covered with all students in First year (DES/NCCA, 2013). In scope, it revisits the topics covered in Fifth and Sixth classes in primary school, while touching on all five strands in the Junior Certificate mathematics syllabus. Upon completion, teachers may 'extend or explore to a greater depth' (p. 33) topics which they feel will benefit the student group in question, based on the progress of the group. It is intended that the CIC will 'lay the foundation for conceptual understanding which learners can build on subsequently' (p. 33). The CIC gives teachers discretion to decide on the order in which topics are introduced. Teachers are encouraged to see the course as a whole, making connections between topics as appropriate. Teachers are also urged to use strategies which encourage the development of students' synthesis and problem-solving skills.

The positive comments from teachers generally concerned the capacity of the CIC to touch on all syllabus strands and the scope for teachers to add in additional topics according to the needs of the class. Teachers further identified the CIC as an opportunity to see where individual students' strengths lie and hence to identify areas in need of additional supports. Much of the negativity around the CIC stemmed from the mixed-ability nature of First year classes. Teachers reported spending a lot of time on basic numeracy skills with First year students because, in their experience, many are coming from primary school without the mathematical skills necessary to engage in Junior Cycle mathematics. Teachers also felt that students with greater abilities in mathematics are not adequately challenged by the CIC and can become bored. In addition, it was reported that some students who find the CIC easy can develop unrealistic expectations about their abilities, and then struggle in Second year. Teachers also felt that the gap between the CIC and Second year Higher level mathematics is too wide, notwithstanding the fact that teachers have the autonomy to increase the attention given to more difficult topics in First year, on a needs basis.

Given the mixed-ability nature of First year classes, teachers may need to differentiate for various student groups in their mathematics classes. However, teachers highlighted barriers to this, such as insufficient instructional time and large classes. In the presence of such barriers, there may be potential for teaching smaller groups of similar ability within a class, a practice that seems to be underutilised in Second year in Ireland according to TIMSS 2015.

Assessment of Mathematics at Junior Cycle

Assessment as it relates to the mathematics performance covers a broad range of issues including the structure and content of tests and other forms of assessment at classroom, school, national and international levels, as well as the outcomes of such assessments. This section focuses on formative and summative assessment as they relate to Junior Certificate mathematics.

The Junior Cycle Framework proposes the introduction of two Classroom-Based Assessments in all subject areas, including mathematics. Administered by schools, on the basis of guidelines to be provided by the NCCA, and assessed by the students' teachers to generate summative descriptors of performance, such assessments are also viewed as an opportunity for teachers to provide formative feedback to students (DES, 2015a). While some teachers in our Focus Group interviews did not engage in the discussion around these assessments, others suggested an emphasis on statistics, and the provision of problems with multiple solutions and varying levels of credit, depending on the sophistication of students' solutions. Such problems can be described as having multiple entry points and multiple representations.

Classroom-Based Assessments may also provide an opportunity for students to engage in solving complex problems that cannot be solved in the context of a short examination paper. The process of mathematical modelling (translating between the real world problems and mathematics in both directions) (e.g., Doerr & English, 2003) could be promoted in the context of solving such problems. Classroom-Based Assessments, and the types of instruction they encourage, may lead to more students working in small groups, and undertaking projects that require at least one week to complete (activities described by PISA as compatible with 'student-orientated instruction').

In our Focus Group meetings, teachers were generally concerned about the introduction of a single two-hour examination, as per the Junior Cycle Framework (DES, 2015a). Although it was acknowledged that a two-hour exam might reduce the pressure on students, concern was expressed that students might not be adequately prepared to undertake two longer papers at Leaving Certificate if they not have the experience of two papers at Junior Certificate. Several teachers raised the question of content coverage, and seemed unconvinced that it would be appropriate to sample from course content to a much greater extent that is currently the case.

A number of teachers called on examiners to indicate the number of marks available for each item, in addition to, or instead of, an indication of the time that should be allocated to each item. Teachers of students who typically take Junior Certificate mathematics at Foundation level expressed concerns that a common paper (for Ordinary and Foundation levels) might be too difficult for these students.

Currently, there is a clear shift towards computer-based assessment (CBA) in international assessments, with PISA having already implemented CBA in most participating countries in 2015, and TIMSS scheduled to introduce CBA in its 2019 assessment. In this context, and considering the broader use of technology in a range of real-life contexts, there is a case to be made for introducing CBA as part of assessment at Junior Cycle. This could involve administration of examinations via computer at some point in the future, but there are other possibilities in the shorter term. For example, Classroom-Based Assessments, or Assessment Tasks linked to the Junior Certificate examination could be administered and completed using CBA. Computer-based tasks could ask students to solve problems by constructing or evaluating mathematical models. Students could be asked to demonstrate their problem-solving skills by presenting outcomes using technology (spreadsheets, interactive charts, tables etc.). Potential uses of technology for assessment (and, as discussed elsewhere, for teaching and learning) should be considered in the course of syllabus review.

Attitudes towards Mathematics among Junior Cycle Students

In TIMSS 2015, 61.3% of students in Second Year in Ireland reported that they like mathematics. However, in the same study, just 48.3% reported that they like learning mathematics (Chapter 3). Students who liked learning mathematics scored significantly higher on TIMSS mathematics than those who did not like learning mathematics.

These findings suggest that some students may have an issue with mathematics learning more so than with mathematics itself. Indeed, over one-half of Second year students (57.5%) in TIMSS 2015 in Ireland reported that they find mathematics boring and over one-third (37.1%) disagreed that they learn many interesting things in mathematics. Additionally, a substantial percentage of students (45.4%) disagreed that their mathematics teacher gives them interesting things to do. In the absence of comparative data from an earlier (pre-Project Maths) cycle of TIMSS, it is difficult to say what impact, if any, Project Maths has had on students' liking of mathematics learning. These data suggest that there continues to be scope for increasing students' enjoyment of mathematics learning.

Focus Group teachers reported that students enjoy the learning and teaching methodologies encouraged by the Project Maths Development Team, but, as described earlier, there may be difficulty implementing them due to time constraints. Teachers reported that these methodologies are easier to implement with Statistics & Probability (the strand generally favoured by students) and more difficult to implement with Algebra and Geometry (strands that students find most difficult). It appears from this evaluation that there is scope for teaching more of the curriculum *'in the spirit of Project Maths'*, and using methodologies that appeal to students. In line with this, data from TIMSS 2015 suggest there is potential for wider use of general teaching practices which promote conceptual understanding, such as relating class content to students' daily lives and encouraging classroom discussions among students.

The teachers consulted also reported that Junior Cycle students respond well to the real-world practicality and problem solving. However, data from international assessments indicate a considerable percentage of students in Ireland do not like solving mathematical problems: 53.8% of students in TIMSS 2015 indicated that they do not like to 'solve mathematics problems'; and 29.8% of students in PISA 2012 reported that they do not like to 'solve complex problems'. Notably, both studies indicated that, compared to the OECD averages, students in Ireland are given less opportunity for deciding their own problem-solving procedures in class.

A large percentage of students (42.8%) in Ireland can be classified as not confident in mathematics and a minority (15.7%) as very confident, based on TIMSS 2015. Students who are very confident or confident in mathematics score significantly higher on TIMSS mathematics than those who are not confident. High levels of mathematics anxiety have also been observed among students in Ireland, with 35.0% of students in TIMSS 2015 reporting that mathematics makes them nervous, and 29.7% of students in PISA 2012 reporting that they get very nervous doing mathematics problems. Hence, there is scope for enhancing mathematics confidence among Junior Cycle students and for lowering their mathematics anxiety, especially in the context of problem solving.

TIMSS 2015 and PISA 2012 both indicate that, compared to female students, male students in Ireland place greater instrumental value on mathematics, but do not differ from female students in their overall liking of mathematics learning. In TIMSS 2015, similar proportions of male (57.5%) and female (57.1%) students in Ireland reported finding mathematics boring, but a greater percentage of male students (66.0%) than female students (59.9%) agreed that they learn interesting things in mathematics. It is worth noting that some teachers consulted felt that the curriculum is somewhat more oriented to male students than female students – firstly because of the kinds of examples used, and secondly, because, in their opinion, female students are more inclined to rote learning. Additionally, PISA 2012 provided evidence that male students are more open to problem solving than female students. In Focus Groups, teachers described female students as less confident and more anxious about mathematics than male students. The view was also expressed that, compared to male students, female students tend to be more concerned with grades. Indeed, in PISA 2012, a greater percentage of female students (69.4%) than male students (55.0%) reported worrying about getting poor grades in mathematics. These gender differences are of particular interest in the context of a larger than average gender difference on PISA 2015 mathematics in Ireland and the lower representation of females among higher achievers in mathematics in PISA and TIMSS. However, it should also be noted that female students outperform male students in Junior Certificate mathematics, especially at Higher level, though at the very highest level (Grade A) (Chapter 4).

Findings of this review also revealed another group of students that would benefit from specific interventions to improve their attitudes to mathematics. In TIMSS 2015, students from lower socioeconomic backgrounds scored lower on average than other students on liking learning mathematics, valuing mathematics and mathematics confidence (see Chapter 3). Also, data indicated that students in SSP (Schools Support Programme) schools under DEIS (Delivering Equality of Opportunity in Schools) like learning mathematics less and have lower mathematics confidence than their counterparts in non-SSP schools.

Professional Development for Junior Cycle Mathematics Teachers

In TIMSS 2015, 86.5% of Second year students were being taught by teachers who had attended six hours or more of formal professional development in the two years prior to TIMSS, compared with an OECD average of 64.3%. It is likely that the high level of participation in Ireland is linked to activities associated with the implementation of Project Maths and also to participation of some teachers in the Professional Diploma in Mathematics for Teaching.

TIMSS 2015 provided information on the kinds of development activities engaged in by mathematics teachers (Chapter 5). In the two years prior to testing, a clear majority (over three-quarters) of students were taught by teachers who participated in professional development relating to mathematics content, pedagogy and curriculum. Fewer than half of students were taught by teachers who participated in professional development related to addressing students' individual needs, or to assessment of mathematics. Around three-quarters of students were taught by teachers who participated in development activities focused on integrating ICTs into mathematics.

Data from TIMSS 2015 also indicate that mathematics teachers in Ireland have high levels of confidence in a range of teaching activities such as assessing student comprehension of mathematics, and making mathematics relevant to students, though not in showing students showing students a variety of problem-solving strategies. TIMSS also shows that around 20% to 25% of students have teachers who are not fully confident where activities such as developing students' higher-order thinking skills, providing challenging tasks for the highest-achieving students, and adapting teaching to engage students' interest are concerned. In Focus Group interviews, some teachers reported that they lacked confidence in the use of ICTs for mathematics teaching and learning, while others came across as being very confident.

The mathematics teachers consulted as part of this review felt they had benefitted from the development activities in which they had engaged since the implementation of Project Maths, and they highlighted the initial 10 sessions in particular. Teachers reported that they would like more frequent and ongoing professional development around Project Maths and would like the 10 initial sessions to be repeated in some form.

Findings of this review suggest other future areas of emphasis for professional development for Junior Cycle mathematics teachers. In particular, they point towards activities that support and reinforce the use of methodologies consistent with Project Maths and strengthen teachers' confidence in their application.

A highly important area is professional development focused on the integration of ICTs into mathematics teaching and learning. Although 65.2% of students in TIMSS were taught by teachers who had participated in professional development activity in this area (compared to an OECD-16 average of 45.1%), it was clear in Focus Group discussions that some teachers lack confidence in the use of ICTs for mathematics teaching and learning and that others are unable to maximise its potential, with several using ICTs to demonstrate mathematics to students, rather than engaging them more directly in exploring mathematics. There is significant scope for enhancing the use of ICTs for mathematics, and teachers need additional support in this area.

Teachers indicated that they value development opportunities that involve collective participation. However, mathematics teachers in Ireland have relatively little interaction with other teachers in development activities. In particular, it is uncommon for teachers to work together to try out new ideas, and it is especially uncommon for teachers to visit other classrooms to learn more about teaching. Hence, this is an area where improvements could be made. Increasing opportunities for teachers to interact is consistent with the literature on teacher professional development that shows collective participation and active learning as elements of effective teacher professional development (Weir, Kavanagh, Kelleher, & Moran, in preparation).

Future Research

The current study drew on data from international assessments and state examinations, which are not designed to tap into changes arising from the implementation of an initiative such as Project Maths. Future evaluations of instructional change might draw on custom-made tests that can be administered to equivalent samples immediately before a revised syllabus is launched for the first time, and again after it has been in place for a number of years. This would eliminate factors which have the potential to complicate the interpretation of findings, such as the transition to computer-based assessment in PISA or changes to questions and marking schemes in the Junior Certificate exam.

The current study drew on limited data in relation to teaching and learning. Data were available from PISA 2012, where students responded to questionnaire items about the teaching and learning they experienced. Data were also available via the Teacher Questionnaire administered to teachers of students in Second year in TIMSS 2015. While it was possible to benchmark the data from these students against OECD average scores, and combine them with observations made by teachers during the Focus Group interviews, no data based on observations of teaching and learning in classrooms were available. Future studies of curriculum reform should ensure that observational data are available.

Looking Ahead

Based on currently-available data, it can be concluded that Project Maths has had a small positive impact on student performance in mathematics, as measured by the PISA and TIMSS studies. It is also clear that there has been a significant impact on teaching approaches to teaching mathematics in schools and on students' attitudes towards mathematics. Nevertheless, there are clear challenges that must be addressed if the progress achieved to date is to be built on. The development of a new specification for Junior Certificate mathematics provides a timely opportunity to build on areas of strength in students' mathematical learning, and to address areas of weakness. The development of the new specification also provides an opportunity to integrate new approaches to assessment that arise from the Junior Cycle Framework.

From a performance perspective, it is clear that Junior Cycle students do well on content areas such as Number in TIMSS and Uncertainty & Data in PISA, and that performance on other areas, such as Space & Shape in PISA, and, to a lesser extent, Algebra and Geometry in TIMSS, needs to be improved. Indeed, it is difficult to see how performance on PISA can improve substantially without addressing weaknesses in Space & Shape. In addition to focusing more strongly on processes such as visualisation and spatial reasoning, there is a need to reconsider the focus on synthetic geometry and proof in the current syllabus, and identify ways in which the teaching of Geometry can be brought into line with other aspects of syllabus. There is a need to ensure continuity between Algebra at Junior Certificate and Leaving Certificate Ordinary levels, both in in terms of syllabus content and teaching approach. There is a need to ensure that high-achieving students in Ireland do better, as they have lagged behind their peers in countries that perform at the same overall level as Ireland in studies like PISA and TIMSS.

From a teacher perspective, it is clear that implementation of the mathematics syllabus under Project Maths has led to substantial changes in teaching methodologies, though for some aspects (such as Statistics & Probability, Number, Problem-solving) more than others (Algebra, Geometry & Trigonometry). Lack of instructional time was cited by teachers as a major impediment to the implementation of the syllabus in line with the methodologies espoused by Project Maths. Given that average annual instructional time in Second and Third years is low relative to other OECD countries, there is a need to identify ways in which instructional time can be increased. Greater flexibility and differentiation in presenting the Common Introductory Course could ensure that better use is made of available instructional time. More integration across content areas could also lead to enhanced learning. While some aspects of mathematics instruction may change as a result of changes to the curriculum (though teachers generally felt that substantial revision was not required at this time), others will emerge as new teaching and assessment approaches are introduced or reinforced. There is evidence from national and international studies that traditional approaches to teaching and learning (memorisation, teacher-directed instruction, an over-emphasis on preparing students to sit examinations) were widely used prior to Project Maths. TIMSS 2015 data, as well as reports by Focus Group teachers, suggest that considerable progress has been made in introducing more meaning-orientated approaches. However, there continues to be scope for development, including a stronger emphasis on student-oriented instruction directed at the needs of students with varying levels of ability. Some of these issues can be addressed through professional development, including stronger collaboration among teachers at school-level. Teacher confidence in areas such as adapting teaching to engage students' interest, providing challenging tasks for higher-achieving students, and developing students' higher-order thinking skills also needs to improve. There is a clear need to ensure that both teachers and students capitalise on the affordances of ICTs.

From a student perspective, reports of Focus Group teachers and students' responses in TIMSS 2015 indicate that a majority of students like mathematics and are enthused about learning it. However, just over 40% of Second years in TIMSS view mathematics as one of their favourite subjects, while just over 50% like solving maths problems. Moreover, students in Ireland are broadly similar to students on average across OECD countries in their liking of mathematics, their confidence in their own mathematics, and their liking of learning mathematics, while they are slightly higher in the extent to which they value and recognise the importance of mathematics for their futures. There is considerable scope for improvement on these measures, most of which are positively correlated with mathematics performance. There is also scope to reduce female students' anxiety about mathematics, and some of this may come from increased confidence and greater success in aspects of mathematics on which they have done less well in the past (spatial reasoning, problem solving).

Finally, the introduction of new assessment arrangements, including Classroom-Based Assessments, offer opportunities for students to engage with aspects of mathematics and approaches to problem solving in mathematics that have not been possible to assess in the past. These new assessments should also offer opportunities for formative feedback that can promote further learning. The transition to a single exam paper in mathematics will present a challenge as only a limited selection of content can be assessed each year. There will also be concerns that students who would have taken the Foundation level paper in the past will not be able to demonstrate their abilities on a new, shorter Ordinary level paper.

Chapter 1: Review of Literature on Project Maths

The development and implementation of revised syllabi under the Project Maths initiative between 2008 (when it was introduced into 24 initial or pilot schools) and 2015 (when all strands were assessed in both the Junior and Leaving Certificate mathematics examinations) represented the first 'root and branch' revision of the post-primary mathematics curriculum in Ireland since the advent of the 'New' or 'Modern' Mathematics curriculum, implemented between 1964 and 1973. This chapter reviews the background, implementation and impact of Project Maths.

1.1. Origins of Project Maths

Project Maths represented the first major revision of post-primary mathematics in Ireland since 'New Maths' was implemented between 1964 and 1973. New Maths, a world-wide development in mathematics education at the time, emphasised unifying themes such as Sets, Relations, and Functions to promote better understanding of the conceptual and logical structure of mathematics.

In Ireland, dissatisfaction with the 'New Maths' curriculum developed in the 1970s, and led to a number of revisions at lower-secondary level. These mainly comprised deletions and additions of specific topics in Number and Algebra, simplification of Geometry, clarifications of rationales and objectives, the addition of a third or Foundation syllabus level (along with the Higher and Ordinary levels) for lower-achieving students and the introduction of calculators into teaching and learning (Oldham 2001; DES/NCCA 2000, 2002).

Prior to Project Maths, several studies identified potential difficulties with mathematics education in Irish post-primary schools. These included:

- a didactic pedagogy with mathematics being taught in a procedural fashion with little emphasis on problem solving (Lyons, Lynch, Close, Sheeran & Boland, 2003);
- an approach to teaching mathematics based on the 'New Mathematics', which elevated abstraction as a core principle in post-primary mathematics teaching (Oldham, 2001);
- an over-reliance on rote learning and a lack of deeper understanding of basic mathematics concepts (SEC, 2003a, 2005);
- a declining interest in mathematics and commitment on the part of students to make the effort required to understand the subject (NCCA, 2005);
- a tendency among students who achieved high scores in mathematics at Junior Certificate to discontinue their study at Higher level for Leaving Certificate, unless they needed it for third level (Smyth & Hannan, 2002);
- a 'very narrow view of achievement' promoted by the Junior and Leaving Certificate mathematics examinations that was at odds with the aims and objectives of mathematics outlined in syllabi (Elwood & Carlisle, 2003);
- performance on international assessments of mathematics that was in the average range (e.g., Beaton et al., 1996; OECD, 2001, 2004);
- low levels of knowledge of mathematics concepts and skills shown by some students proceeding into higher education (O'Donoghue, 2002); and
- negative attitudes towards mathematics on the part of adults and children in Irish society in general (NCCA, 2012A).

A report by Conway and Sloane (2006), commissioned by the National Council for Curriculum and Assessment (NCCA), provided an international context for the revision of mathematics curricula at post-primary level, addressing the relevance of key movements in mathematics education such as Realistic Mathematics Education, Japanese Lesson Study (a central approach to professional development in Japan), and Mathematics in Context. Conway and Sloane (2006) noted that there was no single template for reforming mathematics education at post-primary level. They also noted the complexity of curriculum reform in mathematics, arguing that:

> It is unlikely that merely changing one feature of the system – the mathematics curriculum culture, textbooks or testing/examinations alone – will result in new forms of mathematics education in Irish post-primary classrooms. Even if all three were to be the focus of reform efforts, it is highly unlikely that actual changes in classroom practice or student scores in international tests would be evident on a wide scale for at least five to ten years (p. 203).

1.2. Elements of Project Maths

On its website[7], the National Council for Curriculum and Assessment describes Project Maths as 'an exciting, dynamic development in Irish education. It involves empowering students to develop essential problem-solving skills for higher education and the workplace by engaging teenagers with mathematics set in interesting and real-world contexts'. Elsewhere, the NCCA (2008, p. 1) has described Project Maths as a 'root-and-branch' revision of the mathematics curriculum at both Junior and Leaving Certificate levels.

According to the current Junior Certificate syllabus (DES/NCCA, 2013, p. 6), the aims of Junior Cycle mathematics are to:

- develop the mathematical knowledge, skills and understanding needed for continuing education, for life and for work;
- develop the skills of dealing with mathematical concepts in context and applications, as well as in solving problems;
- support the development of literacy and numeracy skills; and
- foster a positive attitude to mathematics in the learner.

These broad aims are accompanied by objectives designed to support the development of mathematical proficiency – conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. The key mathematics strands or content areas are identified as Statistics & Probability, Geometry & Trigonometry, Number, Algebra, and Functions.

The main structural differences between the revised Project Maths curriculum at Junior Certificate level and its predecessor (the 'pre-2010 curriculum') include the absence of a specific course for Foundation level, and the introduction of learning outcomes for each topic within each strand. In terms of content, the most marked difference is the introduction of Probability for both Higher and Ordinary levels and the expansion of Statistics to cover topics such as sampling (Higher level only) and statistical reasoning (e.g., awareness of misuses of statistics). In addition, the content for each strand in the revised syllabi concludes with a section relating to synthesis and problem-solving skills. This includes learning outcomes such as the justification of conclusions and communication of mathematics, both verbally and in written form. Notably, an additional feature of the Project Maths syllabus is the inclusion of a supplement dealing with Synthetic Geometry that is separate from the

[7] http://www.ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/

rest of the curriculum, with an approach and general philosophy that is more formal and reminiscent of the previous curriculum.

As well as changes to curriculum, Project Maths envisaged changes to instruction. According to the NCCA (2012a, p. 10),

> Project Maths, informed by international trends, develops key skills by promoting a 'collaborative' culture where mathematics is seen as a network of ideas which teacher and students construct together. Learning is seen as a social activity in which students are challenged and arrive at understanding through discussion. Teaching is seen as a non-linear dialogue in which meanings and connections are explored, misunderstandings are recognised, made explicit and students learn from them.

The NCCA (2012, p. 18) also emphasised that students are 'encouraged to think about their strategies, to explore possible approaches and evaluate these, and so build up a body of knowledge and skills that they can apply in both familiar and unfamiliar situations'. Ní Shúilleabháin (2013, p. 23) described this as 'a philosophical shift in Irish post-primary classrooms from a highly didactic approach with relatively little emphasis on problem solving towards a dialogic, investigative problem-focused approach to teaching and learning mathematics'.

A key feature of Project Maths is the implementation of a Common Introductory Course (CIC) in First year to support students in better bridging the gap between primary- and post-primary mathematics education. For example, in Algebra, the CIC provides 'an initial engagement with patterns, relationships and expressions, laying the basis for progression to symbolic representation, equations and formulae' (DES/NCCA, 2013, p. 26). In Geometry, the CIC is intended to enable learners 'to link formal geometrical results to their study of space and shape at primary level' and 'accept the results as true for the purpose of applying them to various contextualised and abstract problems' (p. 17). This is followed by a gradual shift towards establishing constant features or results through common proof. The order in which topics are presented is left to the discretion of teachers, though they are advised to support students in making appropriate connections between topics and strands. According to the syllabus, once the CIC has been completed, 'teachers can decide which topics to extend or explore to a greater depth, depending on the progress being made by the class group' (p. 33).

1.3. Concerns about Project Maths

Implementation of Project Maths began on a phased basis in 24 initial or pilot schools in 2008, and was extended to all schools in 2010, with full implementation completed by 2015, when all Junior Cycle students were examined on all five strands in the Junior Certificate exam (all five strands were assessed in the Leaving Certificate exam in 2014). In both 2008 and 2010, students in First and Fifth years were introduced to new methodologies and revised syllabi, with additional grade levels and topics added in subsequent years. For a number of years, students took examinations in which questions in some strands were based on syllabi under Project Maths, and questions in other strands were based on pre-Project Maths syllabi.[8]

[8] Arising from the phased introduction of Project Maths, in which the final phase at Junior Cycle began with the First year intake of September 2012, 2017 was the first year in which at least some students taking Leaving Certificate Mathematics (those who went from directly from Third year into Fifth year) had covered all five strands of the revised syllabus at Junior Cycle. Only in 2018 will all Leaving Certificate students (those taking and not taking Transition year) have covered all five strands of the revised syllabus at Junior Cycle.

Project Maths has had a mixed reception. The Irish Mathematics Teachers Association (IMTA, 2012) noted that insufficient detail was provided on aspects of course content, giving rise to uncertainty as to whether certain topics were included or not. IMTA also referred to an over-representation of Statistics and of construction in Geometry, and, at Leaving Certificate level, the absence of vectors. IMTA also expressed concern over the literacy demands of Project Maths, arguing that unnecessarily difficult or elaborate language was used in curriculum materials and examinations.

Some commentators (e.g., Grannell, Barry, Cronin, Holland & Hurley, 2011; Kirwan, 2015; Kirwan & Hall 2016) have argued that the Organisation for Economic Co-operation and Development (OECD) and its Programme for International Assessment (PISA) in particular, has had an undue influence on Project Maths. PISA, an international assessment of reading literacy, mathematics and science, is administered to 15-year olds in participating countries every three years. In reporting outcomes, the OECD ranks countries on their overall performance as well as on their performance in specific content areas and processes (see Chapter 2 in this report). According to Kirwan (2015), Project Maths reflects a growing influence of globalisation on education in that it draws heavily on the conceptual framework underlying the PISA assessment framework. However, Kirwan also acknowledges that Project Maths is comprised of two distinct approaches – the abstract, symbolic mathematics of sections of the pre-existing curriculum, as well as a 'PISA-like' approach to pedagogy and real-life problem solving. Indeed, in an analysis of the 2015 Junior Certificate Examination, she reported that just under one-half of questions at each level (Higher, Ordinary and Foundation) could be categorised as 'abstract/symbolic' and just over one-half as 'real-life' problems. She argued that 'adherence to the PISA competency clusters in learning outcomes translates into a two-part approach to assessment which involves a combination of old and new' (p. 323). This interpretation is surprising since all PISA items, including those in the Reproduction cluster, always involve application of mathematical procedures in context.

A report by academics at University College Cork (Grannell et al., 2011) questioned whether Project Maths could adequately prepare students for the depth and breadth of third-level mathematics courses. The report argued that reform in mathematics could have been achieved by training teachers on the existing curriculum, rather than, as the report's authors saw it, lowering the standard of material in the curriculum. The authors called for more intensive training of teachers ('weeks instead of days') and implementation of a more structured approach (based on Singapore mathematics) to solving problems in context. Grannell et al. and Kirkland et al. (2012) strongly questioned the absence of certain topics (e.g., vectors, matrices, sequences and series, aspects of calculus) that had been dropped in the transition to Project Maths, though mainly at Leaving Certificate level. Kirkland et al. also argued that, at the same level, there is too much emphasis on applications and not enough on mathematical foundations, including calculus and linear algebra.

Textbooks prepared for Project Maths have also been commented on. Lubienski (2011) noted substantive differences between textbooks claiming to be based on Project Maths, with one of the two she examined following a didactic pattern (presenting boxed formulae and examples for students to follow) and the other structuring a sequence of investigations through which students discover the formulae. In her review, Lubienski also raised questions about the mathematical thinking underpinning the textbooks she examined and expressed surprise that teachers, rather than mathematics educators or mathematicians, had been responsible for authoring them. In a study that compared a broader range of Project Maths textbooks at Junior Certificate level, O'Keefe and O'Donoghue (2011, p. v) noted that the textbooks 'displayed a genuine attempt to match the intentions of Project Maths, but no one textbook met all the needs of Project Maths'. They also found a mismatch between curriculum expectations (such as integration of content strands) and textbook expectations, minimal emphasis on the integration of ICTs into teaching and learning activities, and lack of consistency between textbooks in the extent to which teaching for understanding and problem solving was promoted.

The NCCA responded to some of the concerns raised about Project Maths in a document issued in October, 2012 – a point at which all strands of Project Maths were being implemented in all schools at both Junior and Senior cycles. The NCCA response clarified an important distinction between, on the one hand, PISA, which focuses on assessing mathematical literacy, and Project Maths, which is concerned with teaching, learning and assessment. It also clarified a distinction between preparing students for mathematics at Third level (a goal often associated with Higher level mathematics at Leaving Certificate) and a broader effort to ensure that the general population of students achieved high levels of general mathematical literacy. The response explained that the omission of certain topics, especially at Leaving Certificate level, was necessary to ensure that students acquired greater conceptual understanding and problem-solving skills around the content that was included. It was also noted that the removal of choice in both the syllabi and examinations at Leaving Certificate level necessitated a reduction of content.

1.4. Systemic Changes and Other Policy Initiatives Coinciding with or Following Implementation of Project Maths

In addition to the implementation of Project Maths, a number of systemic changes have occurred in recent years. An initiative designed to improve take-up of mathematics at Higher level in the Leaving Certificate examination has been to provide 25 bonus points[9] to students achieving a grade D or higher in Leaving Certificate Higher level mathematics from 2012.[10] This has contributed to increases in the proportions of students taking Higher level mathematics at both the Leaving and Junior Certificate, with uptake at Leaving Certificate increasing from 22% in 2012 to 28% in 2016, and uptake at Junior Certificate increasing from 48% to 55% over the same period (SEC, 2012, 2016)[11]. According to the *National Strategy to Improve Literacy and Numeracy 2011-2020* (DES, 2011; DES, 2017), the target for participation at Junior Certificate Higher level mathematics is 60% by 2020.

A consequence of the introduction of bonus points is that it is difficult to disentangle their effect on performance from the effects of Project Maths itself.

One policy change that has not been implemented to date is the introduction of standardised tests in Second year. According to the *National Strategy to Improve Literacy and Numeracy 2011-2020* (DES, 2011), students would be required to sit standardised tests of mathematics (and English reading) at the end of Second year from 2014 onwards. However, subsequent to this, the introduction of standardised tests was linked to Junior Cycle assessment reform, and was set aside to allow a stronger focus on classroom-based assessment (TUI, ASTI & DES, 2015).

[9] The Report of the Project Maths Implementation Support Group (DES, 2010) suggests that the uniform bonus (rather than bonus points on a sliding scale) is designed to ensure that students who might otherwise chose to switch from Higher to Ordinary level are incentivised to take the Higher level exam.

[10] From 2017, bonus points are awarded to students achieving grades H1 to H6 (that is, at least 40%) (see http://www.transition.ie/).

[11] In 2011 (the year before bonus points were introduced), 46% took the Junior Certificate mathematics examination at Higher level, while 16% took the Leaving Certificate mathematics examination at Higher level (SEC, 2011)

Most recently, in its review of the National Strategy, the DES (2017) has set new targets for performance on the PISA mathematics assessment, in which a nationally-representative sample of 15-year olds takes part, along with their counterparts in over 70 countries, every three years. These include:

- a reduction in the proportion of students performing at or below Level 1 in mathematics from 15% in 2015 to 10.5% by 2020
- an increase in the proportion of students performing at Levels 5-6 from 10% in 2015 to 13% by 2020.

Among the steps to be undertaken to achieve these and related targets between now and 2020 according the review are: a greater focus on the development of Algebra and Geometry and on the skill of 'Applying' at post-primary level; the introduction of a revised Junior Cycle mathematics syllabus in September 2018; opportunities for higher-achieving students to reach their potential in mathematics and other areas; and greater integration of ICTs across all aspects of learning, in line with the Digital Strategy for Schools 2015-2020 (DES, 2015a).

Some of these steps reflect changes being implemented as part of a broader Junior Cycle reform that embraces all subject areas, including mathematics (see DES, 2015b). Additional aspects of Junior Cycle reform that may impact on Junior Cycle mathematics over time include:

- The promotion of eight key skills across all curriculum areas, including 'being numerate', which is described as including: expressing ideas mathematically; estimating, predicting and calculating; developing a positive disposition towards investigating, reasoning and problem solving; seeing patterns, trends and relationships; gathering, interpreting and representing data; and using digital technology to develop numeracy skills and understanding.
- Structural changes such as the inclusion of short courses as well as traditional subjects. Such courses could focus on aspects of mathematics that might not otherwise be covered in mathematics classes and may not be of interest to all students.
- Changes to subject specifications that reflect an increased focus on such processes as engaging in research, investigation and experimentation, thinking critically and solving problems, working independently and/or as part of a team, and evaluating one's own learning, either as an individual or in collaboration with others.
- The introduction of Classroom-Based Assessments in short courses that would be conducted in the Second and Third years; written assessment tasks would be linked to a second Classroom-Based Assessment and submitted to the SEC for scoring; and a shorter written state exam at the end of Third year.
- The assessment of mathematics at two levels (instead of three) in the Junior Certificate mathematics examination at the end of Third year – Higher level and Ordinary level.

1.5. Support for Teachers in Implementing Project Maths

The implementation of Project Maths included a significant attempt to upgrade the skills of mathematics teachers to embrace new, discovery-based teaching methods designed to enhance conceptual understanding and problem solving (see, for example, the resources for teachers available at *http://www.projectmaths.ie/*).

A key initiative has been to introduce a Professional Diploma in Mathematics for Teaching that enables 'out-of-field' teachers to meet the Teaching Council of Ireland's requirements to teach mathematics in post-primary schools. In a survey by Ní Riordáin and Hannigan (2009), 48% of mathematics teachers, mainly at Junior Cycle, were found not to have a qualification in mathematics, but were predominantly qualified in science or business studies instead. In 2016, Lane, Faulkner and Smith reported that 550 teachers (three cohorts) had completed the diploma, with a majority of participants (52.5%) indicating satisfaction with the programme despite significant time commitments and related challenges.

The provision of a Professional Diploma in Mathematics for Teaching was separate from the regular inservice training provided to all teachers of mathematics to mark the implementation of Project Maths. This comprised 10 day-long workshops spread over five years, which focused on methodology. In addition, optional evening workshops were available at Education Centres. These dealt with mathematics topics (content) and/or the use of ICTs in teaching and learning mathematics.

Recent activities in the area of professional development, such as the Maths Counts 2017 Conference in Maynooth, illustrate the broad range of activities that are being undertaken throughout the country to further teachers' professional knowledge under the auspices of the Project Maths Development Team, including the implementation of structured problem solving in classrooms through Japanese Lesson Study or research lessons. Such activities are in line with an approach to professional development that supports teachers in developing professional communities of practice where they collaborate and share knowledge (Ní Shuilleabháin, 2013).

1.6. Initial Evaluations of Project Maths

A number of evaluations of Project Maths took place while it was still being developed. The NCCA (2012b) reported on school visits in which they interviewed teachers involved in the implementation of Project Maths in the initial 24 schools. The NCCA reported that:

- The role of mathematics teachers had changed in significant ways as they saw themselves as 'facilitators of learning rather than givers of knowledge' (p. 20)
- Teachers were challenged by the time demands of the Project Maths syllabi, as 'the use of more active teaching methods, characterised by high levels of student involvement, classroom discussion and practical work. . . proved very time consuming' (p. 20). This resulted in some teachers having to lay on extra classes outside core mathematics time to ensure that syllabi were completed.
- While it was found that learning approaches such as group work, classroom discussion and questioning were being used more often by teachers, not all teachers were convinced that such approaches offered additional benefits over more traditional approaches such as 'chalk and talk' and 'drill and practice'.
- There was increased use of the mathematics syllabus as a guide, rather than the textbook or past exam papers, though it was acknowledged that not all teachers were comfortable with language and level of detail provided by the syllabus.
- Collaboration between mathematics teachers within schools had increased. The focus of mathematics department meetings was found to have shifted from concern over timetabling, exams and sequencing issues to 'collaboration around challenging aspects of teaching the syllabus under the new approaches' (p. 21).
- Students now engaged in greater discussion, collaboration and activity within their maths classrooms, though teachers tended to revert to more didactic approaches in examination years.
- A majority of teachers continued to use examinations as the only assessment tool, although some also gained insights into students' thinking and learning through such practices as listening to students explain how they solved a problem, or group discussions on different ways of answering a question.

The report recognised that the syllabus was just one of many factors impacting on teaching and learning mathematics, and identified others as including 'ongoing support (professional development) for teachers, a collaborative mathematics department, organised and accessible resources, a timetable that supports a discursive learning environment, a classroom infrastructure that supports this type of learning, an assessment methodology that reflects the syllabus learning outcomes, and methodologies and external leadership and support from the educational establishment' (p. 22).

The NCCA also commissioned the National Foundation for Educational Research in England to conduct an evaluation of the impact of Project Maths on students' performance and attitudes towards mathematics (see Jeffes et al., 2012, 2013). Separate cohorts of Junior and Senior Cycle students were assessed in spring 2012 and autumn 2012. Key findings from the final report (Jeffes et al., 2013) include:

- Students reported that they frequently undertook activities commonly associated with the Project Maths syllabi (e.g., making connections between mathematics topics, applying mathematics to real-life situations) but they also reported engaging in more traditional activities such as using textbooks and copying from the board.
- While some processes of the Project Maths syllabi were visible in some of the student work reviewed, there did not appear to have been a substantial shift in what teachers asked students to do.
- Students had a good mastery of mathematical procedures, and, to a lesser extent, problem solving and making mathematical representations. However, there was little evidence of students demonstrating reasoning and proof or communication in their written work, or making connections between mathematical topics.
- Student confidence and achievement were associated with examination entry level (Higher, Ordinary and Foundation) and gender, with girls less confident about mathematics than boys, and performing less well on a test developed by the NFER (drawing on released items form international assessments) in Third year (there was no difference in Sixth year).
- While achievement was highest on Statistics & Probability and lowest on Functions, confidence was highest for both these strands and lowest for Number and Algebra.
- While students had developed a general awareness of the importance of mathematics in further study, and of its application, they appeared less certain of what careers will draw on their mathematical skills and knowledge.

Rather discouragingly, the NFER study reported that 'overall, schools following a greater number of strands, or schools having a greater experience of teaching the revised syllabuses, does not appear to be associated with any improvement in students' achievement or confidence' (2013, p. 5). The evaluators did, however, note that their evaluation occurred at a relatively early point in the implementation of Project Maths, and that, over time, there may be a shift from (sometimes new) content to a stronger focus on processes.

In PISA 2012 in Ireland, all initial Project Maths schools were included in the sample of selected schools[12]. Students in these schools achieved higher mean scores than students in non-initial schools (most of whom had not studied under Project Maths) on each PISA mathematics content area and process, and

[12] By 2012, the number of initial Project Maths schools had fallen to 23, as one school had amalgamated with a non-initial school, and was no longer considered to be an initial school. The contribution of Project Maths initial schools to Ireland's overall performance on PISA 2012 was weighted down, to reflect the representation of students in initial schools in the overall population of students.

on the overall performance scale, but none of the differences was statistically significant (Merriman, Shiel, Cosgrove & Perkins, 2014). Worryingly, students in initial schools had significantly higher levels of anxiety about mathematics (as measured by PISA) than their counterparts in non-pilot schools, perhaps because they were the first to experience changes to the Junior Certificate examination arising from the transition under Project Maths. The review also highlighted a discrepancy in performance by gender, with female students outperforming male students on the Junior Certificate mathematics exam, and male students outperforming females on PISA 2012 mathematics items.

Drawing on a survey of mathematics teachers in initial and non-initial schools conducted as part of PISA 2012, Cosgrove, Perkins, Shiel, Fish and McGuinness (2012) reported more frequent use of Information and Communication Technologies (ICTs) in mathematics classes in initial schools, and more positive changes in learning and assessment, though teachers in initial schools were less confident in their teaching. Teachers in initial schools also argued that the problems presented to students in classroom and assessment contexts contained more text and greater linguistic complexity than was the case prior to Project Maths, when teaching and learning mathematics was more formal and less contextualised.

1.7. Project Maths and Preparedness for Mathematics in Higher Education

As noted earlier, a number of educators in the higher education sector have raised issues about absence of certain key content from the Leaving Certificate Project Maths syllabi, especially at higher level. More recently, several reports documenting a drop in standards have been issued by researchers linked to the University of Limerick. For example, Tracy, Faulkner and Prendergast (2016) found a drop in the basic computational skills of beginning undergraduate students who had achieved a Grade C or D on the Leaving Certificate examination. Moreover, the authors linked the decline to the period during which Project Maths was introduced into post-primary schools (2008-2014), though the first group to have studied Algebra under Project Maths would have completed the Leaving Certificate exam in 2013 (due to the phased approach to introducing mathematics strands). Elsewhere, Prendergast and Treacy (2017) have referred to 'Ireland's Algebra Problem', again in the context of low performance among under-graduate students. Their claim is that the promotion of a functions-based approach to algebra in Project Maths rather than the more traditional, rule-based approach, is at fault, though they acknowledge that this argument is weakened by their finding that teachers they interviewed were implementing both functions-based and transformational (rule and procedure-based) approaches.

1.8. Readability of Mathematics Tests and Examinations

As noted earlier, a number of surveys and commentaries on the implementation of Project Maths raised issues about the volume of reading in mathematics textbooks and in examinations, and the linguistic complexity of the problems. This reflected a transition from mainly context-free or abstract mathematics in the past, to a situation in which many questions were embedded in context under the new syllabus (though this was already happening to some extent in the Junior Certificate Ordinary and Foundation level papers prior to Project Maths). The concerns of teachers may relate to a perception that some students can adequately apply their mathematics in situations in which questions are relatively context free, but are at a disadvantage when asked to read a short text, abstract the underlying problem in mathematical terms, solve it, and, in some instances, interpret the results in terms of the original problem. Eivers (2010) attributed the heavy reading load in PISA mathematics (and science) items to a need by PISA to contextualise items and present them in real-life contexts, and argued that this contributed to 'construct-irrelevant variance' that made it difficult to assess whether low performance arose from reading or mathematical difficulties. However, this viewpoint does not acknowledge importance of mathematical modelling as an approach to solving mathematical problems and acquiring associated skills.

A number of studies have been implemented to measure the readability of mathematics assessments, including TIMSS and PISA. Mullis, Martin and Foy (2013) identified four factors that might render mathematics items difficult: number of words (with more words indicative of a greater reading load); vocabulary complexity (with specialised vocabulary likely to be more difficult); complexity of symbolic language such as numerals, symbols and abbreviations; and density or complexity of visual displays such as geometric shapes and figures, models and diagrams. They categorised TIMSS 2011 Grade 4 items into those deemed to have a 'high reading demand', 'medium reading demand', and 'low reading demand'. Fifty percent of items in the Number content area were deemed to have low reading demand, while 85% in the Data content area were judged to have 'high reading demand'. Whereas 59% of items categorised as 'Reasoning' (the highest order process skill) were judged to have high reading demand, 61% categorised as 'Knowing' (the lowest order process skill) were deemed to have a low reading demand. Data for Ireland in the study indicated that, in general, performance in mathematics was related to students' reading proficiency according to a standardised measure of reading (the test underpinning the Progress in International Reading Literacy Study) which was administered in conjunction with TIMSS 2011, and involved the same schools and students. Hence, on average, lower-performing readers in Ireland performed least well on items categorised as having high or medium reading demands. High achievers in reading, on the other hand, performed at about the same level across items with high, medium and low reading demands. It is unclear if these same patterns are evident in TIMSS Grade 8.

Researchers in mathematics education have applied traditional readability formulae to mathematics tests, while acknowledging their drawbacks. Such formulae are typically designed for use with relatively long, continuous texts rather than short items accompanied by diagrams. In addition, differences in estimates of reading difficulty may arise from the application of different formulae. King and Burge (2015) sought to address these issues by applying multiple formulae – mostly developed in the US – to clusters of items administered in PISA 2012. Their results, when averaged across formulae, showed that mean US grade levels ranged from 7.5 to 10.9. These correspond to reading ages extending from 12.3 to 15.5 years. This suggests that some students taking the PISA 2012 tests (for example, those with reading ages below 12 years) might expect to find the reading aspect of the mathematics items to be especially difficult. Using the Flesch-Kincaid formula, Shiel, Cosgrove, Sofroniou and Kelly (2001) reported an average reading difficulty of US Grade 6.8 for mathematics items administered in PISA 2000, with a range of 3.7 grade levels. This is marginally lower than the average of the values reported by King and Burge using the same formula for item clusters in PISA 2012 mathematics (Grade 8.6, range 2.1 grade levels). This suggests that PISA mathematics items have become more complex, in terms of readability, over time. In Chapter 5 in the current study, which discusses changes to the Junior Certificate mathematics examination, the readability of exam items is considered.

1.9. Summary

Project Maths was introduced in 24 pilot post-primary schools in September 2008, and was introduced into all schools in September, 2010 on a phased basis. By June 2015, all students were assessed in all strands of the revised mathematics curriculum in the Junior Certificate mathematics exam. The 2015 Junior Certificate cohort will also be the first to complete their study of mathematics under Project Maths when they take the Leaving Certificate mathematics exam in 2017 (for students who went directly from Third year to Fifth year) or 2018 (for students who took Transition year first).

Several studies have been undertaken into the effects of Project Maths on teaching and learning in schools, on the performance of students, and on their attitudes towards mathematics. A number of these studies were implemented in 2011 or 2012, shortly after Project Maths had extended from the initial 24 schools to all schools. Hence, they provide some useful insights into the initial implementation of Project Maths in schools. However, it is acknowledged that they occurred relatively early in the life of Project Maths, and that it might be some time before its full effects are evident.

The current study is intended to extend and update the initial evaluations completed by the NCCA and the NFER. However, it is not a systematic study of the impact of Project Maths on student performance and attitudes, or of changes in teaching and learning in Project Maths classrooms. Rather, it seeks to draw on existing sources, including current national (Junior Certificate) and international data, to draw inferences about the effects of Project Maths on aspects of teaching and learning mathematics, including students' performance and their attitudes. It also turns an eye towards the future, in that it seeks to situate Project Maths in the context of current changes in Junior Cycle more generally and the forthcoming review of Junior Cycle mathematics.

Chapter 2: Performance on International Studies of Mathematics Achievement

This chapter looks at the performance of Junior Cycle students in Ireland in two international studies – the OECD's Programme for International Student Assessment (PISA) and the International Association for the Evaluation of Educational Achievement's Trends in International Mathematics and Science Study (TIMSS).

2.1. PISA

PISA, an assessment of the mathematical literacy, scientific literacy and reading literacy skills of 15-year olds, has been implemented every three years since 2000. In 2003 and 2012, mathematical literacy was a major assessment domain in PISA, with performance reported by content area and process, as well as overall. In the remaining years (2000, 2006, 2009, 2015), only overall performance was reported. In Ireland, students in all Project Maths initial schools[13] participated in PISA 2012, and their performance was compared with that of students in non-initial schools. PISA 2015 was the first PISA cycle in which all students taking the assessment in Ireland had studied under Project Maths.

2.1.1. The PISA Mathematics Framework

PISA defines mathematical literacy as. . .

> an individual's capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematics concepts, procedures, facts and tools, to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged and reflective citizens (OECD, 2016a, p. 65).

PISA mathematics supports the development of concepts of pure mathematics and the benefits of being engaged in explorations in the abstract world of mathematics. It also emphasises students' capacity to use mathematics in context. It argues that this is equally important for students at or near the end of their formal mathematics education, as well as those planning to engage in further study of mathematics, as it makes mathematics more relevant for all students and increases their motivation. The framework emphasises that mathematical literacy is an attribute along a continuum, with some individuals being more mathematically literate than others, and with potential for growth always there.

PISA identifies four mathematical content areas – Change & Relationships, Shape & Space, Quantity and Uncertainty & Data. It is claimed that these 'meet the requirements of historical development, coverage of the domain of mathematics and the underlying phenomena which motivate its development, and are linked to the major strands of school curricula' (OECD, 2016a, p. 71). In the years in which mathematics is a major assessment domain, performance is reported by mathematical content area. In all cycles, there is a balance of items across content areas.

The content areas can be defined as follows:

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[13] Initial Project Maths schools are the 24 school in which Project Maths was first implemented on a pilot basis, beginning in 2008. When PISA 2012 was implemented, one of the 24 had amalgamated with a non-Initial schools, and hence, there were 23 Initial schools in PISA 2012.

- ***Change & Relationships*** – involves understanding types of change and recognising when these occur in order to use suitable mathematical models to describe and predict change. Mathematically, this involves 'modelling the change and relationships with appropriate functions and equations, as well as creating, interpreting, and translating among symbolic and graphical representations of relationships' (OECD, 2016a, p. 71). Aspects of traditional mathematics content of functions and algebra, including algebraic expressions, equations and inequalities, and tabular and graphic representations are central and must be drawn on to describe, model and interpret change in phenomena. Representations of data and relationships described using statistics are also viewed as important, as is a firm grounding in the basics of number and units.
- ***Space & Shape*** – involves understanding perspective, creating and reading maps, transforming shapes with and without technology, interpreting views of three-dimensional scenes from various perspectives and constructing representations of shapes. Geometry is seen as 'an essential foundation for Space and Shape, but the category extends beyond traditional geometry in content, meaning and method, drawing on elements of other mathematical areas such as spatial visualisation, measurement and algebra' (OECD, 2016a, p. 71).
- ***Quantity*** – involves understanding measurements, counts, magnitudes, units, indicators, relative size, and numerical trends and patterns. Aspects of quantitative reasoning deemed important for mathematical literacy include number sense, multiple representations of numbers, elegance in computation, mental calculation, and estimation and assessment of the reasonableness of results.
- ***Uncertainty & Data*** – includes knowledge of variation in processes, having a sense of the quantification of that variation, acknowledging uncertainty in measurement, and knowing about change. It also involves forming, interpreting and evaluating conclusions drawn in situations where uncertainty is central. The interpretation and presentation of data are viewed by PISA as important elements of uncertainty and data.

PISA identifies three key mathematical processes: Formulating situations mathematically, Employing mathematical concepts, facts, procedures and reasoning, and Interpreting, applying and evaluating mathematical outcomes. These are described as being underpinned by seven mathematical abilities: communication, mathematising, representation, reasoning and argument, devising strategies for solving problems, using symbolic, formal and technical language and operations, and using mathematical tools. The three main processes are:

- ***Formulating*** – identifying opportunities to apply and use mathematics, and seeing that mathematics can be applied to understand or resolve a particular problem or challenge presented. It includes translating a real-world problem into a form amenable to mathematical treatment, providing mathematical structure and representations, and identifying variables and making simplifying assumptions to help solve the problem or meet a challenge
- ***Employing*** – applying mathematical reasoning and using mathematical concepts, procedures, facts and tools to arrive at a mathematical solution. It includes performing calculations, manipulating algebraic expressions and equations or other mathematical models, analysing information in a mathematical manner from mathematical diagrams and graphs, and developing mathematical descriptions and explanations and using mathematical tools to solve problems

- ***Interpreting*** – reflecting on mathematical solutions or results and interpreting them in the context of a problem or challenge. It includes evaluating mathematical solutions or reasoning in relation to the context of the problem and determining whether the results are reasonable and make sense in the situation.

PISA also identifies four broad contexts in which mathematical problems are identified. However, these are for descriptive purposes only, as subscales are not developed for mathematical contexts.

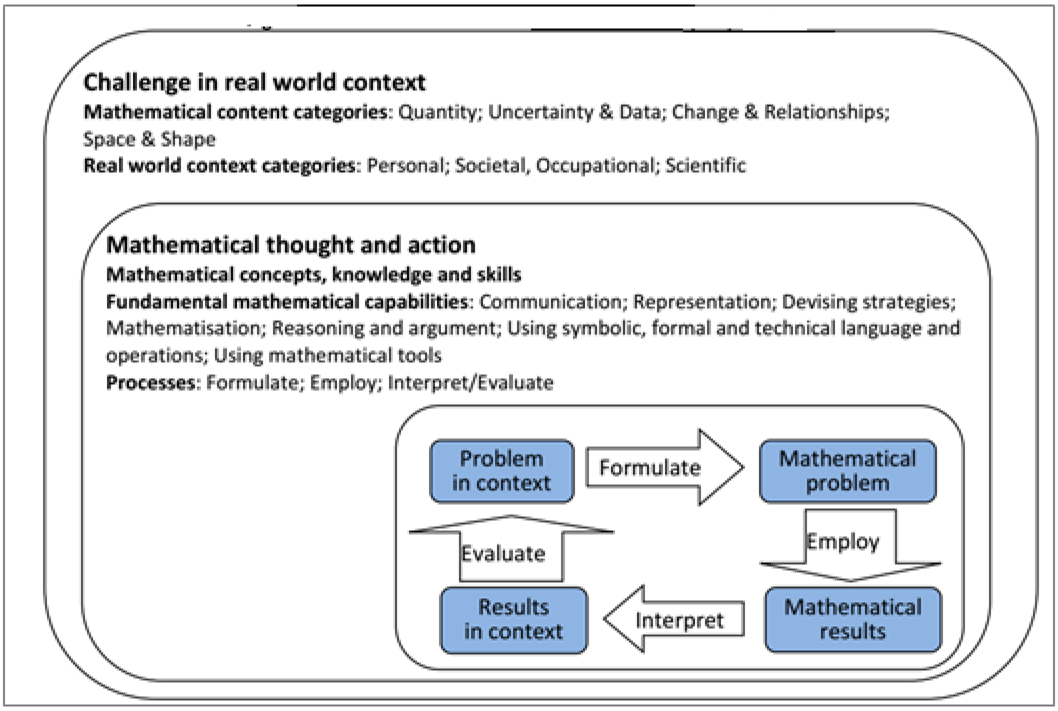
- ***Personal*** – these items focus on activities of one's self, one's family or one's peer group. Such contexts include: food preparation, shopping, games, personal health, personal transportation, sports, travel, personal scheduling, and personal finance.
- ***Occupational –*** these are centred on the world of work and include such activities as measuring, costing and ordering materials for building, payroll/accounting, quality control, scheduling/inventory, design/architecture and job-related decision making.
- ***Societal*** – these focus on community (whether local, national or global), and can involve voting systems, public transport, government, public policies, demographics, advertising, national statistics and economics. The focus of these problems is on a community (rather than a personal) perspective
- ***Scientific*** – these relate to the application of mathematics to the natural world and issues and topics related to science and technology. Contexts include weather or climate, ecology, medicine, space science, genetics, measurement, and the world of mathematics itself.

The elements of the PISA mathematics framework come together in the PISA model of mathematical literacy (Figure 2.1). The outer-most box shows that mathematical literacy occurs in the context of a real-life challenge or problem, described in the framework in terms of the context in which it arises, and the content to which it relates (both described below). The middle box shows that a student needs to draw on mathematics concepts, knowledge and skills to formulate situations mathematically, employ mathematical concepts, facts, procedures and reasoning, and interpret, apply and evaluate mathematical outcomes. The inner box portrays the mathematical modelling cycle described in the PISA framework. This is a simplified rendition of the stages involved in solving mathematical problems in contexts. It begins with a problem situated in a meaningful context. The problem solver formulates the problem drawing on mathematical concepts, in order to make it amenable to mathematical treatment. Following this, the problem solver employs mathematical strategies to obtain mathematical results. The mathematical results are then interpreted and evaluated in terms of the original contextual problem. The extent to which all cycles of the problem-solving process are deployed will depend on the nature of the problem to be solved, and some problems may involve only parts of the cycle.

Table 2.1 provides a breakdown of the distribution of mathematics items in the most recent PISA cycle, PISA 2015. It shows a fairly even distribution across mathematics content areas. However, a greater proportion of items (42%) are described as Employing, compared with Formulating (30%) and Interpreting (28%). Just over 40% of items are multiple-choice, whether simple (selecting one of several options) or complex (responding to a series of Yes/No options). Finally, more items are embedded in societal (38%) and scientific (28%) contexts than in other context types. Whereas PISA

2015 included 69 mathematics items, not all students completed all mathematics items, and some completed none (their scores were estimated or imputed).

Figure 2.1. A model of PISA mathematical literacy in practice



Source: Based on OECD (2016a), Figure 4.1, p. 66

Table 2.1. Distribution of PISA 2015 mathematics items by process, content area, context and item format – number and percent

Component	Number	%	Component	Number	%
Process			Item Format		
Formulating	21	30	Simple Multiple Choice	16	23
Employing	29	42	Complex Multiple Choice	13	19
Interpreting	19	28	Open Response – Human Coded	18	26
Total	69	100	Open Response – Computer-Coded	22	32
			Total	69	100
Content			Context		
Change & Relationships	16	23	Occupational	13	19
Quantity	18	26	Personal	11	16
Space and Shape	17	25	Scientific	19	28
Uncertainty & Data	18	26	Societal	26	38
Total	69	100	Total	69	100

2.1.2. Performance on the Overall PISA Mathematics Scale in 2015

Ireland's mean score of 503.7 on the overall mathematics scale in PISA 2015 was significantly higher than the OECD average of 490.2 (Table 2.2). Ireland ranked 13th out of 35 OECD countries and 18th out of all 70 participating countries/economies. Applying a 95% confidence interval, which takes account of measurement and sampling error, Ireland's true rank in mathematics among the OECD countries was between 10th and 14th and between 15th and 19th among all participating countries/economies.

Singapore significantly outperformed every other participating country/economy in PISA 2015 mathematics, with a mean score of 564.2, and it is among 14 countries/economies that significantly outperformed Ireland. Ireland's mean score did not differ from those of five countries/economies (Belgium, Germany, Poland, Norway and Austria). The remaining 51 countries/economies (19 of which are OECD countries, including Australia, the United Kingdom and the United States) performed significantly less well than Ireland. The mean mathematics score for Northern Ireland (not shown in table) was 493.8 (SE = 4.59, SD = 77.5). This is not significantly different from the mean score for Ireland, nor is it significantly different from the OECD average.

Ireland's standard deviation for mathematics is 79.8, while the OECD average standard deviation is 89.5. This indicates a narrower spread of mathematics achievement in Ireland than on average across OECD countries. Indeed, the spread in Ireland is one of the lowest among OECD countries. Other countries with comparably low standard deviations include Denmark, Estonia, Finland and Norway. The standard deviation for Northern Ireland is also low at 77.5 (SE = 1.95).

The range in mathematics achievement in Ireland (the difference between the 95th and 5th percentiles) is 261.9 points, which is significantly smaller than the corresponding average of 293.3 across OECD countries. This again indicates a narrower range of performance in Ireland.

2.1.3. Trends in Overall Performance on PISA Mathematics Across Cycles

Students in Ireland achieved a mean mathematics score of 502.8 in 2003 and 501.5 in 2006. Performance dropped significantly in 2009, when the mean score was 487.1. There was an improvement again in 2012 (501.5), and the mean score in 2015 was 503.7. Hence, in each cycle except 2009, Ireland's mean score in mathematics ranged between 501.5 and 503.7 (Figure 2.2). Furthermore, while performance in Ireland in 2003, 2006 and 2012 was not significantly different from 2015, performance in 2009 was significantly lower (Shiel et al., 2016b, E-Appendix Table A8.10).

On average across OECD countries, performance dropped significantly from 499.2 in 2003 to 491.4 in 2015, among those countries that participated in PISA 2003 and PISA 2015, and for which valid data were available for both years (ibid, E-Appendix Table A8.10). In 2009, Ireland's mean score was significantly below the corresponding OECD average, while in 2012 and 2015 it was significantly above it.

Figure 2.3 gives the mean score differences on mathematics between 2012 and 2015 for the 30 top-performing countries in PISA 2015, the OECD average difference and the difference for Northern Ireland. Five countries experienced a significant improvement: Sweden (+15.7 points), Norway (+12.4), the Russian Federation (+11.9), Denmark (+11.1), and Slovenia (+8.8). Northern Ireland's score increased by 5.8 score points, but this was not statistically significant.

Eight countries had significant declines in performance between 2012 and 2015 including Korea (-29.7), Hong-Kong (China) (-13.3), Poland (-13.0), the Netherlands (-10.7), Australia (-10.3), and Singapore (-9.3). On average across OECD countries that participated in both PISA 2012 and 2015, there was a non-significant drop in performance (-3.7).

It is noteworthy that the performance of student in Ireland did not change between 2012 and 2015, when performance declined in a number of countries and increased in others. It could be that performance in Ireland would have dropped (due to the transition to computer-based testing) if Project Maths had not been in place. It is also noteworthy that overall performance in Ireland was about the same in 2003 (well before the introduction of Project Maths) and in 2015 (when all students in Ireland taking PISA had studied under Project Maths). This might be interpreted as indicating that performance did not change at all since the introduction of Project Maths. However, as noted, it may be that potential gains in 2015 were, in fact, hidden because of the introduction of computer-based testing.

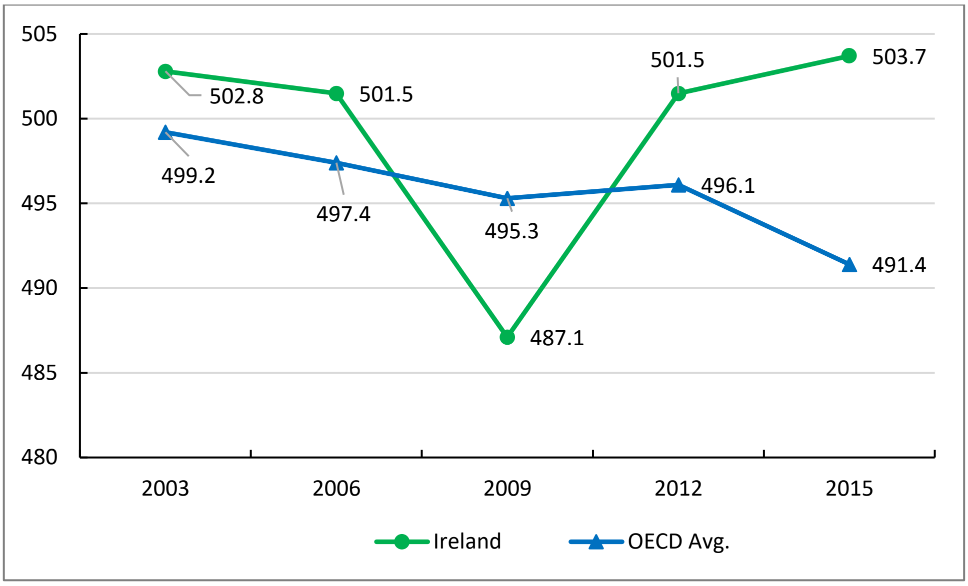
Table 2.2. Mean Scores of top 25 countries on PISA 2015 mathematics and OECD average score

	Mean	SE	SD	SE	IRL
Singapore	564.2	(1.47)	95.4	(0.83)	▲
Hong Kong (C)	547.9	(2.98)	90.1	(1.51)	▲
Macao (China)	543.8	(1.11)	79.9	(1.13)	▲
Chinese Taipei	542.3	(3.03)	102.9	(1.95)	▲
Japan	532.4	(3.00)	88.2	(1.74)	▲
B-S-J-G (China)	531.3	(4.89)	106.0	(2.45)	▲
Korea	524.1	(3.71)	99.7	(1.77)	▲
Switzerland	521.3	(2.92)	95.7	(1.58)	▲
Estonia	519.5	(2.04)	80.4	(1.06)	▲
Canada	515.6	(2.31)	87.7	(1.05)	▲
Netherlands	512.3	(2.21)	91.5	(1.46)	▲
Denmark	511.1	(2.17)	80.6	(1.22)	▲
Finland	511.1	(2.31)	82.2	(1.26)	▲
Slovenia	509.9	(1.26)	87.8	(1.29)	▲
Belgium	507.0	(2.35)	97.4	(1.47)	O
Germany	506.0	(2.89)	89.0	(1.39)	O
Poland	504.5	(2.39)	87.6	(1.67)	O
Ireland	**503.7**	**(2.05)**	**79.8**	**(1.38)**	
Norway	501.7	(2.23)	84.9	(1.10)	O
Austria	496.7	(2.86)	95.1	(1.83)	O
New Zealand	495.2	(2.27)	92.1	(1.31)	▼
Viet Nam	494.5	(4.46)	83.7	(2.71)	▼
Russian Fed.	494.1	(3.11)	83.1	(1.32)	▼
Sweden	493.9	(3.17)	90.1	(1.74)	▼
Australia	493.9	(1.61)	93.1	(1.23)	▼
France	492.9	(2.10)	95.2	(1.53)	▼
United Kingd.	492.5	(2.50)	92.6	(1.36)	▼
Czech Republic	492.3	(2.40)	90.7	(1.74)	▼
Portugal	491.6	(2.49)	95.7	(1.30)	▼
Italy	489.7	(2.85)	93.6	(1.67)	▼
OECD Average	490.2	(0.44)	89.5	(0.26)	▼

(blue)	Significantly above OECD average	▲	Significantly higher than Ireland
(white)	At OECD average	O	Not significantly different from Ireland
--	Significantly below OECD average	▼	Significantly lower than Ireland

OECD countries are in regular font, partner countries/economies are in italics. Argentina, Malaysia and Kazakhstan are omitted, as coverage is too small to ensure comparability (OECD, 2016b). Data for four Argentinian cities are provided.
Shiel et al., (2016a), Table 5.5, p. 90.

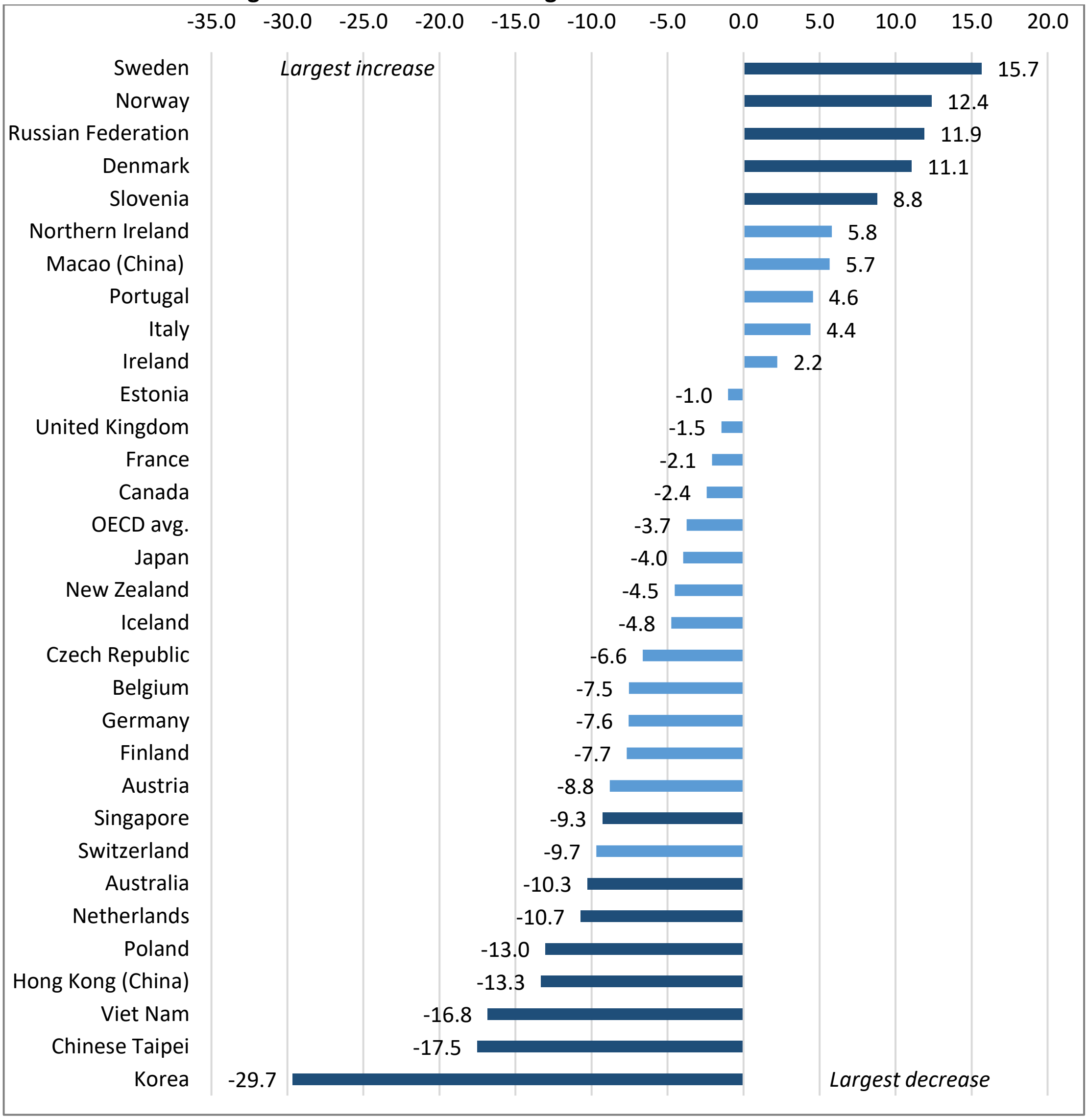
Figure 2.2. Mean scores on overall mathematics scale in Ireland and on average across OECD countries, 2003-2015



OECD data are based on countries that participated in PISA 2003 and subsequent cycles, except for 2009, which draws on the value for countries in PISA 2009 and 2015.

In PISA 2012, it was possible to compare the performance of students in Ireland who were in initial Project Maths schools, and those in non-initial schools (most of whom had not studied under PM). The mean score of students in initial school was 505.3, while the mean score of students in non-PM schools was 501.3. The difference was not statistically significant (Merriman et al., 2014).

Figure 2.3. Mean score difference in mathematics between 2012 and 2015 for the top 30 performing countries/economies on PISA 2015 mathematics that participated in PISA 2012 and PISA 2015, the OECD average difference and the average difference in Northern Ireland



Source: Shiel et al., (2016a), Figure 8.12, p. 155. Significant differences are in bold colour.

In Ireland, the standard deviation fell significantly from 85.3 in 2003 to 79.8 in 2015 (Table 2.3). The average standard deviation across OECD countries also fell significantly between these two years, from 93.4 to 89.4. The inter-decile range (the difference between the 10th and 90th percentiles) in Ireland and on average across OECD countries also fell between 2003 and 2015, by 14.6 and 9.6 points respectively, again indicating a narrowing in achievement. Importantly, in Ireland and on average across OECD countries, standard deviations and inter-decile ranges narrowed significantly between 2012 and 2015.

Table 2.3. Variation in mathematics in Ireland and on average across OECD countries, 2009-2015

	Ireland				OECD			
	Standard deviation		Inter-decile range (90th minus 10th percentile)		Standard deviation		Inter-decile range (90th minus 10th percentile)	
	SD	SE	Score diff.	SED	SD	SE	Score diff.	SED
2003	85.3	(1.26)	220.8	(4.21)	93.4	(0.35)	241.4	(1.05)
2006	82.0	(1.50)	211.6	(4.64)	91.3	(0.37)	234.9	(1.05)
2009	85.6	(1.59)	214.5	(4.58)	91.2	(0.30)	235.5	(0.91)
2012	84.6	(1.26)	218.8	(3.63)	91.7	(0.29)	238.2	(0.91)
2015	79.8	(1.38)	206.3	(4.23)	89.4	(0.28)	231.8	(0.86)
	Diff	SE	Diff	SED	Diff	SE	Diff	SED
2015-2003	**-5.5**	(1.86)	**-14.6**	(5.97)	**-4.0**	(0.45)	**-9.6**	(1.36)
2015-2006	-2.20	(2.04)	-5.3	(6.28)	-1.80	(0.41)	**-3.7**	(1.25)
2015-2009	**5.8**	(2.11)	8.2	(6.23)	-3.80	(1.61)	**-4.0**	(1.21)
2015-2012	**-4.8**	(1.87)	**-12.5**	(5.57)	**-2.3**	(0.40)	**-6.4**	(1.25)

Significant differences in bold. OECD data based on countries that participated in PISA 2003 and subsequent cycles, except 2009, which draws on the value for countries in 2009 and 2015. SE = standard error; SED = standard error of the difference.

2.1.4. Performance on Mathematics Proficiency Levels in PISA 2015

The six proficiency levels used in the PISA 2015 mathematics assessment are the same as those established for the PISA 2012 assessment, when mathematics was the major area of assessment. These range from Level 1 to Level 6, as well as a 'Below Level 1' category (see Table 2.4 for a detailed description of each level). Students performing at Level 1 can answer the most basic PISA mathematics questions, when those questions are in familiar contexts, with all relevant information present, and when the questions call on routine procedures, which are always obvious. Level 6, in contrast, requires students to model complex mathematical problem situations, often presented in non-standard contexts, and to apply their understanding of symbolic and formal mathematical operations and relationships to the most difficult PISA mathematics items. Students performing below Level 1 can complete very direct and straightforward mathematical tasks, typically involving whole numbers and well-defined instructions. Table 2.4 also shows the percentage of students in Ireland and on average across OECD countries performing at that level.

Table 2.4. Summary description of the six levels of proficiency on the mathematics scale and percentages of students achieving each level, in Ireland and on average across OECD countries

Level (Cut-point)	Students at this level are capable of:	Ireland		OECD Avg	
		%	SE	%	SE
6 (669 and above)	Conceptualising, generalising and using information based on their investigations and modelling of complex problem situations; using knowledge in relatively non-standard contexts; linking different information sources and representations and moving flexibly among them; applying their insight and understanding, along with mastery of symbolic and formal mathematical operations and relationships, to develop new approaches and strategies for addressing novel situations; reflecting on their actions and formulating and precisely communicating their actions and reflections regarding their findings, interpretations and arguments, and explaining why they were applied to the original situation. Students at this level are able to successfully complete the most difficult PISA items.	1.5	(0.2)	2.3	(0.1)
5 (607 to less than 669)	Developing and working with models of complex situations, including identifying constraints and specifying assumptions; selecting, comparing and evaluating appropriate problem-solving strategies for dealing with complex problems related to these models; working strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterisations and insights pertaining to these situations; beginning to reflect on their work and formulating and communicating their interpretations and reasoning.	8.3	(0.5)	8.4	(0.1)
4 (545 to less than 607)	Working effectively with explicit models of complex, concrete situations that may involve constraints or making assumptions; selecting and integrating different representations (including symbolic representations) and linking them directly to aspects of real-world situations; using their limited range of skills and reasoning with some insight in straightforward contexts; constructing and communicating explanations and arguments based on their interpretations, arguments and actions.	21.2	(0.7)	18.6	(0.1)
3 (482 to less than 545)	Executing clearly described procedures, including those that require sequential decisions; making sufficiently sound interpretations as a base for building a simple model or for selecting and applying simple problem-solving strategies; interpreting and using representations based on different information sources and reasoning directly from them; showing some ability to handle percentages, fractions and decimal numbers, and to work with proportional relationships; engaging in basic interpretation and reasoning	30.0	(0.9)	24.8	(0.2)
2 (420 to less than 482)	Interpreting and recognising situations in contexts that require no more than direct inference; extracting relevant information from a single source and making use of a single representational mode; employing basic algorithms, formulae, procedures or conventions to solve problems involving whole numbers; making literal interpretations of the results.	24.1	(0.9)	22.5	(0.2)
1 (358 to less than 420)	Answering questions involving familiar contexts where all relevant information is present and the questions are clearly defined; identifying information and carrying out routine procedures according to direct instructions in explicit situations; performing actions that are almost always obvious and follow immediately from the given stimuli.	11.5	(0.7)	14.9	(0.1)
Below Level 1 (less than 358)	Performing very direct and straightforward mathematical tasks, such as reading a single value from a well-labelled chart or table where the labels on the chart match the words in the stimulus and question, so that the selection criteria are clear and the relationship between the chart and the aspects of the contexts depicted are evident; performing arithmetic calculations with whole numbers by following clear and well-defined instructions.	3.5	(0.5)	8.5	(0.1)

Source: OECD (2016b), Figure I.5.7.

In Ireland, 15.0% of students performed below Level 2 on PISA overall mathematics (indicating weak performance), compared with 23.4% on average across OECD countries (Figure 2.4). Indeed, only a small number of countries (Singapore with 7.6%, Japan with 10.7% and Estonia with 11.2%) have fewer students than Ireland performing below Level 2. On the other hand, 9.8% of students in Ireland performed at Levels 5-6 (indicating strong performance in mathematics), compared with an OECD average of 10.7% (Figure 2.5). In Singapore (the country with the highest average performance on PISA), 34.8% performed at Levels 5-6.

Figure 2.4. Percentages of students performing below Proficiency Level 2 on the mathematics scale in Ireland, in selected comparison countries, and on average across OECD and EU countries

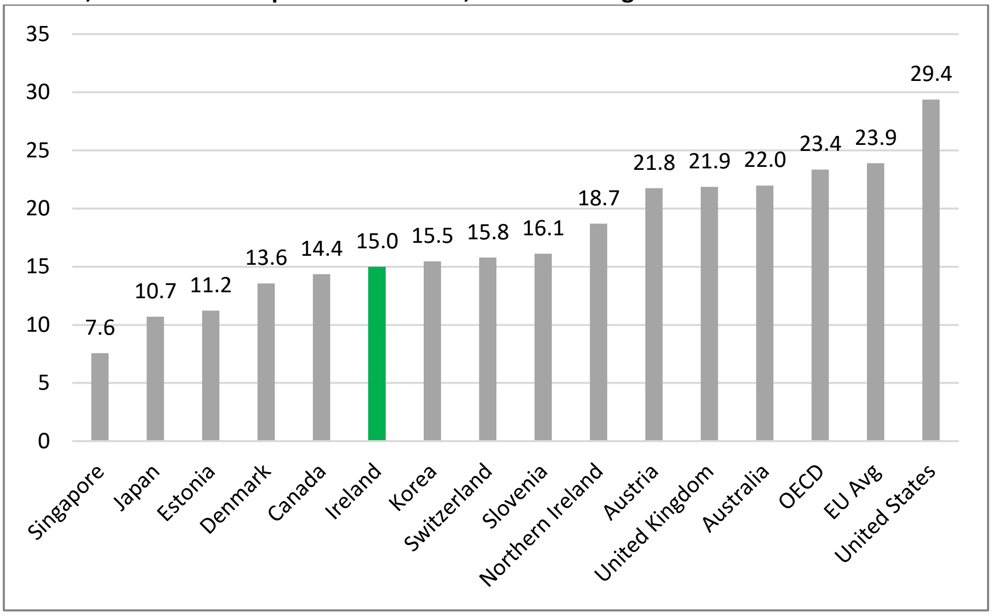
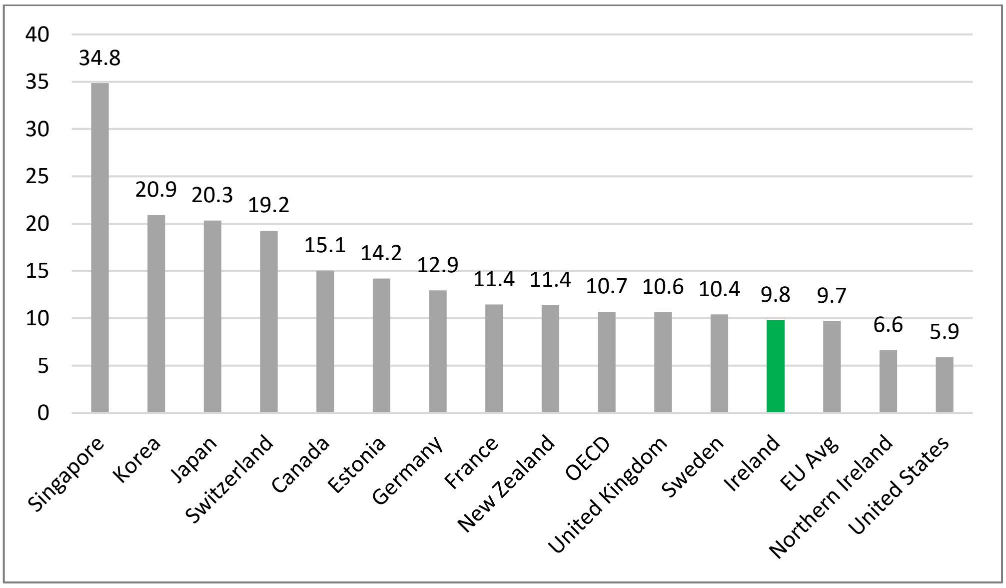


Figure 2.5. Percentages of students performing at or above Proficiency Level 5 on the PISA 2015 overall mathematics scale in Ireland, in selected comparison countries, and on average across OECD and EU countries



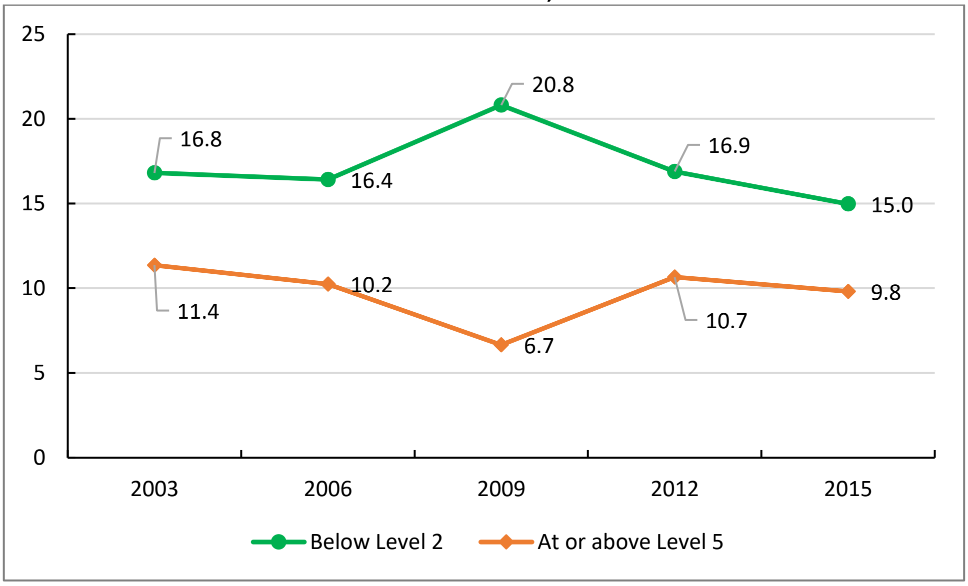
Other countries with higher proportions of students than Ireland performing at Levels 5-6 include Korea (20.9%), Japan (20.3%), Switzerland (19.2%) and Canada (15.1%). All of these countries also have higher average performance on PISA mathematics than Ireland. Hence, in general, lower-performing students in Ireland do well relative to their counterparts in other PISA countries, while higher-performing students do less well. However, Ireland is unique in having a mean score that is significantly above the OECD average, while having about the same proportion of students performing at Levels 5-6 as the corresponding OECD average.

2.1.5. Trends in Performance on Mathematics Proficiency Levels

In Ireland, 16.8% of students performed below Proficiency Level 2 in mathematics in 2003, and roughly equivalent percentages of students performed at this level in subsequent cycles, except 2009, when 20.8% performed below Level 2 (Figure 2.6). In 2015, 15% of students in Ireland performed below Level 2, but this was not statistically significantly different from 2003 (16.8%), or indeed 2012 (16.9%). On average across OECD countries, 21.6% performed below Level 2 in 2003, and this rose to 22.9% in 2015 – the highest percentage across PISA cycles, but not significantly different from any of the earlier cycles (see Shiel et al., 2016b, E-Appendix Table A8.11).

In 2003, 11.4% in Ireland performed at or above Level 5, and roughly equivalent percentages performed at this level in subsequent cycles, except in 2009 when 6.7% performed at or above Level 5 (Figure 2.5). In 2015, 9.8% in Ireland performed at or above Level 5, and this percentage was not significantly different from 2003, or indeed 2012 (10.7%). On average across OECD countries, the percentage performing at or above Level 5 dropped from 14.4% in 2003 to 10.8% in 2015. Moreover, a significantly lower percentage of students on average across OECD countries performed at or above Level 5 in 2015, compared with each previous cycle (Shiel et al.,2016b, E-Appendix Table A8.11).

Figure 2.6. Percentage of students below Proficiency Level 2 and at or above Proficiency Level 5 on overall mathematics in Ireland, 2003-2015



2.1.6. Performance on PISA Mathematics Content Areas – PISA 2012

Since PISA was a minor assessment domain in 2015, data are not available for performance by content area. Hence, this section summarises performance by content area in 2012. It will be recalled that only students in Second and Fifth years (about 20% of PISA participants in Ireland in 2012) had studied under Project Maths. Nevertheless, the fact that Ireland's overall mean scores were broadly similar in PISA 2012 and 2015 suggests that large changes have not occurred in performance across content areas.

The four PISA content areas (Change & Relationships; Space & Shape; Quantity; and Uncertainty & Data) relate to broad parts of the mathematics curriculum found in all countries and economies and as such will reflect differences in course content and curriculum priorities available to 15-year-olds (OECD, 2013b). Tasks on the Change & Relationship subscale involve understanding types of change and recognising when they occur in order to use suitable mathematical models to describe and predict change. The Space & Shape subscale entails understanding perspective, creating and reading maps, transforming shapes with and without technology, interpreting views of three-dimensional scenes from various perspectives, and constructing representations of shapes. The Quantity subscale involves the application of knowledge of number and number operations in a wide variety of settings, while the Uncertainty & Data subscale includes knowledge of variation in processes, uncertainty and error in measurement, and chance. In all PISA cycles, the PISA mathematics test items are split almost evenly across the four content areas; therefore approximately 25% of items address each content area.

The mean scores for students in Ireland and the corresponding OECD average scores for the overall print mathematics scale and the four content area subscales in PISA 2012 are presented in Figure 2.7. In Ireland, performance on the Change & Relationships subscale (501.1) was similar to performance on the overall print mathematics scale (501.5), while performance on the Quantity (505.2) and Uncertainty & Data (508.7) subscales was marginally higher. The mean score for Ireland on the Space & Shape subscale (477.8) was considerably lower than the overall mean print mathematics score. Students in Ireland had significantly higher mean scores on the Change & Relationships, Quantity and Uncertainty & Data subscales compared to the corresponding OECD average scores; however, they performed significantly less well on the Space & Shape subscale.

With the exception of the Space & Shape subscale, there was little variation in the rankings of Ireland's performance in 2012. Among OECD countries, Ireland's performance ranked 10th on the Uncertainty & Data subscale, 12th on the Quantity subscale, 13th on the Change & Relationships subscale and 24th on the Space & Shape subscale.

Figure 2.7. Mean scores of students in Ireland and on average across OECD countries on the PISA overall mathematics scale and on content subscales – 2012

The performance of higher-achieving students (those scoring at the 90th percentile) and lower-achieving students (those scoring at the 10th percentile) can also be considered. Table 2.6 shows the scores of students in Ireland at these markers on each mathematics content area in PISA 2012. The data show that students in Ireland performing at the 10th percentile exceeded the corresponding OECD average on Change & Relationships, Quantity and Uncertainty & Data. At the 90th percentile, students in Ireland had scores that were significantly lower than the corresponding OECD averages on

Change & Relationships and Space & Shape, and a score that was significantly higher on Uncertainty & Data.

Table 2.6: Scores (standard errors) of students in Ireland and on average across OECD countries at the 10th and 90th percentiles on overall mathematics, and on the PISA 2012 content subscales

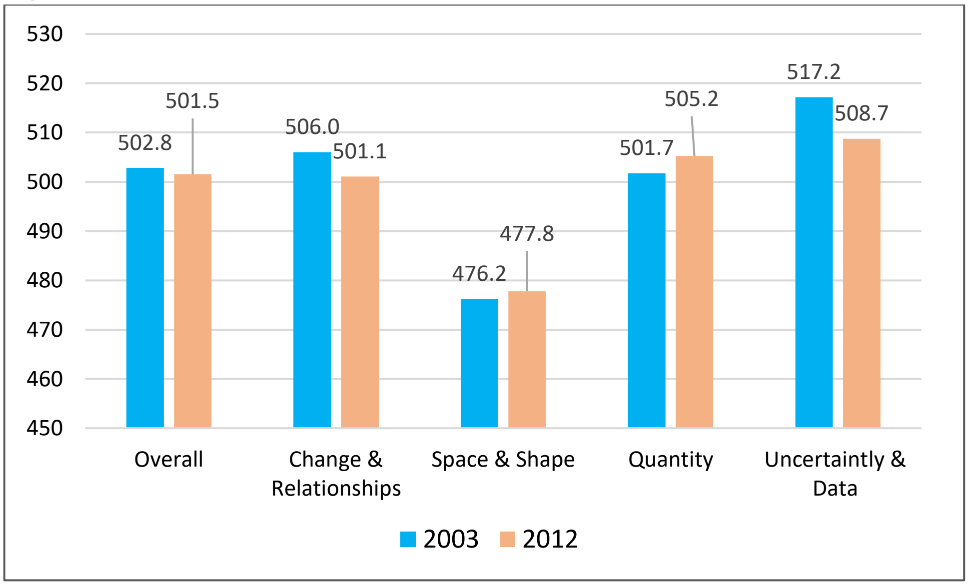
	10th percentile		90th percentile	
	Ireland	OECD Avg.	Ireland	OECD Avg.
Overall	**391.0** (3.63)	375.0 (0.68)	609.8 (2.46)	613.6 (0.71)
Change & Relationships	**389.2** (4.85)	362.2 (0.82)	**612.9** (2.47)	622.5 (0.77)
Space & Shape	356.5 (4.18)	364.8 (0.71)	**598.4** (2.77)	617.9 (0.84)
Quantity	**385.5** (4.57)	369.1 (0.76)	623.5 (3.11)	620.4 (0.70)
Uncertainty & Data	**395.0** (4.36)	372.5 (0.71)	**619.4** (2.46)	613.2 (0.70)

Scores that are significantly different from the corresponding OECD average score are in bold.

2.1.7. Trends in Performance on PISA Mathematics Content Areas

Trend data for PISA mathematics content areas are available for two cycles only – 2003 and 2012. Although performance is highest on the Uncertainty & Data subscale in both cycles, students in Ireland performed significantly less well on this subscale in 2012 than in 2003 (508.7 versus 517.2; Perkins, Shiel, Merriman, Cosgrove, & Moran et al., 2013b; E-appendix Table A7.3). Although performance was higher on each of two remaining content areas in 2012 compared with 2003 (Quantity and Space & Shape), and lower on one (Change & Relationships), none of the changes was statistically significant. There has also been little change in the position of Ireland's mean scores on the content area subscales relative to the corresponding OECD average scores: students in Ireland had significantly higher mean scores on the Change & Relationships and Uncertainty & Data subscales compared to the corresponding OECD average scores, but had significantly lower mean scores on the Space & Shape subscale in both cycles. The mean score of students in Ireland on the Quantity subscale was not significantly different from the OECD average in 2003, but was significantly above it in 2012.

Figure 2.8. Mean scores on PISA mathematics content areas – Ireland – 2003 and 2012



Although scaled scores on PISA mathematics content areas were not issued in 2015, it is possible to monitor performance across content areas with reference to percent correct scores on item clusters. Table 2.7 provides percent correct scores by content area for 2012 and 2015 for 69 items presented in both years (on paper in 2012 and on computer in 2015).

Table 2.7. Percent correct scores on PISA mathematics content areas in 2012 and 2015 – Ireland and OECD average (based on 69 common items)

	Ireland (% Correct)	OECD Average (% Correct)
Change & Relationships		
2012	47.3	45.4
2015	47.9	42.9
Difference (2015-2012)	0.6	-2.5
Space & Shape		
2012	34.4	37.3
2015	32.4	32.0
Difference	-2.0	-5.3
Quantity		
2012	60.7	59.2
2015	57.3	52.8
Difference	-3.4	-6.5
Uncertainty & Data		
2012	51.1	47.7
2015	51.6	44.9
Difference	0.5	-2.8
All 69 trend items		
2012	48.6	47.6
2015	47.5	43.3
Difference	-1.1	-4.3

The data in Table 2.10 also demonstrate the relatively difficulty of Space & Shape questions for students in Ireland, and on average across OECD countries, with average percent correct scores ranging from the mid to low 30s. On the other hand, Quantity is relatively easier for students, with average scores in Ireland ranging from 57% to 61%, and from 53% to 58% on average across OECD countries.

It can be concluded that, in broad terms, the profile of strengths and weaknesses on PISA mathematics content areas (based on scaled scores) in 2012 held in 2015, notwithstanding small differences that may have arisen due to the transition to computer-based testing.

2.1.8. Performance on Mathematics Process Subscales – PISA 2012

Three mathematical processes were assessed in PISA 2012: Formulating situations mathematically; Employing mathematical concepts, facts, procedures and reasoning; and Interpreting, applying and evaluating mathematical outcomes. Formulating situations mathematically involves recognising an opportunity to use mathematics in a real-world context, and translating the problem into formal mathematical language. The Employing process involves the application of mathematical concepts, facts, procedures and reasoning to mathematically-formulated problems, to obtain mathematical results, while the Interpreting process involves translating mathematical solutions back into the original problem context and evaluating whether the solution makes sense.

Each mathematical item was classified according to the prevalent process required to solve the problem. Just under 30% of mathematics items assess the Formulating process, while approximately 45% assess the Employing process and about a quarter of items assess the Interpreting process.

Figure 2.9 presents the mean scores for students in Ireland and the corresponding OECD average scores for the overall mathematics scale and each of the three process subscales for print mathematics in 2012. Students in Ireland performed best on the Interpreting subscale, obtaining a mean score of 506.8, which is significantly above the corresponding OECD average score (497.0). Students in Ireland also had a significantly higher mean score on the Employing subscale (502.3) compared to the corresponding OECD average (493.4), while the performance of students in Ireland on the Formulating subscale (492.4) did not differ significantly from the average across OECD countries (491.6).

Figure 2.9. Mean scores of students in Ireland and on average across OECD countries on PISA overall mathematics scale and process subscales – 2012

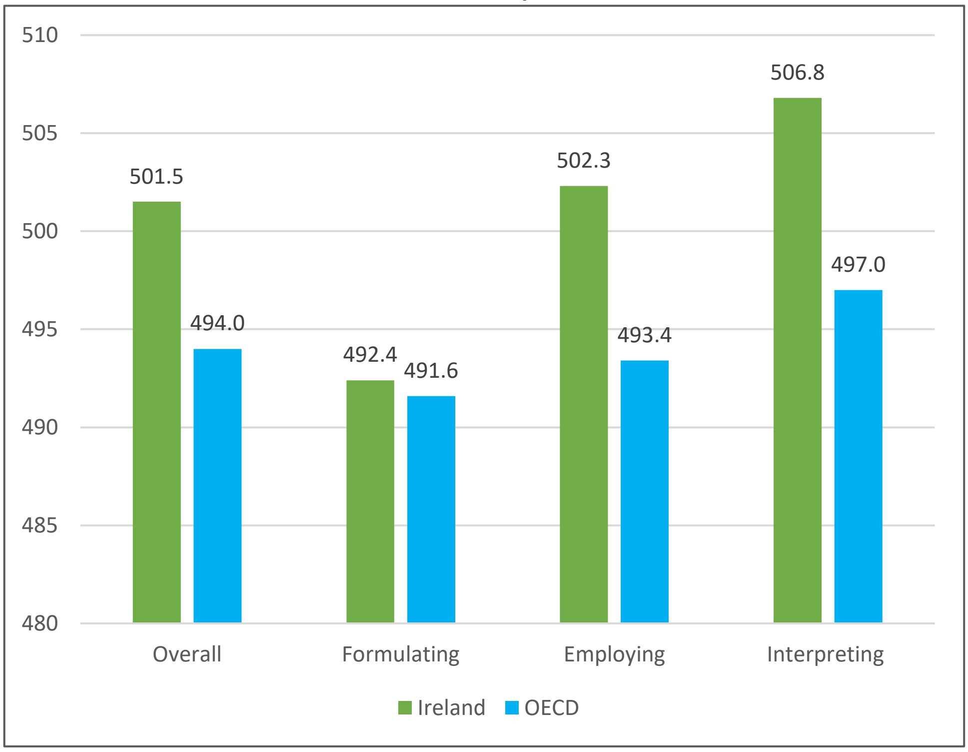


Table 2.10 provides data on the performance of higher achievers (those scoring at the 90th percentile) and low achievers (those scoring at the 10th percentile) on the PISA 2012 mathematics process subscales. Students scoring the 10th percentile in Ireland achieved significantly higher scores on Employing and Interpreting than the corresponding average scores across OECD countries, while students scoring the 90th percentile achieved a significantly lower score on Formulating, and marginally lower scores on Employing and Interpreting (Table 2.8).

Table 2.8. Scores (standard errors) of students in Ireland and on average across OECD countries at the 10th and 90th percentiles on overall mathematics, and on the PISA 2012 process subscales

	10th percentile		90th percentile	
	Ireland	OECD Avg.	Ireland	OECD Avg.
Overall	**391.0** (3.63)	375.0 (0.68)	609.8 (2.46)	613.6 (0.71)
Formulate	369.4 (4.41)	361.7 (0.75)	**614.7** (3.06)	623.8 (0.85)
Employ	**393.7** (4.62)	375.5 (0.69)	608.8 (2.99)	611.1 (0.69)
Interpret	**389.0** (4.58)	373.0 (0.73)	621.7 (2.54)	622.4 (0.70)

Scores that are significantly different from the corresponding OECD average score are in bold.

2.1.9. Trends in Performance on Mathematics Process Subscales

It is not possible to report on trends in performance on the mathematics process subscales. That is because the PISA mathematics process categories changed between 2003 and 2012. In PISA 2003, the PISA process categories were described as:

- The ***Reproduction Cluster*** – reproducing representations, facts and definitions; interpreting simple, familiar representations; performing routine computations; and solving routine problems.
- The ***Connections Cluster*** – integrating and connecting across content, situations and representations; non-routine problem solving and translation; interpretation of problem situations and mathematical statements using multiple well-defined methods; and engaging in simple mathematical reasoning.
- The ***Reflection Cluster*** – complex problem solving and posing, reflecting on and gaining insight into mathematics; constructing original mathematical approaches; communicating complex arguments and complex reasoning; using multiple complex methods; and making generalisations.

The 2003 competency clusters were set aside in 2012, and replaced by the process categories of Formulating, Employing and Interpreting (see above). Moreover, the OECD did not scale performance by mathematics cluster in 2003. Instead, the categorisation was used to describe item properties.

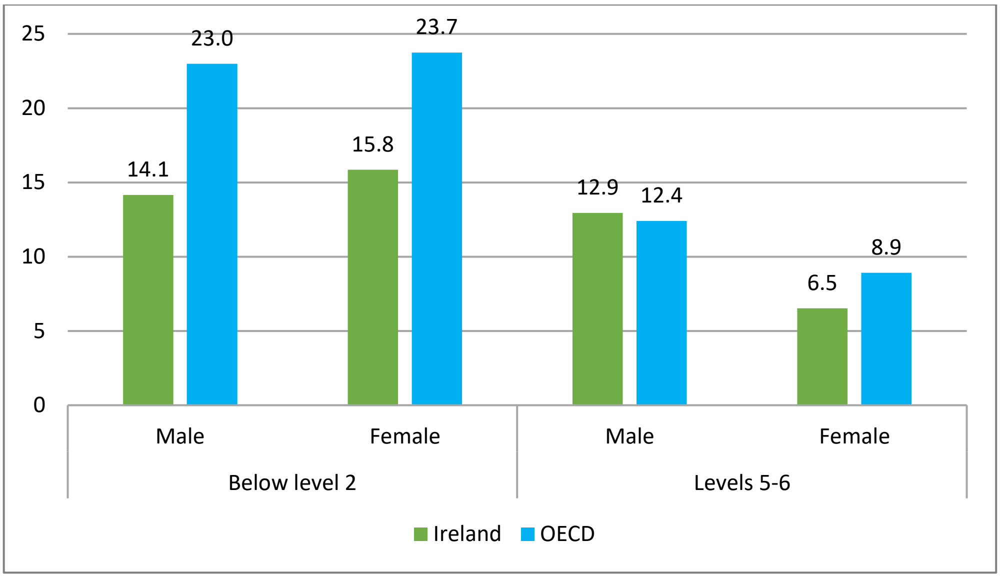
2.1.10. Gender Differences on PISA 2015 Mathematics

In PISA 2015, male students in Ireland achieved a mean score of 511.6 on the overall mathematics scale, while females achieved a mean score of 495.4. The difference, 16.1 score points in favour of male students, is statistically significant (see Shiel et al., 2016b, Table 5.8). The average difference in favour of male students across OECD countries was 7.9 score points, and this was also statistically significant. Other countries with relatively large gender differences (all in favour of male students) were Germany (16.6 points), Japan (13.8), Switzerland (12.0), and the United Kingdom (11.6). In Korea, Singapore and Sweden, female students had marginally higher mean scores than males, but differences were not statistically significant.

Differences can also be interpreted with respect to proficiency levels. In 2015, the proportions of male and female students achieving below Level 2 were broadly equivalent (14.1% of males, 15.8% of females) (Figure 2.10), and were lower than on average across OECD countries, where the respective proportions are also broadly similar (23.0% and 23.7% respectively). In Ireland, 12.9% of male students, and 6.5% of female students performed at Levels 5-6 in 2015. On average across OECD countries, there are also more male (12.4%) than female (8.9%) students performing at these levels.

Hence, equivalent proportions of male students in Ireland and on average across OECD countries performed at Levels 5-6, while fewer females in Ireland than on average across OECD countries performed at these levels.

Figure 2.10. Percentages of male and female students achieving below Proficiency level 2 and at or above Proficiency level 5 on the PISA 2015 overall mathematics scale, in Ireland and on average across OECD countries



Gender differences were also observed on PISA mathematics content areas when they were assessed in 2012. In Ireland and on average across OECD countries, male students outperformed females on all four content subscales. The largest difference in Ireland was in Space & Shape, where males outperformed females by 24.8 score points. The differences were broadly similar for Change & Relationships (13.4), Quantity (14.1) and Uncertainty & Data (14.2). On average across OECD countries, differences (again, all in favour of males) were smaller than in Ireland – Shape and Space (15.3), Quantity (10.8), Change & Relationships (10.6) and Uncertainty & Data (8.7).

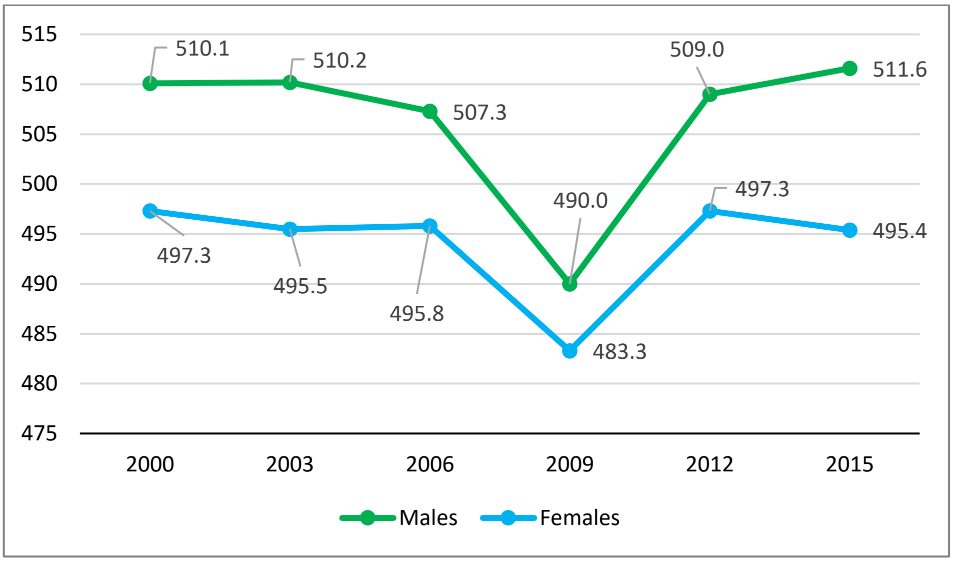
In Ireland and on average across OECD countries in PISA 2012, male students also outperformed females on the three mathematics processes subscales. In Ireland, the largest difference was on Formulating (20.4), with smaller differences on Interpreting (17.1) and Employing (12.8). Again, differences in favour of males were smaller on average across OECD countries on all three subscales – Formulating (15.7), Employing (9.4), and Interpreting (9.3).

2.1.11. Trends in Gender Differences on PISA Mathematics

In all but one cycle since PISA 2000, male students in Ireland achieved significantly higher overall mathematics scores than females (Figure 2.11). The difference of 6.7 score points in 2009 was not statistically significant. As per Figure 2.9, the gender difference was largest in 2015 (16.1 points).

There was some variation in the mean scores of male and female students in Ireland on the content area subscales between 2003 and 2012. For example, both male and female students had lower mean scores on the Change & Relationships and Uncertainty & Data subscales in 2012 compared to 2003, while males had a higher mean score on the Quantity subscale in 2012 (512.1) compared with 2003 (506.1). However, none of these differences was statistically significant.

Figure 2.11. Mean scores of male and female students on PISA overall mathematics in Ireland, 2000-2015



2.2. TIMSS

TIMSS, the Trends in International Maths and Science Study, is implemented by the International Association for the Evaluation of Educational Achievement (IEA) on a four-year cycle, with the most recent assessment implemented in 2015. Students in Grade 8 (Second Year) in Ireland participated in TIMSS in 1995 and in 2015, while those in Grade 4 (Fourth class) participated in 1995, 2011, and 2015. The focus of this section is on the performance of students in Second year.

2.2.1. TIMSS Mathematics Assessment Framework

The TIMSS mathematics framework has two dimensions: a content dimension specifying the subject matter to be assessed, and a cognitive dimension, specifying the thinking processes to be assessed.

Content Domains

At Grade 8 (Second year in Ireland), 50% of items are attributed to Number, 30% to Algebra, 20% to Geometry and 20% to Data & Chance.

The Number content area consists of three broad topics: whole numbers; fractions, decimals and integers (rational numbers); and ratio, proportion and percent. Examples of aspects of Number that are assessed include:

- Demonstrate understanding of whole numbers and operations (e.g., the four arithmetic operations; place value; and the commutative, associative, and distributive properties).
- Find and use multiples or factors of numbers, identify prime numbers, and evaluate powers of numbers and square roots of perfect squares up to 144.
- Identify, compare, or order rational numbers (fractions, decimals, and integers) using various models and representations (e.g., number line); and know that there are numbers that are not rational.
- Identify and find equivalent ratios; and model a given situation by using a ratio and divide a quantity in a given ratio.
- Convert among percentages, proportions, and fractions.

Three key topics are identified in Algebra: expressions and operations; equations and inequalities; and relationships and functions. TIMSS expects students to solve real-world problems using algebraic models and explain relationships involving algebraic concepts. Students are expected to go beyond memorisation to understand that when there is a formula about two quantities, and one is known, the other can be calculated. This conceptual understanding is extended to linear equations for calculations about things that expand at constant rates (e.g., slope) and quadratic expressions to study motion such as the paths of traveling objects. Functions are studied to find out what will happen to a variable over time, including when the variable will reach its highest or lowest value. Examples of aspects of Algebra that are assessed include:

- Simplify algebraic expressions involving sums, products, and powers of expressions; and compare expressions to determine if they are equivalent.
- Use expressions to represent problem situations
- Solve linear equations, linear inequalities, and simultaneous linear equations in two variables
- Generalize pattern relationships in a sequence, or between adjacent terms, or between the sequence number of the term and the term, using numbers, words, or algebraic expressions.
- Identify functions as linear or non-linear; contrast properties of functions from tables, graphs, or equations; and interpret the meanings of slope and y-intercept in linear functions.

In Geometry, TIMSS expects students to analyse the properties and characteristics of a variety of two- and three-dimensional figures and be competent in geometric measurement (perimeters, areas, and volumes). They are also expected to solve problems and provide explanations based on geometric relationships. Three topics in Geometry are assessed: geometric shapes; geometric measurement; and location and movement. Specific aspects of Geometry that are assessed include:

- Identify geometric properties of two- and three-dimensional shapes, including line and rotational symmetry.
- Relate three-dimensional shapes with their two-dimensional representations (e.g., nets, two-dimensional views of three-dimensional objects).
- Use geometric properties, including the Pythagorean Theorem, to solve problems.
- Select and use appropriate measurement formulas for perimeters, circumferences, areas, surface areas, and volumes; and find measures of compound areas.
- Locate points and solve problems involving points in the Cartesian plane
- Recognise and use geometric transformations (translation, reflection, and rotation) of two-dimensional shapes.

In Data & Chance, TIMSS expects students to be able to read and extract the important meaning from a variety of visual displays and to be familiar with the statistics underlying data distributions (e.g., mean, median, mode, and spread) and how these relate to the shape of data graphs. Students are also expected to understand how creators of charts and graphics can misrepresent the truth. Students are expected to have an initial grasp of some concepts related to probability. Key topics in this content area include: Characteristics of data sets; data interpretation; and chance. Specific aspects of Data & Chance that are assessed include:

- Identify and compare characteristics of data sets including mean, median, mode, range, and shape of distributions (in general terms).

- Use and interpret data sets to solve problems (e.g., make inferences, draw conclusions, and estimate values between and beyond given data points).
- Given a process designed to be random, determine the probabilities of possible outcomes.

Cognitive Domains/Processes

Thirty-five percent of items are attributed to the cognitive process of Knowing, 40% to Applying and 25% to Reasoning. Hence, about two-thirds of items require students to use applying and reasoning skills.

The Knowing domain includes procedures for solving problems, especially those encountered frequently in everyday life. According to TIMSS, students need to be efficient and accurate in using a variety of computational procedures and tools, and to see that particular procedures can be used to solve entire classes of problems, rather than just individual problems. Key elements are:

- Recall definitions, terminology, number properties, units of measurement, geometric properties, and notation
- Recognise numbers, expressions, quantities, and shapes
- Recognise entities that are mathematically equivalent
- Classify/order numbers, expressions, quantities, and shapes by common properties
- Compute – carry out algorithmic procedures for +, –, ×, ÷, or a combination of these with whole numbers, fractions, decimals, and integers. Carry out straightforward algebraic procedures
- Retrieve information from graphs, tables, texts, or other sources
- Measure – use measuring instruments, and choose appropriate units of measurement

The Applying domain involves the application of mathematics in a range of contexts. In some items aligned with this domain, students need to apply mathematical knowledge of facts, skills, and procedures or understanding of mathematical concepts to create representations. Problem solving is deemed to be central to the Applying domain, with an emphasis on more familiar and routine tasks. Problems may be set in real-life situations, or in purely mathematical contexts, such as numeric or algebraic expressions, functions, equations, geometric figures, or statistical data sets. Key elements of Applying include:

- Determine efficient/appropriate operations, strategies, and tools for solving problems for which there are commonly used methods of solution
- Represent/model – display data in tables or graphs; create equations, inequalities, geometric figures, or diagrams that model problem situations; and generate equivalent representations for a given mathematical entity or relationship
- Implement strategies and operations to solve problems involving familiar mathematical concepts and procedures.

The Reasoning domain, according to TIMSS, involves logical, systematic thinking. It includes intuitive and inductive reasoning based on patterns and regularities that can be used to arrive at solutions to problems set in novel or unfamiliar situations, including purely mathematical problems and those that have real-life settings. Both of these problem types are viewed as involving transfer of knowledge and skills to new situations. Key elements of Reasoning include:

- Analyse - determine, describe, or use relationships among numbers, expressions, quantities, and shapes

- Integrate/synthesise - link different elements of knowledge, related representations, and procedures to solve problems
- Evaluate alternative problem-solving strategies and solutions.
- Draw conclusions – make valid inferences on the basis of information and evidence
- Generalise – make statements that represent relationships in more general and more widely applicable terms
- Justify - provide mathematical arguments to support a strategy or solution.

All TIMSS items are categorised by content domain and process. Performance is reported on an overall mathematics scale, as well as subscales corresponding to each content area and process.

2.2.2. Performance on the Overall TIMSS Mathematics Scale in 2015

Students in Ireland achieved a mean score of 523.5 on the TIMSS 2015 overall mathematics scale (Table 2.19). Six countries (Singapore, Korea, Chinese Taipei, Hong Kong SAR, Japan and the Russian Federation) achieved significantly higher mean scores. Ireland was placed in a group of six countries (including Kazakhstan, Canada, the United States, England and Hungary) whose mean scores were not significantly different from one another. The remaining 13 countries in TIMSS 2015 had significantly lower mean scores than Ireland.

Ireland's mean score was significantly higher than the TIMSS international average score of 481.6. It was also significantly higher than the mean score of the 16 participating OECD countries (513.2) (95% CI around the mean score difference = 4.7 to 15.9).

It is noteworthy that the standard deviation in Ireland (73.9 score points) is well below the corresponding OECD-16 average of 83.4 and the international average of 85.1. This points to a relative narrow range in achievement in Ireland. Other countries in this situation include Canada (69.8) and Slovenia (69.2). As noted earlier, a narrow standard deviation can be interpreted a pointing to greater equity in learning outcomes.

2.2.3. Trends in Performance on TIMSS Mathematics at Grade 8

Students in Grade 8 in Ireland participated in one previous cycle of TIMSS – in 1995. It is possible to compare performance between 1995 and 2015 because TIMSS reports performance on the same underlying scale, even though most, if not all, of the 1995 items had been retired by 2015. The mean score of students in Ireland in 1995 was 518.9 (SE = 4.83). While Ireland's mean score was marginally higher in 2015, at 523.5 (SE = 2.73), the difference is not significantly different (95% CI around the mean score difference = -16.5 to 5.5).

2.2.4. Performance on TIMSS 2015 Mathematics Benchmarks

Similar to proficiency levels in PISA, TIMSS reports the percentages of students scoring at different benchmarks on overall mathematics. In Ireland, 6.8% of students achieved at the Advanced TIMSS benchmark (Table 2.10, Figure 2.9), compared with an average of 10.7% among participating OECD countries. Students performing at the High benchmark can be described as those who can apply and reason in a variety of problem situations. In Ireland, 31.5% performed at the High benchmark, indicating that they can apply their understanding and knowledge in a variety of relatively complex mathematical situations. On average across participating OECD countries in TIMSS 2015, 24.5% performed at this benchmark. In Ireland, 37.5% performed at the Intermediate benchmark, indicating that they can apply basic mathematical knowledge in a variety of situations. The corresponding estimate across participating OECD countries was 31.6%. Eighteen percent of students in Ireland performed at the Low TIMSS benchmark, indicating that they have some knowledge of whole numbers and basic graphs, while a further 6% performed below the Low benchmark, and no information is available on the mathematical skills of these students. The corresponding average estimates for students in participating OECD countries were 21.9% and 11.2% respectively. Hence, in all, 24% of students in Ireland performed at or below the Low benchmark, compared with 30.6% on average across participating OECD countries.

Broadly similar outcomes are evident in PISA and TIMSS when we consider the proportions of high and low performers. Ireland tends to have relatively fewer high performers and relatively few low performers compared with the average for participating OECD countries, with large proportions of students situated at the medium levels such as Levels 2-4 in PISA and the High and Intermediate benchmarks in TIMSS.

Table 2.9. Mean scores of top 25 countries on TIMSS 2015 mathematics, OECD-16 and international average scores

Country	Mean	SE	SD	SE	IRL
Singapore	621.0	3.2	82.1	2.2	▲
Korea (Rep. of)	605.7	2.6	85.3	1.1	▲
Chinese Taipei	599.1	2.4	97.2	1.7	▲
Hong Kong SAR	594.2	4.6	78.4	2.8	▲
Japan	586.5	2.3	88.9	1.3	▲
Russian Federation	538.0	4.7	81.7	1.8	▲
Kazakhstan	527.8	5.3	93.2	2.3	O
Canada	527.3	2.2	69.8	1.3	O
Ireland	523.5	2.7	73.9	2.3	
United States	518.3	3.1	83.3	1.6	O
England	518.3	4.2	79.8	2.6	O
Slovenia	516.3	2.1	69.2	1.4	▼
Hungary	514.4	3.8	93.4	2.2	O
Norway (Grade 9)	511.5	2.3	70.1	1.2	▼
Lithuania	511.3	2.8	77.3	1.5	▼
Israel	510.9	4.1	102.0	2.3	▼
Australia	505.0	3.1	82.4	1.9	▼
Sweden	500.7	2.8	72.0	1.9	▼
Italy	494.4	2.5	74.5	1.8	▼
Malta	493.5	1.0	88.4	0.9	▼
New Zealand	492.7	3.4	87.9	2.0	▼
Malaysia	465.3	3.6	86.6	2.1	▼
United Arab Emirates	464.8	2.0	97.9	1.5	▼
Turkey	457.6	4.7	105.4	2.8	▼
Bahrain	454.0	1.4	80.3	1.4	▼
OECD-16 Average	513.2	0.8	83.4	0.5	▼
International Average	481.6	0.5	85.1	0.3	▼

(blue)	Significantly above International average	▲	Significantly higher than Ireland
(white)	At International average	O	Not significantly different from Ireland
(orange)	Significantly below International average	▼	Significantly lower than Ireland

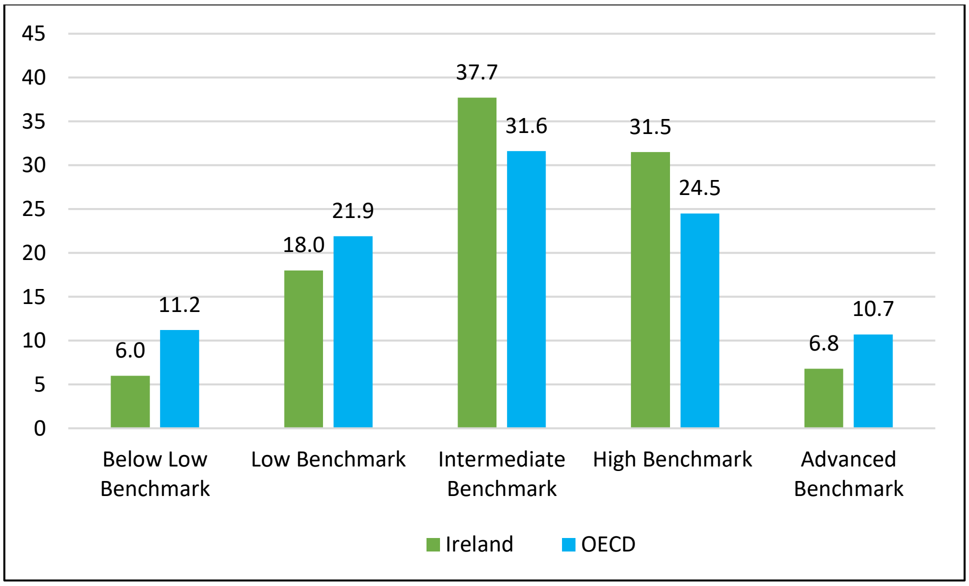
Italics: OECD country in TIMSS 2015. England was included in computing OECD average scores since data for the UK as a whole were not available.

Table 2.10. Summary description of the benchmarks on the TIMSS 2015 grade 8 mathematics scale and percentages of students achieving each level, in Ireland and on average across participating OECD countries

Benchmark (Cut-point)	Students at this level are capable of:	Ireland		OECD-16 Avg.	
		%	SE	%	SE
Advanced (625)	Students can apply and reason in a variety of problem situations: • Solve a variety of fraction, proportion, and percent problems, and justify their conclusions • Use their knowledge of geometric figures to solve a wide range of problems about area • Demonstrate an understanding of the meaning of averages and solve problems involving expected values	6.8	0.8	11.2	0.3
High (550)	Students can apply their understanding and knowledge in a variety of relatively complex situations: • Use information to solve problems involving different types of numbers and operations • Relate factions, decimals and percentages to each other • Show basic procedural knowledge related to algebraic expressions • Solve a variety of problems with angles including those involving triangles, parallel lines, rectangles and similar figures. • Interpret data in a variety of graphs and solve simple problems involving outcomes and probabilities	31.5	1.2	21.9	0.3
Intermediate (475)	Students can apply basic mathematical knowledge in a variety of situations: • Solve problems involving negative numbers, decimals, percentages, and proportions • Show some knowledge of linear expressions and two-and three-dimensional shapes • Read and interpret data in graphs and tables • Show some basic knowledge of chance	37.7	1.3	31.6	0.3
Low (400)	Students have some knowledge of whole numbers and basic graphs.	18.0	1.0	21.5	0.3
Below Low	There is insufficient information on which to base a description of the mathematical skills of these students.	6.0	0.8	10.7	0.2

Sources: Clerkin et al. (2016); Mullis et al. (2016)

Figure 2.9. Percentages of Students at Key TIMSS Mathematics Benchmarks, Ireland and OECD-16 Average (2015)



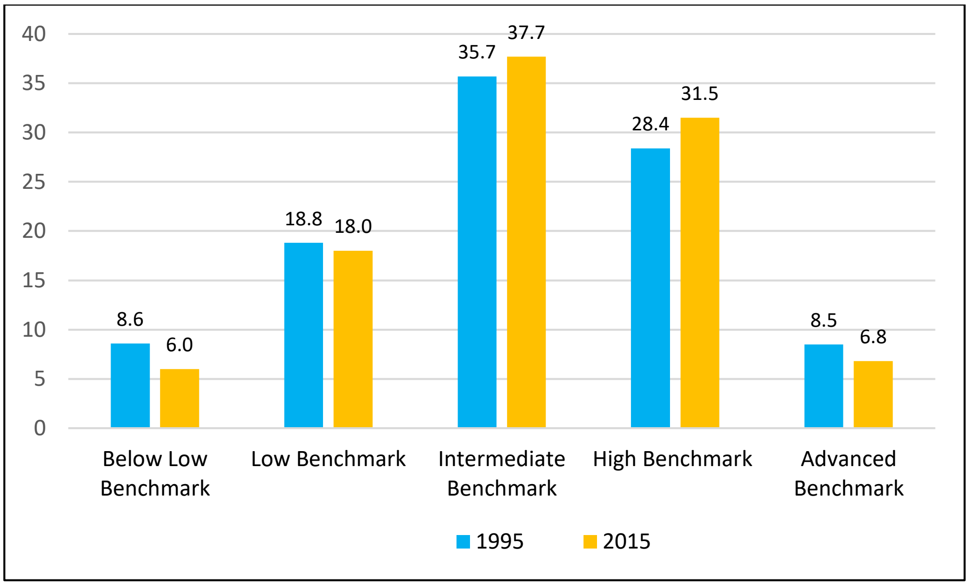
It might be noted that a number of countries in TIMSS 2015 had large proportions of students performing at the Advanced benchmark, including Singapore (54%), Korea (43%) and Hong Kong SAR (37%). Even among countries performing at about the same average level as Ireland, the United States

(10%) and England (10%) had marginally more students performing at the Advanced benchmark than Ireland (7%).

2.2.5. Trends in Performance on TIMSS Benchmarks

Data are available on trends in performance on TIMSS mathematics benchmarks for 1995 and 2015 (Figure 2.10). In line with a slightly but not significantly higher mean score in Ireland in 2015, compared with 1995, there are small non-significant increases in the proportions performing at High and Intermediate benchmarks (Clerkin et al., 2016), and non-significant reductions at the Low and Advanced benchmarks. This essentially signals a shift in performance away from the extremities of the distribution towards the upper-middle.

Figure 2.10. Percentages of Students in Grade 8 in Ireland at Key TIMSS Mathematics Benchmarks, 1995 and 2015



2.2.6. Performance on TIMSS 2015 Mathematics Content Areas

As noted earlier, TIMSS comprises four content areas at Grade 8: Number (64 items across all booklets), Algebra (61), Geometry (43) and Data and Chance (41). Figure 2.12 shows the average score for Ireland and for participating OECD countries on each content area and on the TIMSS mathematics test as a whole. Relative to their performance on the mathematics test as a whole (523.5), students in Ireland achieved significantly higher mean scores on Number (544.5) and Data & Chance (533.8), and significantly lower scores on Algebra (501.0) and Geometry (503.0) (Clerkin et al., 2016).

Ireland's mean scores on Number and Data & Chance were also higher than the corresponding OECD-16 average scores in these content areas, while Ireland's mean scores on Algebra and Geometry were not significantly different from the corresponding OECD-16 average scores. A number of countries had particularly high scores in Algebra, including OECD countries Korea (612.1) and Japan (595.9). Among countries that performed at a level similar to Ireland on the overall scale, performance on Algebra was stronger (Mullis, Martin, Foy & Hooper, 2016).

Figure 2.11. Means scores on TIMSS 2015 content areas and overall – Ireland and OECD-16 average

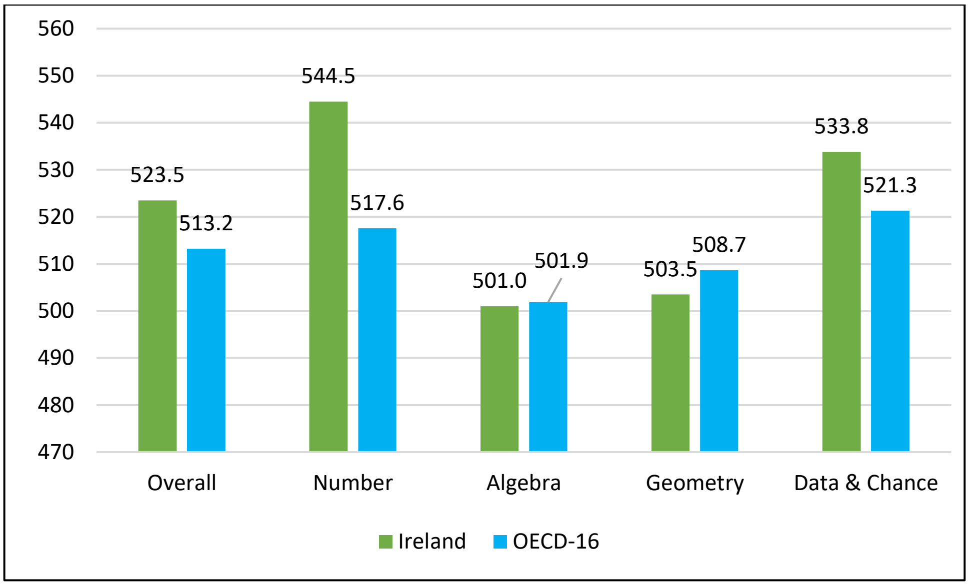


Table 2.11 examines the performance of higher-achieving students (those scoring at the 90th percentile) and lower-achieving students (those scoring at the 10th percentile) in Ireland and on average across the OECD-16 on the TIMSS content subscales. The table shows that students in Ireland at the 10th percentile achieved a score that is significantly higher than the corresponding OECD-16 average on Number, and that their scores on Algebra, Geometry and Data & Chance are not significantly different from the corresponding OECD average scores. The score of students at the 90th percentile in Ireland on Number is also significantly higher than the corresponding OECD-16 average score, but their scores on Algebra and Geometry are significantly lower.

Table 2.11. Scores (standard errors) of students in Ireland and on average across OECD countries at the 10th and 90th percentiles on overall mathematics, and on the TIMSS 2015 content subscales

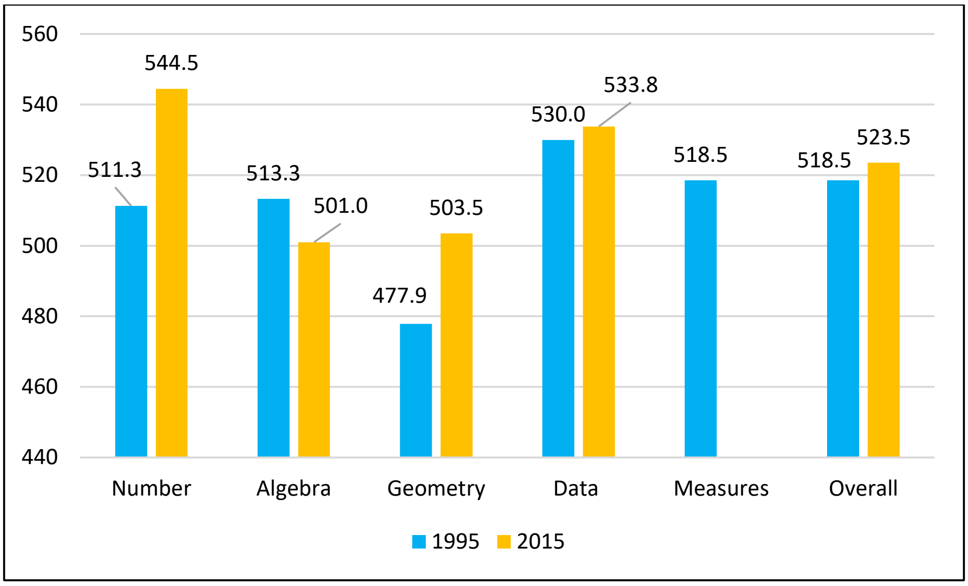
	10th percentile		90th percentile	
	Ireland	OECD-16 Avg.	Ireland	OECD-16 Avg.
Overall	**426.0** (6.0)	404.5 (1.3)	612.3 (4.0)	617.5 (1.0)
Number	**431.8** (6.7)	401.0 (12.3)	**648.7** (3.5)	631.0 (1.1)
Algebra	399.3 (5.9)	387.4 (1.23)	**597.0** (3.4)	614.1 (1.2)
Geometry	393.5 (5.7)	392.7 (1.3)	**607.9** (4.2)	622.3 (1.3)
Data & Chance	412.8 (7.6)	397.7 (1.6)	647.4 (4.4)	639.8 (1.3)

Scores that are significantly different from the corresponding OECD average score are in bold.

2.2.7. Trends in Performance on TIMSS Mathematics Content Areas

Drawing on data from TIMSS 1995 and 2015, it is possible to examine trends in performance in Ireland on mathematics content areas (Figure 2.12). It should be noted, however, that the Measures content area featured in 1995, but not 2015, with Measures items integrated into other content areas in that year. Figure 2.12 points to a sizeable increase in Number (called Fractions and Number Sense in 1995). On the other hand, Algebra was lower by 13 points in 2015 than in 1995, while Geometry improved by 25 points between these years. Data & Chance (called Data Representation, Analysis & Probability in 1995), was about the same in both years.

Figure 2.12. Means scores on TIMSS 2015 content areas in Grade 8 in Ireland, 1995 and 2005



Note: For 2015, Measures was integrated into Number.

2.2.8. Performance on TIMSS 2015 Mathematics Cognitive Subscales

As noted earlier, TIMSS also describes performance in three cognitive domains – Knowing (based on 69 test items), Applying (94) and Reasoning (46). Figure 2.13 shows the average score for Ireland and for participating OECD countries on each cognitive domain and on the TIMSS mathematics test as a whole. Students in Ireland achieved a mean score on Knowing items (523.5) that was significantly higher than on the TIMSS test as a whole, and a mean score on Applying (520.4) that was significantly lower, while Ireland's mean score on Reasoning was not significantly different than on the test as a whole (Clerkin et al., 2016). The difference between Ireland's mean score and the mean score of students on average across the OECD-16 is greatest on the Knowing scale, and lowest on Reasoning.

Figure 2.13. Means scores on TIMSS 2015 cognitive scales and overall – Ireland and OECD-16 Average

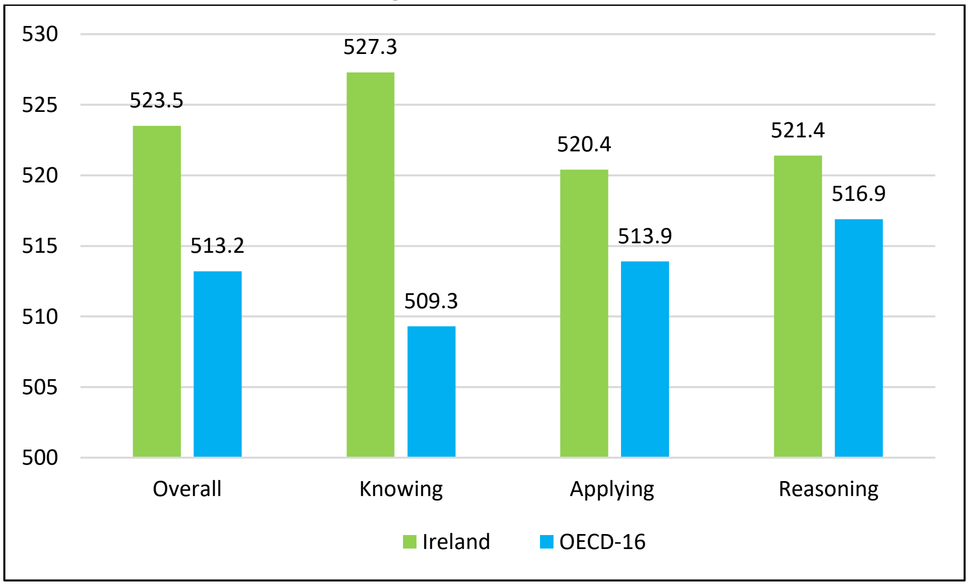


Table 2.12 provides data on the performance of higher-achieving students (those scoring at the 90th percentile) and lower-achieving students (those scoring at the 10th percentile) in Ireland and on average across the OECD-16 on TIMSS 2015 cognitive processes. Students in Ireland scoring at the 10th percentile have significantly higher scores than students on average across the OECD-16 on all three cognitive domains (Knowing, Applying and Reasoning), with a relative strength on Knowing. Students at the 90th percentile in Ireland have a significantly lower mean score on Reasoning than students on average across the OECD-16, and their scores do not differ significantly from the OECD average on either Knowing (higher by 3 points) or Applying (lower by 5 points).

Table 2.12. Scores (standard errors) of students in Ireland and on average across OECD countries at the 10th and 90th percentiles on overall mathematics, and on the TIMSS 2015 cognitive subscales

	10th percentile		90th percentile	
	Ireland	OECD-16 Avg.	Ireland	OECD-16 Avg.
Overall	**426.0** (6.0)	404.5 (1.3)	612.3 (4.0)	617.5 (1.0)
Knowing	**427.9** (5.9)	399.8 (1.3)	618.6 (3.5)	615.8 (1.1)
Applying	**417.5** (6.5)	404.2 (1.2)	614.9 (3.6)	619.5 (1.1)
Reasoning	**421.1** (5.6)	404.1 (1.3)	**616.0** (3.6)	626.4 (1.2)

Scores that are significantly different from the corresponding OECD average score are in bold.

No trend data on performance by cognitive level are available for TIMSS.

2.2.9. Gender Differences on TIMSS 2015 Mathematics Scales

In TIMSS 2015, male students in Ireland achieved a mean score of 526.1, while females achieved a score of 520.9. The 5.2 score-point difference was not statistically significant. On average across the OECD-16, male students (514.6) had a significantly higher mean score than females (511.7).

Figure 2.14 shows the proportions of male and female students performing at the different TIMSS 2015 performance benchmarks. Differences between the proportions of male and female students at each benchmark are small and not statistically significant. However, it is noteworthy that marginally more males (8.5%) than females (5.1%) performed at the Advanced benchmark. On average across the OECD, gender differences were also small at each TIMSS benchmark.

Performance by gender can also be examined with reference to mathematics content domain and cognitive process (Figures 2.15, 2.16). There was significant difference in favour of male students (mean = 549.1) over females (540.0) on Number. Small differences in favour of male students on Geometry and Data & Chance, and in favour of females on Algebra, were not statistically significant. On average across the OECD-16 countries in TIMSS, there was a large significant difference (12.9 score points) in favour of males on Number, a smaller significant difference in favour of males on Data & Chance (3.3), a small significant difference in favour of females on Algebra (5.4), and a non-significant difference in favour of females on Geometry (1.6).

Figure 2.14. Percentages of male and female students in Ireland at each TIMSS overall mathematics benchmark in 2015

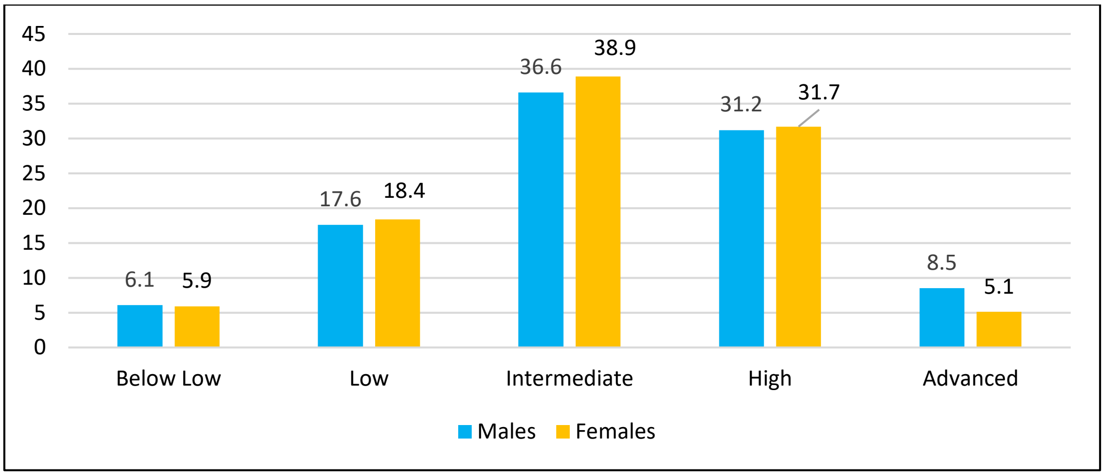
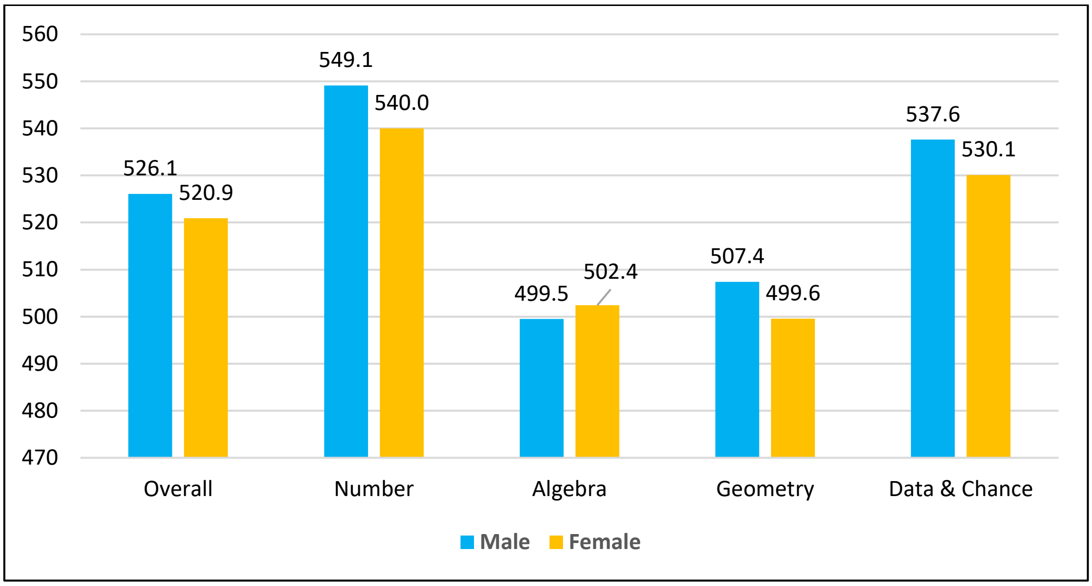
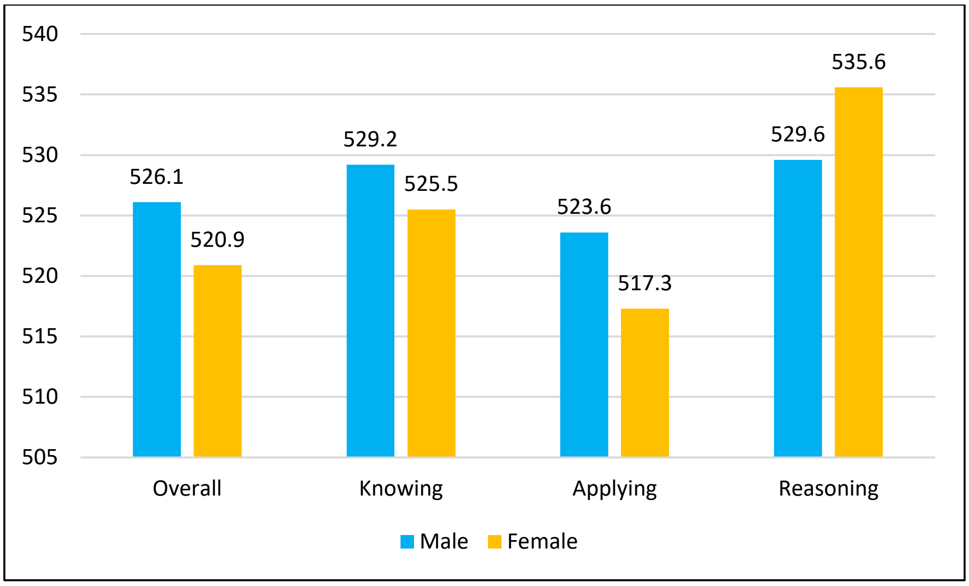


Figure 2.15. Means scores of male and female students in Ireland on TIMSS 2015 mathematics content subscales



Finally, gender differences on TIMSS can be examined with reference to cognitive processes (Figure 2.16). While male students in Ireland had marginally higher scores on the Knowing and Applying subscales, and females had a marginally higher score on Reasoning, none of the differences was statistically significant. This is in line with the finding that, although males had an overall mean score that was significantly higher than females, the difference was not statistically significant. On average across the OECD-16 in TIMSS, there was a statistically significant difference in favour of male students on the Knowing scale (by 4.4 score points), a non-significant difference in favour of females on the Applying scale (by 1.2 points), and a non-significant difference in favour of males on Reasoning (by 2 points).

Figure 2.16. Means scores of male and female students in Ireland on TIMSS 2015 mathematics cognitive subscales



2.2.10. Comparing PISA and TIMSS Outcomes

In 2015, both PISA and TIMSS were implemented in Ireland. In considering the outcomes of the mathematics components of the two assessments, the following should be noted:

- PISA assesses mathematics as a major assessment domain every nine years; the last year for which detailed data on mathematics are available is 2012, though mathematics was assessed as a minor domain in PISA 2015. TIMSS assesses mathematics as a major domain in each cycle; hence, detailed information on mathematics performance is available for 2015, including performance on content and process subscales.
- PISA assesses 15-year old students. These are spread across grade levels. In 2015, about 60% were in Third year, 27% in Transition year, 11% in Fifth year, and fewer than 2% in Second year. TIMSS is a grade-based study. All participants are at the same grade levels (Grades 8 or Second year post-primary in Ireland).
- PISA collects context information (mainly questionnaire data) from school principals, parents and students. At Grade 8, TIMSS collects context data from school principals, students and teachers. In addition, TIMSS National Research Co-ordinators are responsible for completing a Curriculum Questionnaire, which asks about the organisation and content of the mathematics (and science) curriculum.
- PISA aims to measure how well students are prepared to meet the challenges they may encounter in their future lives, and, while it is informed by the content of school curricula, the focus is on student ability to apply knowledge and skills in real-life situations. TIMSS aims to measure school-based learning and the extent to which students have mastered essential mathematics concepts, content and procedures.
- PISA content is presented in clusters. Students see a stimulus (such as a text or graphic) and a set of accompanying questions. Some follow a multiple-choice format while others are constructed response items that require answers of varying lengths. Much of the TIMSS test consists of short stand-alone questions, with minimal accompanying text. In a small number of cases, a short stimulus text is presented with two to three associated questions.

- In 2015 PISA was delivered on computer in most participating countries, including Ireland. In 2015 TIMSS was delivered on paper in all participating countries. For the next TIMSS cycle (2019), countries will have the option of electronic testing.
- Students are allocated two hours to complete the PISA tests, with students taking some combination of mathematics, reading and science items (with an emphasis on the major assessment domain in each cycle). Students are allocated 90 minutes to take the TIMSS test, with a short break in the middle. Half of the time is allocated to mathematics, and half to science.

Table 2.13 provides a ranking of countries that participated in both PISA 2015 and TIMSS 2015 (Grade 8) mathematics. Singapore, Hong-Kong, the Republic of Korea, Chinese Taipei and Japan were the five top performing countries in mathematics in both TIMSS and PISA. In addition to these five countries, the Russian Federation significantly outperformed Ireland on TIMSS mathematics and Canada and Slovenia significantly outperformed Ireland on PISA mathematics. Students in Ireland performed at a similar level to students in the United States and England on TIMSS mathematics, but significantly outperformed students in the United States and the United Kingdom on PISA mathematics. While the Russian Federation achieved a mean mathematics score that was significantly higher than Ireland's mean mathematics score in TIMSS, it performed significantly less well than Ireland on PISA mathematics. On the other hand, Slovenia performed significantly less well than Ireland on TIMSS mathematics, but had a significantly higher mean mathematics score than Ireland on PISA mathematics.

2.3. Summary

In considering how the outcomes of PISA and TIMSS might inform an evaluation of the impact of Project Maths on student performance, a number of points should be noted:

- In 2015, PISA transitioned from paper-based testing to computer-based testing. Hence, possible mode effects may have impacted on the outcomes in 2015, compared with earlier cycles. This is evidenced by the fact that 8 of the top-performing countries in PISA 2012 mathematics experienced significant declines in performance in 2015, while an additional 5 experienced significant increases. Ireland's performance did not change.
- While the focus of PISA is on mathematical literacy, and solving problems in real-life contexts, the focus of TIMSS is on mathematics school-based learning, and students' mastery of content, concepts and procedures.
- Mathematics was a minor assessment domain in PISA 2015. Hence, data are not available on mathematics content areas and processes/cognitive domains. However, these are available for PISA 2012, when Third- and Transition-year students (85% of the sample) had not studied under Project Maths.
- While all students in Ireland taking TIMSS 2015 (administered in Second year) had studied under Project Maths, there are no recent trend data on TIMSS. Students in Second year in Ireland took part in TIMSS 1995.

Hence, care needs to be exercised in drawing conclusions about the effects of Project Maths on student performance using data from these international studies.

2.3.1. Overall Performance on Mathematics

On the whole, performance on PISA did not change substantively between 2000 and 2015. There was a dip in 2009 and a recovery in 2012, but this does not seem to be related to Project Maths, since all students taking PISA 2009 in Ireland and most students taking PISA 2012 had studied under the pre-Project Maths syllabus.

Table 2.13. Countries that participated in TIMSS and PISA 2015, ranked in descending order of their performance in mathematics, and with reference to their position relative to Ireland's mean score.

Mathematics

TIMSS	PISA
Singapore	Singapore
Korea, Rep. of	Hong Kong SAR
Chinese Taipei	Chinese Taipei
Hong Kong SAR	Japan
Japan	Korea, Rep. of
Russian Fed.	Canada
Canada	Slovenia
Ireland	**Ireland**
United States	Norway
England	New Zealand
Slovenia	Russian Fed.
Hungary	Sweden
Norway	Australia
Lithuania	United Kingdom
Israel	Italy
Australia	Malta
Sweden	Lithuania
Italy	Hungary
Malta	Israel
New Zealand	United States
UAE	UAE
Turkey	Chile
Georgia	Turkey
Lebanon	Thailand
Qatar	Georgia
Thailand	Qatar
Chile	Lebanon
Jordan	Jordan

Mean score significantly above Ireland's
Mean score not significantly different from Ireland's
Mean score significantly below Ireland's

While Ireland's mean score on overall mathematics was not significantly different from the OECD average in 2000, 2003 and 2006, it was significantly above the OECD average in 2012 and 2015. However, this arises because the OECD average dropped in 2012 and again in 2015. As noted above, with the exception of 2009, Ireland's mean score in mathematics has varied within a very narrow range.

The stability in performance in Ireland between 2012 and 2015 (coinciding with the introduction of computer-based tests for all students in PISA), is noteworthy, given that 13 of the top-performing countries in mathematics in 2012 saw a change in their performance in 2015. Ireland's stable performance may mask a small positive increase arising from Project Maths, but cancelled out by a negative mode effect. This would be consistent with the finding by Merriman et al. (2014) that students in Project Maths initial schools had a higher mean score (by 4 points) than students in non-initial schools in PISA 2012, though the difference was not statistically significant.

Although the overall mean score of students in Second year in Ireland on TIMSS mathematics increased by 4.6 score points between 1995 and 2015, the change is not statistically significant. Again, this is broadly consistent with the view that performance at Junior Cycle in particular may have increased a little since the introduction of Project Maths, but not to any large extent.

Although increasing performance on international assessments is not an aim of Junior Cycle mathematics, there is value in considering strengths and weaknesses linked to the performance of students in Ireland, coupled with the views of teachers (see Chapter 6), as these can provide insights into areas where a change in emphasis might be needed.

2.3.2. Performance on Space & Shape and Geometry

In both 2003 and 2012, when mathematics was a major assessment domain in PISA, students in Ireland performed at a level that was below the average for OECD countries on Space & Shape. Given that overall performance on PISA 2015 was similar to 2012, there is no evidence to indicate that performance on Space & Shape has improved. Moreover, female students in Ireland do considerably less well than male students on Space and Shape (there was a 25 points difference in favour of males in 2012). More positively, however, Merriman et al. (2014) reported that that students in initial Project Maths schools in PISA 2012 achieved a higher mean score on Space & Shape (485.5) compared with non-initial schools (477.4), although the difference was not statistically significant.

According to Merriman et al. (2014), male students in Initial Project Maths schools performed on Space & Shape at a level that was not significantly different from the OECD average for male students in 2012, whereas female students performed at a level that was still significantly below the OECD average for females. This outcome is indicative of the potential of Project Maths to further enhance performance on Space & Shape, which also suggesting that female students may need even more intensive intervention.

Items on PISA Space & Shape span a range of activities such as understanding perspective, creating and reading maps, transforming shapes with and without technology, interpreting views of three-dimensional scenes from various perspectives, and constructing representations of shapes. Geometry is viewed by PISA as being central to Space & Shape, though aspects of other content areas such as Spatial Visualisation, Measurement, Number and Algebra are also drawn on. The manipulation and interpretation of shapes in settings such as dynamic geometry software and Global Position System (GPS) tools are included in the domain, though not necessarily represented in current PISA Space & Shape items.

Consideration should be given as to what elements of PISA Space & Shape are important, and whether students would benefit from interacting with these elements, with possible carryover to other mathematics content areas, as well as other subjects such as science (e.g., Wai, Lubinski & Benbow, 2009).

There might also be value in providing direct instruction in visual-spatial skills ('spatially-enriched education') to students. Greater use of software linked to spatial reasoning, geometry and functions could help with visualisation skills, including manipulation of moving shapes. Indeed, Uttal et al. (2013)

have shown that computer programmes can be effective in developing students' spatial reasoning skills, and improving their performance in both mathematics and science. There might also be a benefit to many students if teachers were to build further on visual approaches to teaching fundamental mathematics concepts (Sobanski, 2002), though some aspects of this are already built into Project Maths.

More detailed input on Space & Shape could be provided in the context of supplementary work in mathematics (for example, computer-based activities assigned for homework), Transition Year mathematics modules, and short courses with an emphasis on mathematics. Opportunities for integrating concepts associated with Space & Shape could also be availed of in subjects like geography (e.g., location of cities in relation to one another, map reading, orientation, grid references, latitude and longitude, time zones), history (location, buildings, archaeology, sense of time and space), science (shape in natural phenomena, properties of particles), and literature (timescales, direction) (see Fox & Surtees, 2010). In devising interventions, particular attention needs to be given to ensuring consistency of methodology and terminology across subjects. Attention also needs to be given to addressing female students' development of spatial concepts and spatial reasoning.

It is noteworthy that Geometry was identified in TIMSS 2015 as an area of relative weakness (along with Algebra) for students in Ireland, as performance on Geometry was lower than performance on the TIMSS test as a whole. Performance in Ireland was also below the average for OECD countries in TIMSS, though the difference was not statistically significant. However, perhaps surprisingly in light of the large gender difference in favour of male students in Ireland on PISA 2012 Space & Shape, female students in Ireland outperformed males on TIMSS Geometry, albeit to a non-significant degree. This suggests that TIMSS is less reliant on the types of visualisation and spatial reasoning tasks found in PISA Space & Shape, where larger gender differences arise.

Another issue emerging from both PISA and TIMSS is that higher achievers in Ireland (those scoring the 90th percentile) perform relatively less well compared with higher achievers in other countries on Space & Shape and Geometry.

These findings (and those relating to the balance across content areas on the Junior Certificate examination – see Chapter 6) suggest a need to increase the relative emphasis on Geometry at Junior Cycle, and to broaden it to include a stronger focus on visualisation and spatial reasoning. There is also value in continuing to emphasise visual linkages across mathematics content areas (such as between Geometry and Algebra). As noted later in this report, consideration also needs to be given to the strong focus on synthetic (Euclidian) geometry in the current syllabus, which remains isolated from the rest of the curriculum, which may result in some students memorising proofs rather than developing deep understanding.

2.3.3. Performance on Change & Relationships and Algebra

The two content domains, Change & Relationships in PISA and Algebra in TIMSS broadly cover the same content, including algebra, functions and aspects of measurement (e.g., speed). As noted above, while performance on Change & Relationships in PISA fell by 5 score points between 2003 and 2012, the change was not statistically significant and students in Ireland still performed above the OECD average in this domain in 2012. Performance on Algebra in TIMSS was at about the same level as the OECD-16 average in 2015, while Algebra, along with Geometry, was identified as an area of relative weakness for students in Ireland. It may be that the relatively stronger performance of students in Ireland on Change & Relationships arises because it is less abstract than in TIMSS or in Junior Cycle, and students can draw on their knowledge of Algebra and Functions to solve problems in that domain.

The data from TIMSS, in particular, point to underperformance on Algebra, and a need to strengthen performance in that area.

2.3.4. Performance of High- and Low-Achieving Students

While the mean score of students in Ireland on PISA mathematics was significantly higher than the corresponding OECD average score in 2015, the proportion of higher-achieving students (those achieving at Levels 5-6 on the PISA proficiency scales) (9.8%) was a little below the OECD average proportion (10.7%). More than 20% of students in east-Asian countries such as Korea and Japan perform at Levels 5-6. Non-Asian countries including Switzerland (19.2%), Canada (15.1%) and Estonia (14.2%) also have larger proportions of students than Ireland performing at these levels. Moreover, the proportion of higher-achieving students in Ireland has also been relatively low in previous PISA cycles including 2003 (11.4%), 2006 (10.2%), and 2012 (10.7%).

TIMSS 2015 also points a low proportion of higher-achieving students in Ireland. Although Ireland's overall mean score on TIMSS 2015 was significantly higher than the corresponding OECD-16 average, fewer students in Ireland (6.8%) than on average across the OECD-16 (10.7%) performed at the Advanced TIMSS benchmark.

While there are fewer students in Ireland than on average across OECD countries performing at the highest levels of proficiency in PISA, significantly fewer students than on average across OECD countries perform at the lowest levels of proficiency (at or below Level 1), with 15.0% of students doing so in 2015, compared to an OECD average of 23.4%. This indicates that Ireland's relatively strong overall performance on PISA mathematics arises because of a low proportion of low performers rather than a high proportion of high performers. It may be that, in recent years, priority has been given to addressing the needs of lower-achieving students, and that there should now be more emphasis on supporting higher-achieving students to reach their potential. This, in turn, might contribute to higher overall standards at Leaving Certificate level.

These data point to a need to raise the performance of higher-achieving students in Ireland. As noted this chapter, areas of specific weakness among higher-achieving students are Change & Relationships / Algebra, and Shape & Space / Geometry. However, aspects of Algebra and Geometry are probably more abstract for students in general, than other content areas, and efforts to strengthen Algebra and Geometry will need to address the appropriate level of abstraction.

2.3.5. Gender Differences in Performance

In PISA, male students in Ireland significantly outperformed females in all cycles from 2000 to 2015, except in 2009, when the difference in favour of males was not statistically significant. In 2015, Ireland had one of the largest gender differences among participating countries – 16.1 score points, compared with an OECD average of 7.9 score points. Moreover, female students were less strongly represented among higher achievers (those performing at proficiency levels 5-6) in 2015 (6.5%) compared with males (12.9%). In PISA 2012, there was a difference of 24.8 score points in favour of males in Space and Shape, with differences ranging from 13.4 to 14.2 points in favour of males in the other PISA content areas (Change & Relationships Quantity and Uncertainty & Data). Finally, in the same cycle, males outperformed females in all three PISA mathematics processes categories.

Gender differences are less-pronounced on TIMSS 2015 than on PISA, and, although male students in Ireland achieved a higher overall score than females (by 5.2 score points) in PISA, the difference was not statistically significant. However, male students did achieve a significantly higher mean score than females on average across the OECD-16, albeit by 3 score points. More male students in Ireland (8.5%)

than females (5.9%) performed at the Advanced TIMSS benchmark, though the difference was not statistically significant. Male students in Ireland achieved a significantly higher mean score than females on the Number content area, by 9.1 score points. Differences in other content areas, including a small difference in favour of females on Algebra, were not statistically significant.

The relatively large gender differences on PISA mathematics contrast with the small differences observed on TIMSS, as well as some differences in favour of females. Differences between the studies may arise because of the focus in PISA on problem solving, and the inclusion of Shape & Space items (for example, involving manipulation of 3-D shapes) that may favour male students. It is less clear why Ireland has one of the largest gender differences across OECD countries, though again this may arise because of the low performance of female students on Space & Shape. This suggests that any increased focus on visualisation and spatial reasoning in the Junior Cycle syllabus needs to address the difficulties encountered by female students in this area.

Chapter 3: Student Factors Associated with Mathematics Performance

3.1. Introduction

This chapter addresses student factors associated with performance in mathematics, drawing on data from the TIMSS 2015 study, as well as findings from the PISA 2012 study in which mathematics was a major assessment domain. Firstly, students' attitudes to mathematics are described using data gathered from Second year (Eight grade) students in April/May 2015 as part of TIMSS 2015 in Ireland. By then, the Project Maths initiative had been fully implemented for all Junior Cycle students who participated in this study. The data therefore provide an insight into the attitudes of students who studied only this curriculum for mathematics at Junior Cycle. Secondly, findings relating to students' attitudes to mathematics in PISA 2012 are summarised, drawing on published analyses (Perkins, 2013a; Perkins & Shiel, 2016a). Although 23 initial Project Maths schools (those that first implemented the revised curriculum in the period 2008 to 2010) were included in PISA 2012, most of the students who participated in PISA 2012 in Ireland had studied under the 2000 syllabus. However, the findings are indicative of the attitudes of 15-year-olds to mathematics at the time and hence provide a useful point of reference.[14]

3.2. Student Attitudes to Mathematics in TIMSS 2015

TIMSS 2015 asked students about how much they like learning mathematics, how much they value mathematics, how confident they feel in mathematics, and the extent to which they report experiencing engaging teaching in mathematics classes. Four scales were constructed, each with a centrepoint of 10 and standard deviation of 2 (Mullis et al., 2016). This section describes these attitudinal factors for Second year students in Ireland and considers them alongside other characteristics of interest that tend to be associated with performance (gender, school SSP (DEIS) status, and parents' highest level of education, with the latter serving as an indicator of student socio-economic status). Additionally, scale scores for Ireland are compared with an international average calculated using data for the 16 OECD countries that participated in TIMSS 2015.[15]

3.2.1. Students' Liking of Mathematics Learning

In TIMSS 2015, students were asked to indicate their agreement or disagreement with a set of statements about the extent to which they like learning mathematics (Figure 3.1). In Ireland, the majority of students agreed ('a little' or 'a lot') that they like mathematics (61.3%) and that they enjoy learning mathematics (63.2%), with around one quarter of students agreeing 'a lot' with these statements. Around one-in-five students (19.4%) disagreed a lot that they like mathematics, and 16.1% disagreed a lot that they enjoy learning mathematics. Almost a quarter of students agreed a lot that they find mathematics boring (23.6%) and that they wish that they did not have to study mathematics (22.2%). However, the majority of students agreed to some extent that they learn many interesting things in mathematics (62.9%). While over one-third of students agreed a little or a lot with the statements that mathematics is one of their favourite subjects (38.1%) and that they look forward to mathematics class (38.1%), a majority of students (61.8%) disagreed with both statements. Over

[14] The contributions of students in initial schools were weighted to reflect their representation in the population for the analyses reported here and in Chapter 2, unless otherwise stated.
[15] Australia, Canada, Chile, Hungary, Ireland, Israel, Italy, Japan, Korea, New Zealand, Norway, Slovenia, Sweden, Turkey, United Kingdom, and United States.

half of students in Ireland disagreed to some extent that they like to solve mathematics problems (53.9%), and that they like any schoolwork that involves numbers (56.8%).

Figure 3.1. Percentages of students agreeing/disagreeing with statements about their liking for learning mathematics, Ireland, TIMSS 2015 – Grade 8

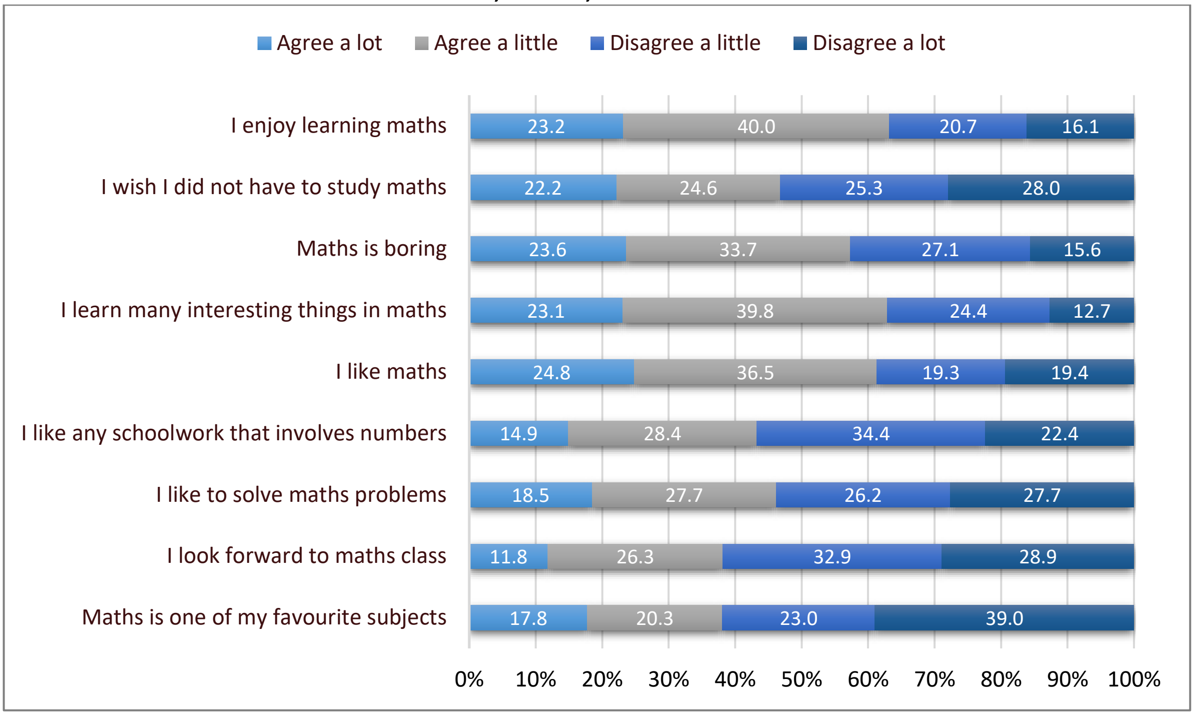
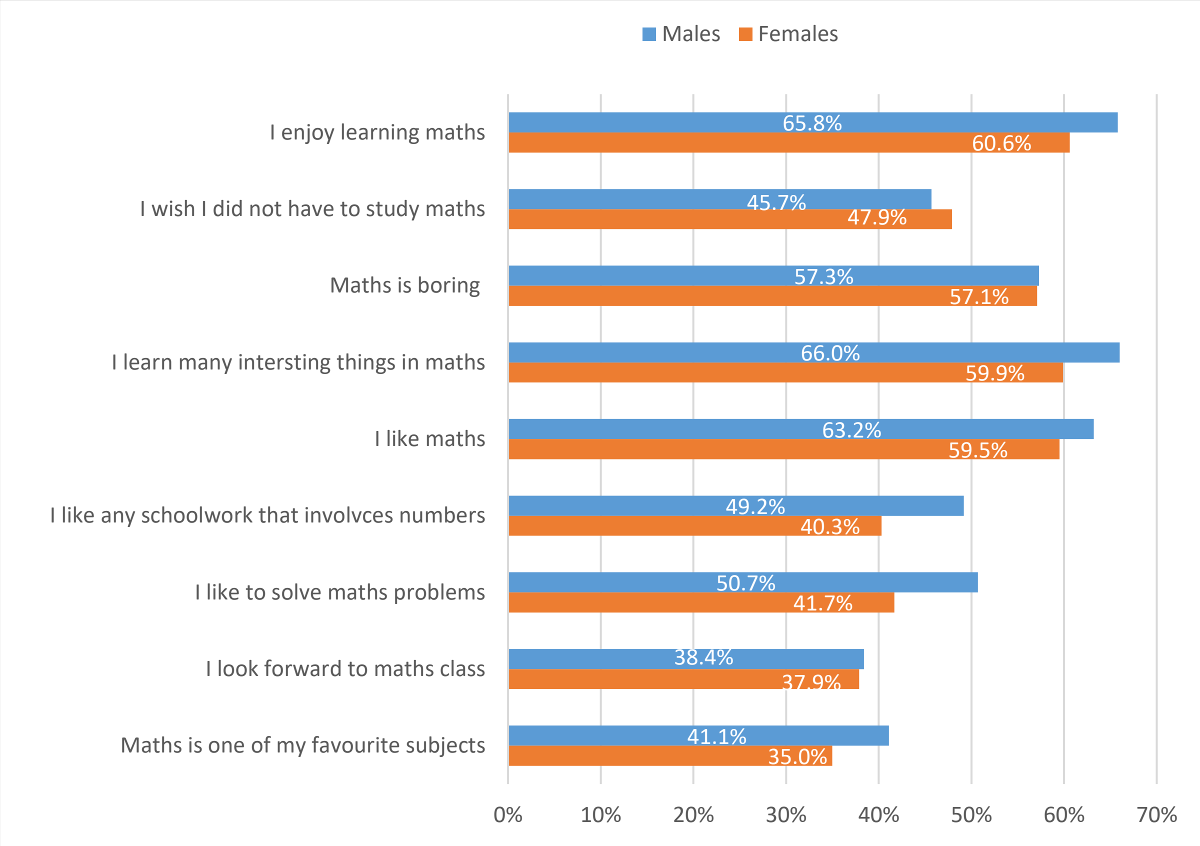


Figure 3.2 presents the percentages of male and female students agreeing (a little and a lot) with the various statements about their liking for learning mathematics. Overall, greater percentages of male students than female students agreed that they like mathematics (63.2% for males and 59.5% for females) and that they enjoy learning mathematics (65.8% for males and 60.6% for females). Similar percentages of male and female students agreed that mathematics is boring (57.5% for males and 57.1% for females), whereas a greater percentage of male students (66.0%) than female students (59.9%) agreed that they learn interesting things in mathematics. The greatest differences between male and female students are in the percentages agreeing that they like to solve mathematics problems (50.7% for males and 41.7% for females) and agreeing that they like any schoolwork that involves numbers (49.2% for males and 40.3% for females). Slightly more female students (47.9%) than male students (45.7%) agree that they wish they did not have to study mathematics.

Figure 3.2. Percentages of male and female students in Second year agreeing a little or a lot with statements about their liking for learning mathematics, Ireland, TIMSS 2015 – Grade 8



Using the nine statements about liking learning mathematics, TIMSS developed a scale ('Students Like Learning Mathematics') with higher scores indicating greater liking for learning mathematics. Students in Ireland had a mean score of 9.3 on the scale, which is significantly lower than the OECD-16 average of 9.4 (Table 3.1). A significant moderate positive correlation (.32) is observed between scores on students' liking for learning mathematics scale in Ireland and performance on overall mathematics.

Table 3.1. Mean scores on the students' liking for learning mathematics scale in Ireland and across 16 OECD countries on average, TIMSS 2015 – Grade 8

	Students' Liking for Learning Mathematics		
	Mean	SE	SD
Ireland	**9.3**	(0.05)	1.94
OECD 16 (ref)	9.4	(0.01)	1.87

Significantly different mean scores are in bold. OECD 16 = 16 participating OECD countries.

Male students in Ireland score higher on the students' liking for learning mathematics scale than do female students, but the difference is not statistically significant (Table 3.2). Across the 16 OECD countries on average, male students score significantly higher on the students' liking for learning mathematics scale than do female students. On average, male students in Ireland score significantly lower on the scale than do male students across the 16 participating OECD countries. However, female students in Ireland do not differ from their OECD-16 counterparts on average scale scores (Table 3.3).

Table 3.2. Mean scores on the students' liking for learning mathematics scale by gender, in Ireland and across 16 OECD countries on average, TIMSS 2015 – Grade 8

	Students' Liking for Learning Mathematics							
	Ireland				OECD 16			
	%	Mean	SE	SD	%	Mean	SE	SD
Males (ref)	49.6	9.4	(0.07)	1.93	50.4	9.6	(0.02)	1.88
Females	50.4	9.3	(0.05)	1.94	49.7	**9.3**	(0.01)	1.85

Significantly different mean scores are in bold. (Males vs. Females, Ireland and OECD).

Table 3.3. Mean scores on the students' liking for learning mathematics scale compared for males and for females in Ireland and across 16 OECD countries on average, Ireland, TIMSS 2015 – Grade 8

	Students' Liking for Learning Mathematics							
	Males				Females			
	%	Mean	SE	SD	%	Mean	SE	SD
Ireland	49.6	**9.4**	(0.07)	1.93	50.4	9.3	(0.05)	1.94
OECD 16 (ref)	50.4	9.6	(0.02)	1.88	49.7	9.3	(0.01)	1.85

Significantly different mean scores are in bold. OECD 16 = 16 participating OECD countries.
(Male vs. Male, Female vs. Female, Ireland and OECD).

Table 3.4 presents mean scores on the students' liking for learning mathematics scale by parents' highest level of education. Students whose parents' highest level of education is at least upper secondary school had significantly higher mean scores on the scale than students whose parents' highest education is some primary, lower secondary or no school.

Table 3.4. Mean scores on the students' liking for learning mathematics scale by parents' highest education level, Ireland, TIMSS 2015 – Grade 8

	Students' Liking for Learning Mathematics				
	%	SE	Mean	SE	SD
University or higher	33.1	(1.11)	**9.6**	(0.07)	1.93
Post-secondary but not university	21.0	(0.72)	**9.4**	(0.08)	1.95
Upper secondary	16.2	(0.68)	**9.1**	(0.10)	1.97
Lower secondary	4.5	(0.52)	8.9	(0.13)	1.79
Some primary, lower secondary, or no school (ref)	2.3	(0.26)	8.5	(0.24)	2.14
Don't know	22.9	(0.90)	**9.3**	(0.07)	1.86

Significantly different mean scores are in bold.

Using scale scores, TIMSS classified students into three groups: those who very much liked learning mathematics, those who liked learning mathematics, and those who did not like learning mathematics.[16] In Ireland, students who did not like learning mathematics (51.7%) outnumbered those who liked learning mathematics (34.7%) and those who very much liked learning mathematics (13.6%) (Table 3.5). Students who do not like learning mathematics score significantly lower on average on the TIMSS overall mathematics scale than do students who like learning mathematics. Students who very much like learning mathematics scored significantly higher on average on overall mathematics than did students who like learning mathematics. The mean mathematics score of

[16] Students who very much like learning mathematics have a scale score of at least 11.4 points, which corresponds to their agreeing a lot with five of the nine statements and agreeing a little with the remaining four, on average. Students who do not like learning mathematics have a score no higher than 9.4, which corresponds to their disagreeing a little with five of the nine statements and agreeing a little with the other four, on average. All other students are classified as having a liking for learning mathematics (Mullis et al., 2016, Exhibit 10.4).

students who very much liked learning mathematics is over one-half of a standard deviation higher than that of those who did not like learning mathematics.

Table 3.5. Percentages of students who very much like learning mathematics, like learning mathematics and do not like learning mathematics, and average achievement in overall mathematics, Ireland, TIMSS 2015 – Grade 8

	Overall Mathematics				
	%	SE	Mean	SE	SD
Very much like learning maths	13.6	0.71	**561.9**	(4.59)	71.75
Like learning maths (ref)	34.7	0.92	537.1	(3.13)	71.93
Do not like learning maths	51.7	1.23	**504.6**	(2.81)	69.92

Significantly different mean scores are in bold.

A similar percentage of male students (14.0%) as female students (13.3%) very much liked learning mathematics (Table 3.6). However, a greater percentage of female students (55.3%) than male students (48.0%) did not like learning mathematics. In all three categories (very much like learning mathematics, like learning mathematics, and do not like learning mathematics), male students had slightly higher mean scores on overall mathematics than female students, but the differences are not significant.

Table 3.6. Percentages of students who very much like learning mathematics, like learning mathematics and do not like learning mathematics, and average achievement on overall mathematics by gender and gender differences, Ireland, TIMSS 2015 – Grade 8

		Overall Mathematics				
		%	SE	Mean	SE	SD
Very much like learning maths	Males (ref)	14.0	(1.12)	563.2	(6.63)	72.77
	Females	13.3	(0.72)	560.5	(5.80)	70.62
Like learning maths	Males (ref)	38.1	(1.28)	540.0	(4.35)	73.40
	Females	31.4	(1.33)	533.6	(3.38)	69.96
Do not like learning maths	Males (ref)	48.0	(1.67)	505.1	(4.11)	72.99
	Females	55.3	(1.58)	504.1	(2.98)	67.17

Significantly different mean scores are in bold.

Students in schools participating in the SSP under DEIS (20.6% of students) had a significantly lower score on the students' liking for learning mathematics scale than students whose schools are not part of the SSP (79.4% of students) (Table 3.7).

Table 3.7. Mean scores on the students' liking for learning mathematics scale by school SSP (DEIS) status, Ireland, TIMSS 2015 – Grade 8

	Overall Mathematics				
	%	SE	Mean	SE	SD
In SSP	20.6	(2.79)	**9.09**	(0.12)	1.99
Non-SSP (ref)	79.4	(2.79)	9.37	(0.05)	1.92

Significantly different mean scores are in bold.

In both SSP and non-SSP schools, a substantial proportion of students (at least 50%) did not like learning mathematics (Table 3.8). However, the percentage of students who did not like learning mathematics is slightly greater in SPP schools (56.3%) than in non-SSP schools (50.4%). Non-SSP schools have a slightly greater percentage of students who very much liked learning mathematics (14.5%) than SSP schools (10.4%). There is little difference between students attending SSP (33.3%) and non-SSP schools (35.1%) in the percentages classified as liking learning mathematics (Table 3.8).

Average mathematics performance of students in SSP schools is lower (by more than one-half standard deviation) than that of students in non-SSP schools at all three levels of liking mathematics learning (Table 3.8), reflecting lower overall performance in such schools.

Table 3.8. Percentages of students in SSP (DEIS) and non-SSP schools who very much like learning mathematics, like learning mathematics, and do not like learning mathematics, and average achievement on overall mathematics, Ireland, TIMSS 2015 – Grade 8

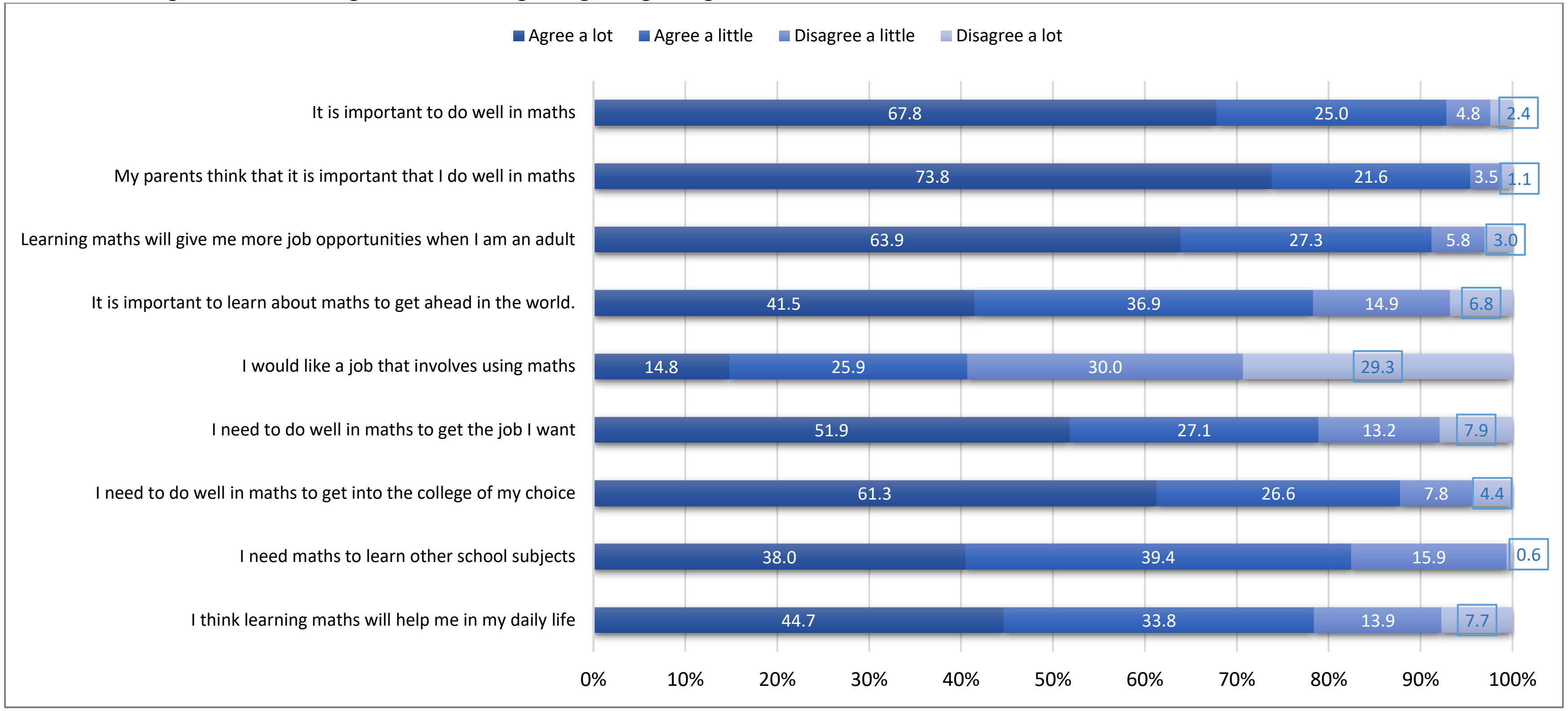
		Overall Mathematics				
		%	SE	Mean	SE	SD
Very much like learning maths	In SSP	10.4	1.27	**514.0**	(12.05)	82.01
	Non SSP (ref)	14.5	0.81	570.8	(4.85)	65.92
Like learning maths	In SSP	33.3	2.13	**486.1**	(8.80)	78.44
	Non SSP (ref)	35.1	0.98	549.6	(3.02)	64.32
Do not like learning maths	In SSP	56.3	2.66	**466.5**	(7.39)	78.05
	Non SSP (ref)	50.4	1.30	515.6	(2.95)	63.23

Significantly different mean scores are in bold.

3.2.2. Students' Valuing of Mathematics

TIMMS 2015 also asked students to indicate their level of agreement/disagreement with statements about the extent to which they value mathematics. These questions typically focus on the utilitarian or instrumental value of mathematics, rather than a liking for learning mathematics or a liking of mathematics for its own sake. Nearly three-quarters of students in Ireland (73.8%) agreed a lot that their parents think it is important that they do well in mathematics, while more than two-thirds of students (67.8%) agreed a lot that it is important to do well in mathematics (Figure 3.3). 'Agree a lot' was also selected by large percentages of students in relation to the statements: 'learning maths will give me more job opportunities when I am an adult' (63.9%); 'I need to do well in maths to get into the college of my choice' (61.3%); 'I need to do well in maths to get the job I want' (51.9%); and 'it is important to learn about maths to get ahead in the world' (41.5%). Some 40.7% of students agreed a little or a lot that they would like a job that involves using mathematics, with 14.8% agreeing a lot with this statement. Overall, 59.3% of students disagreed that they would like a job that involves using mathematics, and some 29.3% disagreed a lot.

Figure 3.3. Percentages of students agreeing/disagreeing with statements about their value of mathematics, Ireland, TIMSS 2015 – Grade 8



There was less agreement among female students than among male students that it is important to learn about mathematics to get ahead in the world (81.5% for males and 75.3% for females), that mathematics will help them in their daily life (80.8% for males and 76.2% for females), that they need to do well in mathematics to get the job they want (81.2% for males and 76.7%) and to get into the college of their choice (90.1% for females and 85.7% for females) (Figure 3.4). While similar proportions of male and female students agreed that they need mathematics to learn other school subjects and that it is important to do well in mathematics, a noticeably smaller percentage of female students than male students agreed that they would like a job that involves using mathematics (46.1% of males and 35.5% of females).

Figure 3.4. Percentages of male and female students agreeing a little or a lot with statements about their value of mathematics, Ireland, TIMSS 2015 – Grade 8

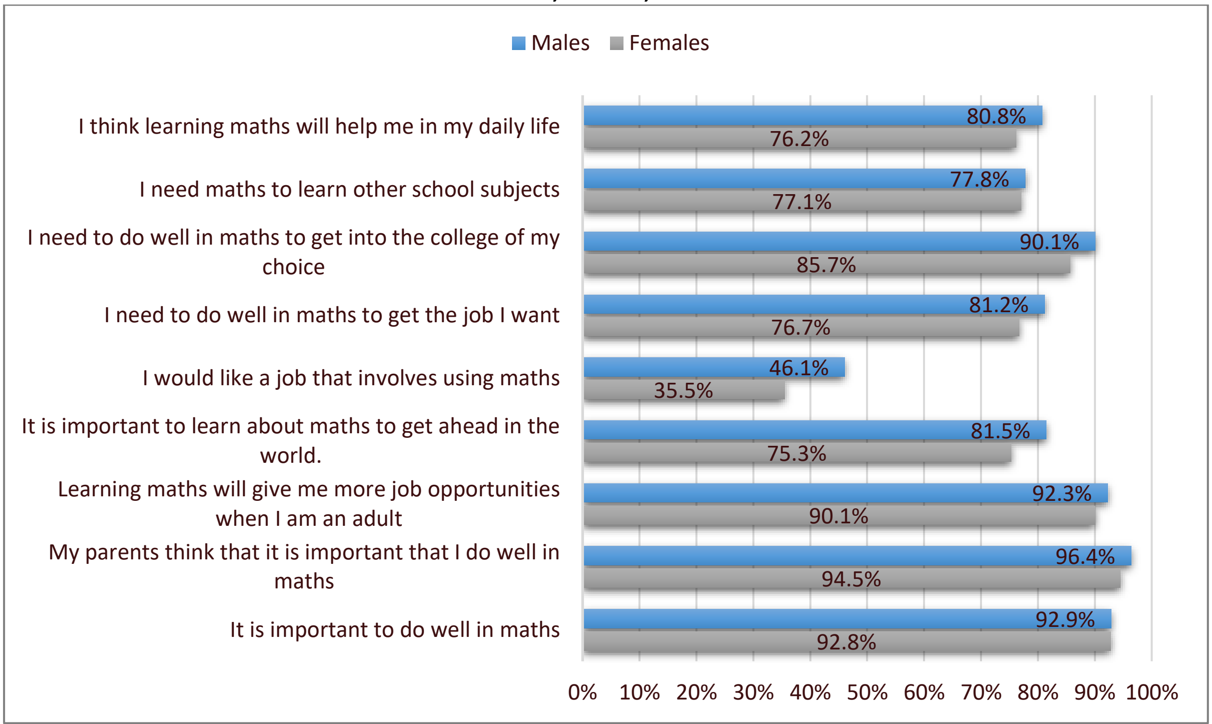


Table 3.9 presents mean scores on the Students Value Mathematics scale, where higher scores represent a greater valuing of mathematics. The mean score of students in Ireland on the scale is significantly higher than the mean score of students on average across the 16 OECD countries that participated in TIMSS. Hence students in Ireland placed a greater value on mathematics than did students on average across the OECD-16. Students Value Mathematics has a weak-to-moderate positive and significant correlation (.15) with performance on overall mathematics in Ireland.

Table 3.9. Mean scores on the Students Value Mathematics scale in Ireland and across 16 OECD countries on average, TIMSS 2015 – Grade 8

	Students Value Mathematics		
	Mean	SE	SD
Ireland	**9.8**	(0.04)	1.83
OECD 16 (ref)	9.6	(0.01)	1.84

Significantly different mean scores are in bold. OECD 16 = 16 participating OECD countries.

In Ireland, and on average across the 16 OECD countries in TIMSS 2015, female students achieved a significantly lower score than male students on the Students Value Mathematics scale (Table 3.10).

However, female students in Ireland achieved a significantly higher score on the scale than did female students across the 16 OECD countries on average (Table 3.11). Similarly, male students in Ireland achieved a higher score on the Students Value Mathematics scale than did male students across the participating OECD countries on average (Table 3.11).

Table 3.10. Mean scores on the Students Value Mathematics scale by gender, in Ireland and across 16 OECD countries on average, TIMSS 2015 – Grade 8

	Students Value Mathematics							
	Ireland				OECD 16			
	%	Mean	SE	SD	%	Mean	SE	SD
Males (ref)	49.6	10.0	(0.05)	1.80	50.3	9.8	(0.01)	1.94
Females	50.4	**9.6**	(0.05)	1.85	49.7	**9.5**	(0.01)	1.71

Significantly different mean scores are in bold. OECD 16 = 16 participating OECD countries. (Male vs. Female, Ireland, OECD)

Table 3.11. Mean scores on the Students Value Mathematics scale compared for males and for females in Ireland and across 16 OECD countries on average, TIMSS 2015 – Grade 8

	Students Value Mathematics							
	Males				Females			
	%	Mean	SE	SD	%	Mean	SE	SD
Ireland	49.6	**10.0**	(0.05)	1.80	50.4	**9.6**	(0.05)	1.85
OECD 16 (ref)	50.3	9.8	(0.01)	1.94	49.7	9.5	(0.01)	1.71

Significantly different mean scores are in bold. OECD 16 = 16 participating OECD countries.
(Male vs. Male, Female vs. Female, Ireland and OECD).

In Ireland, students whose parents' highest education level is at least upper secondary scored higher on the Students Value Mathematics scale than students whose parents' highest completed education level is some primary, lower secondary or no school (Table 3.12).

Table 3.12. Mean scores on the Students Value Mathematics scale by parents' highest education level, Ireland, TIMSS 2015 – Grade 8

	Students Value Mathematics				
	%	SE	Mean	SE	SD
University or higher	33.1	(1.11)	**10.1**	(0.05)	1.78
Post-secondary but not university	21.0	(0.72)	**9.8**	(0.07)	1.86
Upper secondary	16.2	(0.68)	**9.7**	(0.07)	1.82
Lower secondary	4.5	(0.52)	9.3	(0.12)	1.76
Some primary, lower secondary, or no school (ref)	2.3	(0.26)	8.9	(0.29)	2.18
Don't know	22.9	(0.90)	**9.7**	(0.06)	1.80

Significantly different mean scores are in bold.

TIMSS created a value of mathematics index, with students categorised as either strongly valuing mathematics, valuing mathematics, or not valuing mathematics.[17] In Ireland, a substantial proportion of students strongly valued mathematics (40.9%), while a minority (11.4%) did not value mathematics (Table 3.13). Compared to those who did not value mathematics, those who valued mathematics or

[17] Students who strongly value mathematics have a scale score of at least 10.3, corresponding to their agreeing a lot with five of the nine statements and agreeing a little with the remaining four, on average. Students who do not value mathematics have a scale score no higher than 7.7, which corresponds to their disagreeing a little with five of the nine statements and agreeing a little with the other four, on average. All other students value mathematics (Mullis et al., 2016).

strongly valued mathematics achieved significantly higher mean scores on overall mathematics (Table 3.13).

Table 3.13. Percentages of students who strongly value mathematics, value mathematics and do not value mathematics, and average achievement on overall mathematics, Ireland, TIMSS 2015 – Grade 8

	Overall Mathematics				
	%	SE	Mean	SE	SD
Strongly value maths	40.9	(0.86)	**533.9**	(3.32)	74.61
Value maths	47.7	(0.82)	**520.5**	(3.11)	73.02
Do not value maths (ref)	11.4	(0.54)	500.6	(4.63)	69.20

Significantly different mean scores are in bold.

In Ireland, more male students (45.0%) than female students (36.9%) strongly valued mathematics (Table 3.14). Conversely, more female students (13.5%) than male students (9%) did not value mathematics. Male and female students who valued mathematics scored similarly to each other on overall mathematics (Table 3.14). Students who did not value mathematics also did not differ on overall mathematics on the basis on gender (Table 3.14). Among students who strongly valued mathematics, male students achieved a higher score than female students on overall mathematics, but the difference is not significant.

Table 3.14. Percentages of students who strongly value mathematics, value mathematics and do not value mathematics, and average achievement on overall mathematics by gender, Ireland, TIMSS 2015 – Grade 8

		Overall Mathematics				
		%	SE	Mean	SE	SD
Strongly value maths	Males (ref)	45.0	(1.17)	538.1	(4.71)	73.8
	Females	36.9	(1.19)	528.8	(3.53)	75.2
Value maths	Males (ref)	45.7	(1.21)	520.3	(4.89)	78.0
	Females	49.6	(1.01)	520.6	(2.76)	68.2
Do not value maths	Males (ref)	9.3	(0.65)	500.6	(6.45)	70.4
	Females	13.5	(0.79)	500.6	(5.50)	68.3

Significantly different mean scores are in bold.

Students in SSP schools and non-SSP schools did not differ from one another in their average scores on the Students Value Mathematics scale (Table 3.15). Similar proportions of students in SSP schools (39.6%) as non-SSP schools (41.3%) strongly valued mathematics (Table 3.16). Also, in both types of schools, around one-in-ten students did not value mathematics (11.4% both in SSP and in non-SSP schools). On average, students in non-SSP schools scored significantly higher on overall mathematics than students in SSP schools irrespective of how much they valued mathematics (Table 3.16).

Table 3.15. Mean scores on the Students Value Mathematics scale by school SSP (DEIS) status, Ireland, TIMSS 2015 – Grade 8

	Students Value Mathematics				
	%	SE	Mean	SE	SD
In SSP	20.6	(2.78)	9.76	(0.08)	1.89
Non-SSP (ref)	79.5	(2.78)	9.82	(0.04)	1.82

Significantly different mean scores are in bold.

Table 3.16. Percentages of students in SSP (DEIS) and non-SSP schools who strongly value mathematics, value mathematics and do not value mathematics, Ireland, TIMSS 2015 – Grade 8

		Overall Mathematics				
		%	SE	Mean	SE	SD
Strongly value maths	In SSP	39.6	(1.46)	**488.3**	(8.12)	80.86
	Non SSP (ref)	41.3	(1.01)	545.2	(3.29)	68.42
Value maths	In SSP	49.1	(2.93)	**473.7**	(8.31)	79.11
	Non SSP (ref)	47.3	(0.86)	533.0	(3.14)	65.86
Do not value maths	In SSP	11.4	(1.35)	**459.5**	(11.45)	77.42
	Non SSP (ref)	11.4	(0.59)	511.2	(4.74)	62.67

Significantly different mean scores are in bold.

3.2.3. Students' Mathematics Confidence

TIMSS 2015 assessed Second year students' mathematics confidence by asking them to indicate their agreement/disagreement with a set of statements, such as 'I usually do well in maths', and 'maths makes me nervous' (Figure 3.5). The majority of students in Ireland (74.1%) agreed (a little or a lot) that they usually do well in mathematics, with some 29.0% of students strongly agreeing (agreeing a lot) with this statement. Around 58% of students agreed (a little and a lot) that they learn things quickly in mathematics, and 47.8% agreed a little or a lot that they are good at working out difficult mathematics problems. Some 39.8% of students similarly agreed that mathematics is more difficult for them than for their classmates, and 38.7% reported that mathematics is harder for them than any other subject. While 52.5% of students agreed a little or a lot that mathematics is not one of their strengths, 26.4% disagreed and 21.1% disagreed a lot. Similarly, while 52.2% of students agreed that mathematics makes them confused, 47.6% disagreed and 21.6% strongly disagreed (disagree a lot). Around one-in-ten students (11.2%) strongly agreed (agreed a lot) that mathematics makes them nervous. Overall, however, the majority of students (65.0%) disagreed to some extent that mathematics makes them nervous. More than half of students (57.2%) agreed that their teacher tells them they are good at mathematics, while 42.9% disagreed with this statement.

Female students were less likely than male students to agree (a little or a lot) that they usually do well in mathematics (76.4% for males and 42.0% for females), that their teacher tells them they are good at mathematics (59.1% for males and 55.2% for females), that they learn things quickly in mathematics (61.1% for males and 54.6% for females), and that they are good at working out difficult mathematics problems (55.5% for males and 40.3% for females) (Figure 3.6). Greater percentages of female students than male students reported that mathematics is not one of their strengths (48.8% for males and 56.1% for females), that mathematics made them confused (48.6% for males and 55.7% for females), and that mathematics made them nervous (30.8% for males and 34.2% for females). Female students were also more likely than male students to agree that mathematics is harder for them than any other subject (34.6% for males and 42.8% for females).

Figure 3.5. Percentages of students agreeing/disagreeing with various statements about their confidence in mathematics, Ireland, TIMSS 2015 – Grade 8

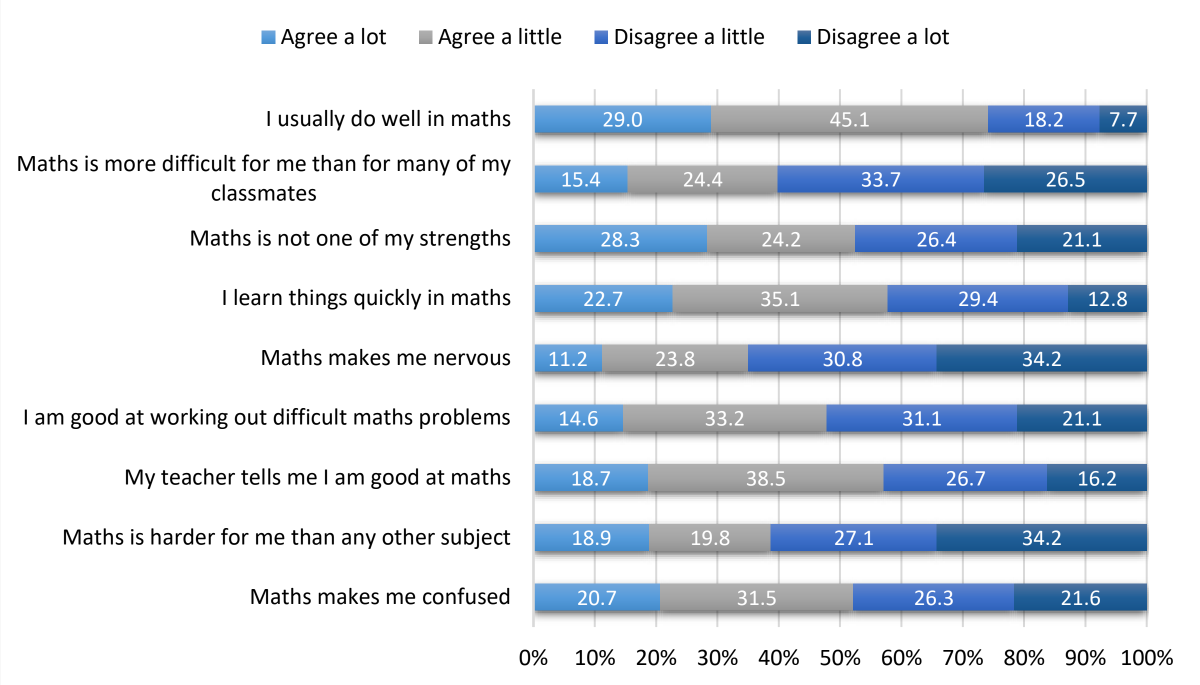


Figure 3.6. Percentages of male and female students agreeing a little or a lot with statements about their confidence in mathematics, Ireland, TIMSS 2015 – Grade 8

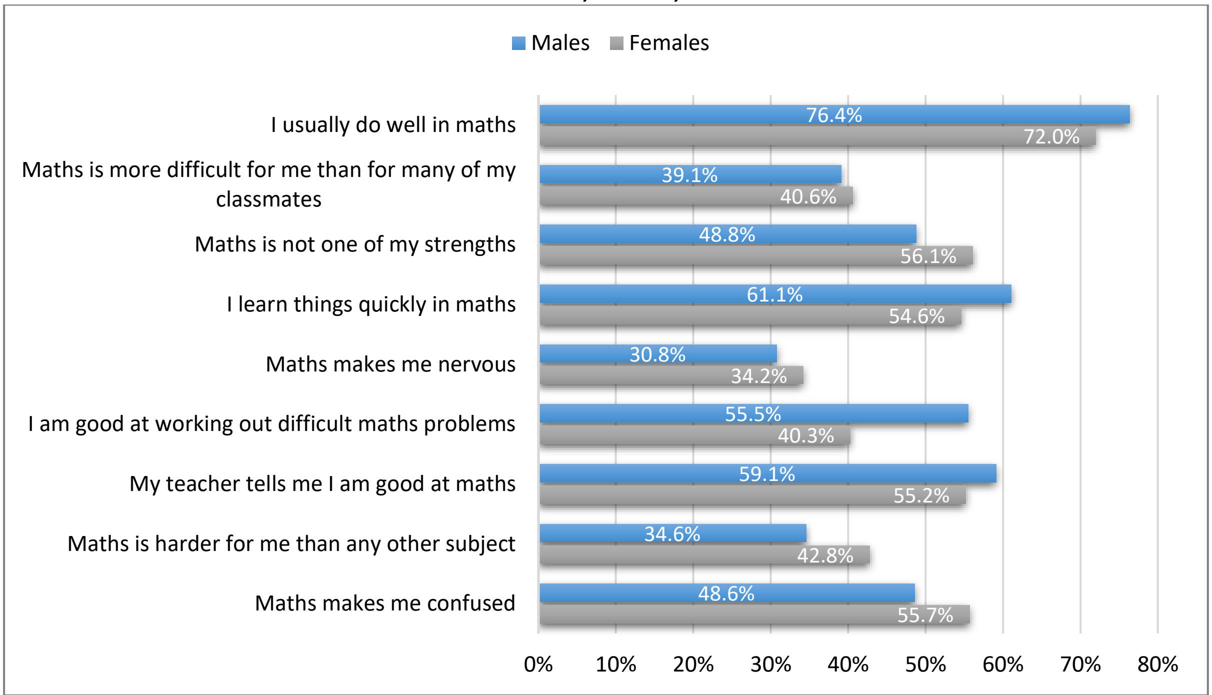


Table 3.17 presents mean scores on the 'Students Confident in Mathematics' scale, with higher scores indicating greater mathematics confidence. The mean score of students in Ireland on the students' confidence in Mathematics scale does not differ from the mean score of students across the 16 OECD countries participating in TIMSS, on average.

Table 3.17. Mean scores on the students' confident in mathematics scale in Ireland and across 16 OECD countries on average, TIMSS 2015 – Grade 8

	Students' Confidence in Mathematics		
	Mean	SE	SD
Ireland	10.0	(0.05)	2.14
OECD 16 (ref)	10.0	(0.01)	2.16

Significantly different mean scores are in bold. OECD 16 = 16 participating OECD countries.

In Ireland, and across the 16 OECD countries on average, female students had a significantly lower mean score than male students on the Students' Confident in Mathematics scale (Table 3.18). However, female students in Ireland did not differ from female students across the 16 OECD countries on average in their mathematics confidence (Table 3.19). Additionally, male students in Ireland do not differ significantly from their male OECD counterparts in mathematics confidence either (Table 3.19).

Table 3.18. Mean scores on the students' confidence in mathematics scale by gender, in Ireland and across 16 OECD countries on average, TIMSS 2015 – Grade 8

	Students' Confidence in Mathematics							
	Ireland				OECD 16			
	%	Mean	SE	SD	%	Mean	SE	SD
Males (ref)	49.6	10.3	(0.07)	2.08	50.3	10.3	(0.02)	2.11
Females	50.4	**9.8**	(0.05)	2.18	49.7	**9.8**	(0.01)	2.18

Significantly different mean scores are in bold. OECD 16 = 16 participating OECD countries. (Male vs. Female, Ireland, OECD)

Table 3.19. Mean scores on the students' confidence in mathematics scale compared for males and for females in Ireland and across 16 OECD countries on average, TIMSS 2015 – Grade 8

	Students' Confidence in Mathematics							
	Males				Females			
	%	Mean	SE	SD	%	Mean	SE	SD
Ireland	49.6	10.3	(0.07)	2.08	50.4	9.8	(0.05)	2.18
OECD 16 (ref)	50.3	10.3	(0.02)	2.11	49.7	9.8	(0.01)	2.18

Significantly different mean scores are in bold. OECD 16 = 16 participating OECD countries.
(Male vs. Female, Ireland, OECD)

Students whose parents' highest education level is at least upper secondary school achieved higher mean scores on the Students' Confidence in Mathematics scale than did students whose parents' highest level of education is some primary school, lower secondary, or no school (Table 3.20).

Table 3.20. Mean scores on the students' confidence in mathematics scale by parents' highest education level, Ireland, TIMSS 2015 – Grade 8

	Students' Confidence in Mathematics				
	%	SE	Mean	SE	SD
University or higher	33.1	(1.11)	**10.3**	(0.08)	2.28
Post-secondary but not university	21.0	(0.72)	**10.2**	(0.08)	2.19
Upper secondary	16.2	(0.68)	**9.8**	(0.11)	2.07
Lower secondary	4.5	(0.52)	9.5	(0.16)	1.85
Some primary, lower secondary, or no school (ref)	2.3	(0.26)	9.3	(0.20)	2.06
Don't know	22.9	(0.90)	**9.8**	(0.05)	1.94

Significantly different mean scores are in bold.

Using students' responses to the mathematics confidence items, TIMSS categorised students according to whether they are very confident in mathematics, confident in mathematics, or not confident in mathematics.[18] In Ireland, 15.7% of students are described as feeling very confident in mathematics, 41.5% as confident in mathematics, and 42.8% as not confident in mathematics (Table 3.21). Students who are confident and students who are very confident in mathematics both had significantly higher mean scores on overall mathematics than did students who are not confident in mathematics (Table 3.21). The score difference in mathematics between students who are very confident and students who are not confident in mathematics was more than one standard deviation (91.2 scale score points). Additionally, the students' confidence in mathematics scale had a moderate-to-strong positive and significant correlation (.44) with overall mathematics in Ireland.

Table 3.21. Percentages of students who are very confident, confident and not confident in mathematics and average achievement in overall mathematics, Ireland, TIMSS 2015 – Grade 8

	Overall Mathematics				
	%	SE	Mean	SE	SD
Very confident in mathematics	15.7	(0.76)	**583.0**	(3.95)	62.17
Confident in mathematics	41.5	(0.87)	**534.1**	(2.90)	67.30
Not confident in mathematics (ref)	42.8	(1.01)	491.8	(3.18)	67.54

Significantly different mean scores are in bold.

A greater percentage of male students than female students in Ireland were very confident (17.7% of males and 13.7% of females) or confident (43.6% of males and 39.6% of females) in mathematics, whereas a greater percentage of female students than male students were not confident in mathematics (38.7% for males and 46.8% for females) (Table 3.22). No significant differences are observed in scores on overall mathematics between male and female students who were very confident in mathematics, between male and female students who were confident in mathematics, or between male and female students who were not confident in mathematics (Table 3.22).

Table 3.22. Percentages of students who are very confident, confident and not confident in mathematics and average achievement in overall mathematics by gender, Ireland, TIMSS 2015 – Grade 8

		Overall Mathematics				
		%	SE	Mean	SE	SD
Very confident in mathematics	Males (ref)	17.7	(1.10)	588.8	(5.31)	62.96
	Females	13.7	(0.75)	575.6	(4.61)	60.32
Confident in mathematics	Males (ref)	43.6	(1.22)	533.8	(3.83)	67.08
	Females	39.6	(1.19)	534.4	(3.20)	67.54
Not confident in mathematics	Males (ref)	38.7	(1.38)	489.5	(4.81)	70.54
	Females	46.8	(1.37)	493.6	(3.15)	64.93

Significantly different mean scores are in bold.

Students in SSP schools scored significantly lower on the students' confidence in mathematics scale than did students in non-SSP schools (Table 3.23). In SSP schools, 11.8% of students were very confident in mathematics compared to 16.7% in non-SSP schools (Table 23). Similar proportions of students in SSP schools (40.7%) and non-SSP schools (41.8%) were confident in mathematics. SSP

[18] Students who are very confident in mathematics have a scale score of at least 12.1, which corresponds to their agreeing a lot with five of the nine mathematics confidence statements and agreeing a little with the remaining four, on average. Students who are not confident in mathematics have a score no higher than 9.5, which corresponds to their disagreeing a little with five of the nine statements and agreeing a little with the remaining four, on average. All other students are classified as confident in mathematics (Mullis et al., 2016).

schools (47.5%) had a greater percentage of students who are not confident in mathematics than non-SSP schools (41.6%).

Table 3.23. Mean scores on the students' confidence in mathematics scale by school SSP (DEIS) status, Ireland, TIMSS 2015 – Grade 8

	Students Confident in Mathematics			
	%	Mean	SE	SD
In SSP	20.5	**9.8**	(0.10)	2.13
Non-SSP (ref)	79.5	10.1	(0.05)	2.17

Significantly different mean scores are in bold.

Table 3.24. Percentages of students in SSP (DEIS) and non-SSP schools who are very confident, confident and not confident in mathematics and average achievement in overall mathematics, Ireland, TIMSS 2015 – Grade 8

		Overall Mathematics				
		%	SE	Mean	SE	SD
Very confident in mathematics	In SSP	11.8	(1.50)	**545.1**	(10.03)	74.06
	Non-SSP (ref)	16.7	(0.83)	589.9	(4.27)	57.09
Confident in mathematics	In SSP	40.7	(1.84)	**490.9**	(7.64)	74.41
	Non-SSP (ref)	41.8	(0.97)	545.0	(3.08)	60.72
Not confident in mathematics	In SSP	47.5	(1.90)	**450.0**	(8.26)	73.45
	Non-SSP (ref)	41.6	(1.06)	504.1	(3.12)	60.39

Significantly different mean scores are in bold.

3.2.4. Students' Views on Engaging Teaching in Mathematics Classes

TIMSS was also interested in students' views on engaging teaching in mathematics classes and asked them to indicate their level of agreement/disagreement with a set of ten statements about their experiences of engaging teaching practices, such as 'my teacher is good at explaining' and 'my teacher listens to what I have to say'. Students in Ireland indicated a high level of agreement overall with the many of the statements (Figure 3.7). For example, most students agreed (a little and a lot) that they know what their teacher expects them to do (89.1%), that their teacher listens to what they have to say (83.7%), that their teacher tells them how to do better when they make a mistake (82.8%), and that their teacher is easy to understand (75.8%), and around half of students agree a lot with these statements. The highest level of disagreement (a little or a lot) was with the statement 'my teacher gives me interesting things to do', with which 31.9% of students disagreed a little, and 13.5% disagreed a lot. Additionally, more than a quarter of students disagreed a little or a lot with the statements 'I am interested in what my teacher says' (28.9%) and 'my teacher does a variety of things to help us learn' (28.1%). More than one third of students (37.2%) similarly disagreed that their teacher lets them show what they have learned.

Male and female students in Ireland reported broadly similar levels of agreement with the various statements about their experiences of engaging teaching in mathematics classes (Figure 3.8). Small differences between male and female students in their views on engaging teaching relate to their agreement with the statements 'my teacher does a variety of things to help us learn' (74.2% of males and 69.7% of females agree a little or a lot), 'my teacher has clear answers to my questions' (75.5% of males and 72.4% of females), 'my teacher gives me interesting things to do' (55.7% of males and 53.5%

Figure 3.7. Percentages of students agreeing/disagreeing with statements about engaging teaching in mathematics classes, Ireland, TIMSS 2015 – Grade 8

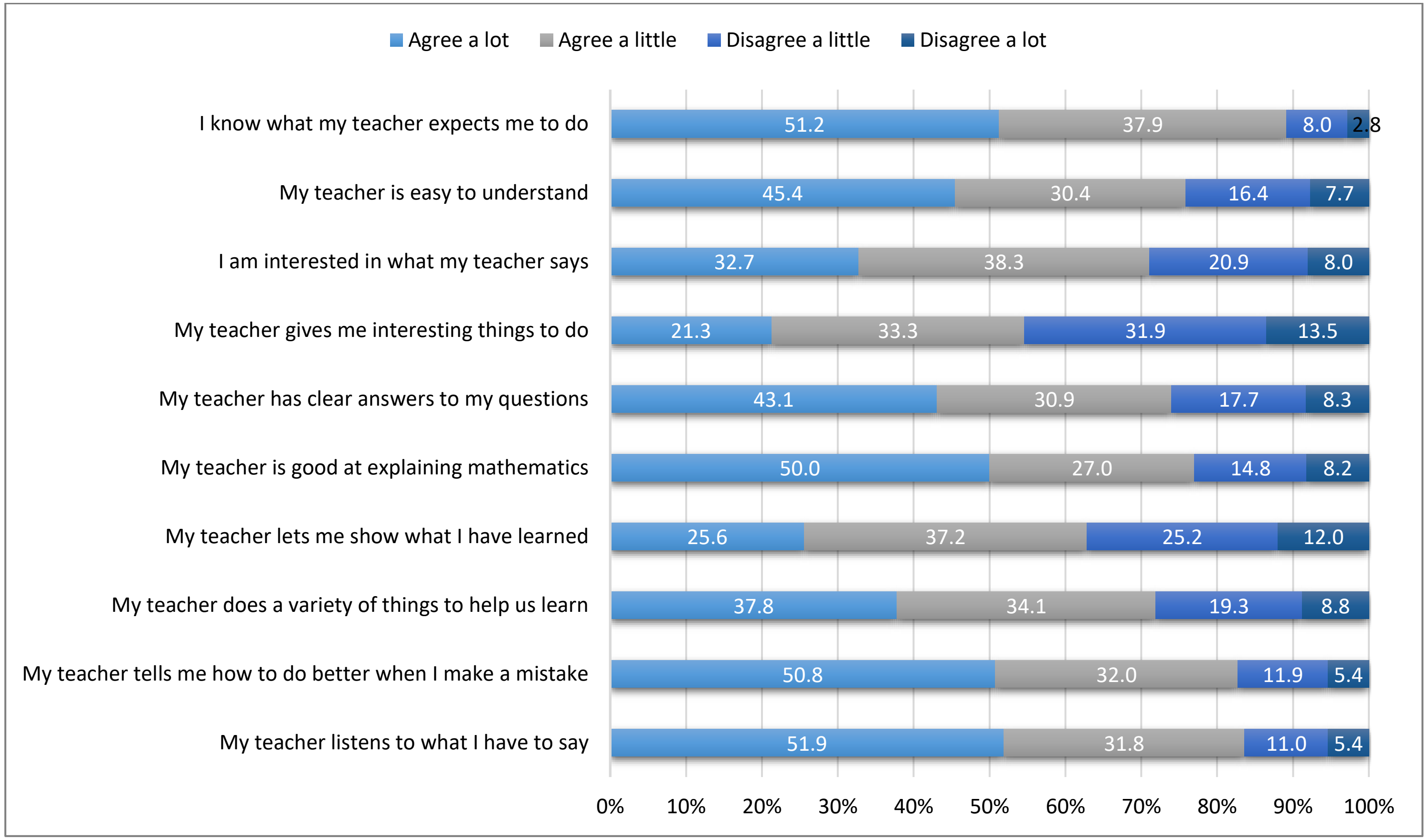
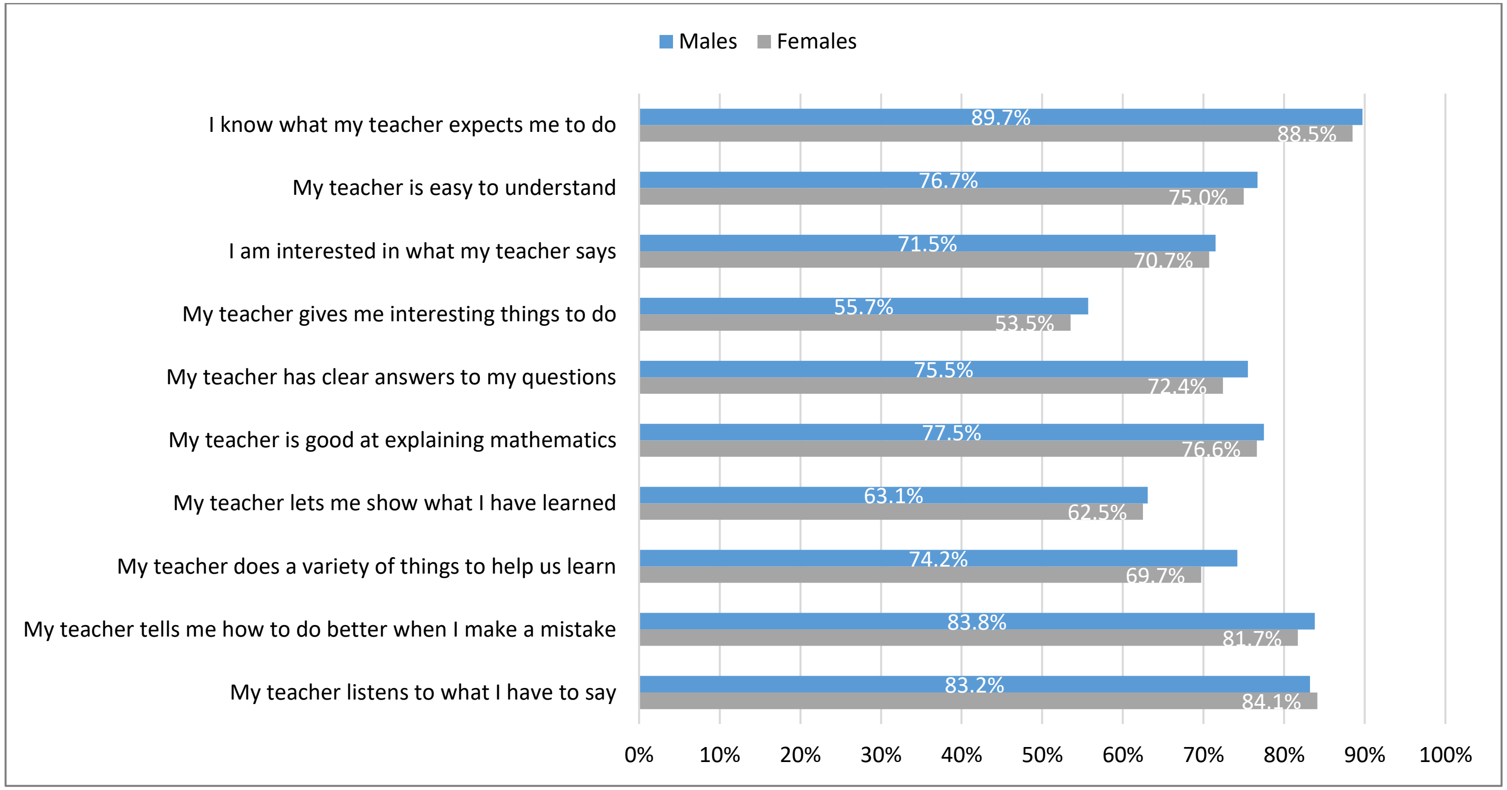


Figure 3.8. Percentages of male and female students agreeing a little or a lot with statements about engaging teaching in mathematics classes, Ireland, TIMSS 2015 – Grade 8



of females), and 'my teacher tells me how to do better when I make a mistake' (83.8% of males and 81.7% of females).

A 'Students' Views on Engaging Teaching in Mathematics' scale was constructed using the ten items, with higher scores indicating more engaging teaching in mathematics classes, from students' perspectives. On average, students in Ireland did not score differently on the scale to students across the 16 OECD countries (Table 3.25). The correlation between Students' Views on Engaging Teaching in Mathematics and performance on overall mathematics in Ireland (.04) is not significant.

Table 3.25. Mean scores on the students' views on engaging teaching in mathematics scale in Ireland and across 16 OECD countries on average, Ireland, TIMSS 2015 – Grade 8

	Students' Views on Engaging Teaching in Mathematics		
	Mean	SE	SD
Ireland	9.7	(0.06)	1.93
OECD 16 (ref)	9.6	(0.02)	1.85

Significantly different mean scores are in bold. OECD 16 = 16 participating OECD countries.

In Ireland, and across the 16 OECD countries on average, male and female students did not differ from one another in terms of their scores on the engaging teaching in mathematics scale, indicating that, overall, they felt they experience a similar level of engaging teaching in mathematics classes (Table 3.26). Also, male students in Ireland did not differ on the scale from male students across the 16 OECD countries on average, nor did female students in Ireland differ from female students in participating OECD countries on average (Table 3.27).

Table 3.26. Mean scores on the students' views on engaging teaching in mathematics scale by gender, Ireland, TIMSS 2015 – Grade 8

	Students' Views on Engaging Teaching in Mathematics							
	Ireland				OECD 16			
	%	Mean	SE	SD	%	Mean	SE	SD
Males (ref)	49.6	9.7	(0.08)	1.94	50.4	9.6	(0.02)	1.89
Females	50.4	9.7	(0.07)	1.93	49.7	9.6	(0.02)	1.80

Significantly different mean scores are in bold. OECD 16 = 16 participating OECD countries. (Male vs. Female, Ireland, OECD).

Table 3.27: Mean scores on the students' views on engaging teaching in mathematics scale for male students and for female students, in Ireland and across 16 OECD countries on average, Ireland, TIMSS 2015 – Grade 8

	Students' Views on Engaging Teaching in Mathematics							
	Males				Females			
	%	Mean	SE	SD	%	Mean	SE	SD
Ireland	49.6	9.7	(0.08)	1.94	50.4	9.7	(0.07)	1.93
OECD 16 (ref)	50.4	9.6	(0.02)	1.89	49.7	9.6	(0.02)	1.80

Significantly different mean scores are in bold. OECD 16 = 16 participating OECD countries. (Male vs. Male, Female vs. Female, Ireland, OECD).

In Ireland, students did not differ in their scores on the Engaging Teaching in Mathematics scale on the basis of their parents' highest educational level (Table 3.28).

Table 3.28. Mean scores on the students' views on engaging teaching in mathematics scale by parents' highest education level, Ireland, TIMSS 2015 – Grade 8

	Students' Views on Engaging Teaching in Mathematics				
	%	SE	Mean	SE	SD
University or higher	33.1	(1.11)	9.7	(0.08)	2.01
Post-secondary but not university	21.0	(0.72)	9.7	(0.09)	1.90
Upper secondary	16.2	(0.68)	9.7	(0.09)	1.84
Lower secondary	4.5	(0.52)	9.5	(0.13)	1.89
Some primary, lower secondary, or no school (ref)	2.3	(0.26)	9.3	(0.22)	2.19
Don't know	22.9	(0.90)	9.7	(0.07)	1.89

Significantly different mean scores are in bold.

Using students' responses to the items addressing their views on engaging teaching, TIMSS constructed an index of Engaging Teaching in Mathematics Classes, with students categorised as experiencing very engaging teaching, engaging teaching, or less than engaging teaching.[19] Using this index, 37.1% of students in Ireland experienced very engaging teaching in mathematics classes, 40.6% of students experience engaging teaching, and 22.4% experienced less than engaging teaching (Table 3.29). Students who experienced very engaging teaching in mathematics classes scored significantly higher on overall mathematics than students who experienced less than engaging teaching (Table 3.29).

Table 3.29. Percentages of students reporting very engaging, engaging, and less than engaging teaching in mathematics and average achievement in overall mathematics, Ireland, TIMSS 2015 – Grade 8

	Overall Mathematics				
	%	SE	Mean	SE	SD
Very engaging teaching	37.1	(1.39)	**527.7**	(3.30)	74.57
Engaging teaching	40.6	(0.97)	523.4	(3.41)	74.47
Less than engaging teaching (ref)	22.4	(1.08)	517.5	(3.78)	71.69

Significantly different mean scores are in bold.

Similar percentages of male students as female students in Ireland were classified as experiencing very engaging teaching in mathematics classes, while slightly more female students than male students were classified as experiencing less than engaging teaching (21.4% for males and 23.3% for females) (Table 3.30). There are no significant gender differences on overall mathematics among students who experienced very engaging teaching, among students who experienced engaging teaching, and among students who experienced less than engaging teaching.

[19] Students are deemed to have experienced very engaging teaching in mathematics classes, if they have a score of at least 10.4, which corresponds to their agreeing a lot with five of the ten statements and agreeing a little with the other five, on average. Students are deemed to have experienced less than engaging teaching if they have a score no higher than 8.2, which corresponds to their disagreeing a little with five of the ten statements and agreeing a little with the other five, on average. All other students experienced engaging teaching in mathematics (Mullis et al., 2016).

Table 3.30. Percentages of males and female students reporting very engaging, engaging, and less than engaging teaching in mathematics and average achievement in overall mathematics, Ireland, TIMSS 2015 –Grade 8

		Overall Mathematics				
		%	SE	Mean	SE	SD
Very engaging teaching	Males (ref)	36.9	(2.08)	531.5	(5.10)	74.49
	Females	37.2	(1.68)	523.9	(3.84)	74.43
Engaging teaching	Males (ref)	41.7	(1.28)	527.6	(5.12)	77.64
	Females	39.5	(1.35)	519.1	(3.36)	70.77
Less than engaging teaching	Males (ref)	21.4	(1.42)	515.8	(5.13)	75.82
	Females	23.3	(1.45)	519.0	(4.88)	67.68

Significantly different mean scores are in bold.

Students in Ireland did not differ from one another in their views on engaging teaching in mathematics classes on the basis of whether or not their school is in the SSP under DEIS (Table 3.31). The proportion of students experiencing very engaging teaching in mathematics classes is slightly greater in non-SSP schools (37.5%) than in SSP schools (35.4%) (Table 3.32). The proportion of students experiencing less than engaging teaching in mathematics classes was also slightly greater in non-SSP schools (22.7%) than in SSP schools (20.9%). In all three categories of engaging teaching, students in SSP schools have significantly lower scores on overall mathematics than do students in non-SSP schools (Table 3.32).

Table 3.31. Mean scores on the students' views on engaging teaching in mathematics scale by school SSP (DEIS) status, Ireland, TIMSS 2015 – Grade 8

	Overall Mathematics				
	%	SE	Mean	SE	SD
In SSP	20.6	(2.79)	9.7	(0.07)	1.91
Non-SSP (ref)	79.4	(2.79)	9.7	(0.09)	1.94

Significantly different mean scores are in bold.

Table 3.32. Percentages of students in SSP (DEIS) and non-SSP schools who report very engaging, engaging, and less than engaging teaching in mathematics, in Ireland, Ireland, TIMSS 2015 – Grade 8

		Overall Mathematics				
		%	SE	Mean	SE	SD
Very engaging teaching	In SSP	35.4	(2.01)	**483.5**	(8.02)	77.51
	Non-SSP (ref)	37.5	(1.66)	538.4	(3.34)	69.70
Engaging teaching	In SSP	43.7	(1.86)	**476.7**	(8.62)	82.46
	Non-SSP (ref)	39.7	(1.01)	536.8	(3.40)	66.23
Less than engaging teaching	In SSP	20.9	(1.78)	**471.3**	(9.98)	78.52
	Non-SSP (ref)	22.7	(1.29)	528.5	(3.81)	65.29

Significantly different mean scores are in bold.

Table 3.33 presents the correlations among the four attitudes to mathematics variables described thus far in this chapter. The correlations are in the moderate to strong range and are significant. The strongest correlation is between 'Students Like Learning Mathematics' and 'Students Confident in Mathematics' (.677).

Table 3.33. Correlations among student attitude to mathematics variables, Ireland, TIMSS 2015 – Grade 8

		Students Like Mathematics			
		1	2.	3.	4.
1.	Students Like Learning Mathematics		**.544**	**.677**	**.500**
2.	Students Value Mathematics	**.544**		**.392**	**.379**
3.	Students Confident in Mathematics	**.677**	**.392**		**.354**
4.	Students' Views on Engaging Teaching in Mathematics Classes	**.500**	**.379**	**.354**	

Significant correlations are in bold (two-tailed). Df = 150 (number of variance strata associated with JRR (jackknife repeated replication) method of variance estimation).

3.3. Students' Attitudes in Mathematics in PISA 2012

PISA 2012 assessed students' motivation for mathematics learning, as well as their mathematics self-beliefs. It should be noted that, for a majority of students in PISA 2012 in Ireland (those in Third and Transition years), the Project Maths initiative had not been introduced in their schools when they began First year. In fact, these students were in the last cohorts to study under the pre-2010 Junior Certificate syllabus (having started First year in 2008 or 2009).

3.3.1. Motivational Factors

PISA 2012 was interested in intrinsic motivation, or students' liking of mathematics learning, and their instrumental motivation, or the extent to which they view mathematics as useful for their future life, study, or careers. In PISA 2012, students in Ireland scored significantly above the OECD averages on the indices of intrinsic and instrumental motivation for mathematics learning (see Perkins et al., 2013a). Male students scored significantly higher on instrumental motivation than female students, but not on intrinsic motivation.

Four items comprised the scale assessing intrinsic motivation for mathematics learning. Some 37% of students in Ireland agreed or strongly agreed that they do mathematics because they enjoy it, 33.3% that they enjoy reading about mathematics, and 40.2% that they look forward to mathematics class. Around half of students in Ireland (49.6%) agreed or strongly agreed with the statement 'I am interested in the things I learn in maths'.

The instrumental motivation for mathematics learning scale also had four items. The majority of students in Ireland agreed or strongly agreed that learning mathematics is worthwhile for them because it will improve their career prospects and chances (88.3%); that making an effort in maths is worth it because it will help them in what they want to do later on (79.9%); that they will learn many things in maths that will help them get a job (75.6%); and that maths is an important subject for them because they need it for what they want to study later on (66.2%).

A third motivational factor of interest in PISA 2012 was perseverance with problem solving. Students in Ireland scored significantly above the OECD average on this scale, and male students scored significantly higher than female students. Perseverance was assessed with five items describing perseverance behaviours. Students were asked to indicate the extent to which each reflects their behaviour. For example, one-third of students in Ireland (33.3%) indicated 'when confronted with a problem I do more than is expected of me' as 'very much like me' or 'like me', and almost two-thirds of students (61.2%) described 'when confronted with a problem, I give up easily' as 'not like me at all' or 'not like me'.

In PISA 2012 in Ireland, correlations of motivation indices and mathematics performance (print mathematics) were in the weak to moderate ranges: .24 for intrinsic motivation and mathematics; .14 for instrumental motivation and mathematics; and .26 for perseverance and mathematics.

3.3.2. Mathematics Self-Beliefs

PISA 2012 assessed three aspects of mathematics self-belief: mathematics self-efficacy, which refers to students' beliefs that they can solve mathematical tasks; mathematics self-concept, which refers to students' belief in how well they are doing in mathematics; and mathematics anxiety (Perkins & Shiel, 2016a). Students in Ireland did not differ from students across OECD countries on average in terms of their mathematics self-concept or self-efficacy, but their mathematics anxiety was significantly above the OECD average (see Perkins et al., 2013a). In addition, male students in Ireland had significantly greater mathematics self-efficacy and self-concept scores and significantly lower mathematics anxiety scores than female students.

Mathematics self-efficacy was assessed by asking students about the confidence with which they can undertake various maths tasks; for example, 53.0% of students in Ireland reported that they would feel confident or very confident in calculating the petrol consumption rate of a car (Perkins et al., 2013a). Mathematics self-concept was assessed by asking students to indicate their level of agreement with five statements. Among students in Ireland, 61.4% agreed or strongly agreed that they get good grades in mathematics and 46.5% agreed or strongly agreed that they learn mathematics quickly (Perkins et al., 2013a). Some 34.3% of students in Ireland agreed or strongly agreed that they have always believed that mathematics is one of their best subjects, and 34.2% that in their mathematics class, they understand even the most difficult work. Two-in-five students (39.9%) agreed or strongly agreed that they are 'just not good at mathematics' (Perkins et al., 2013a).

Mathematics anxiety was assessed with five items.[20] Around 70% of students (69.8%) in Ireland agreed or strongly agreed with the statement: 'I often worry that it will be difficult for me in mathematics classes'. Additionally, 29.7% of students in Ireland agreed or strongly agreed that they get very nervous doing mathematics problems; 62.1% that they worry they will get poor grades in mathematics; 36.0% that they get very tense when they have to do mathematics homework; 29.7% that they get very nervous doing mathematics problems; and 28.0% that they feel helpless when doing a mathematics problem (Perkins et al., 2013a). More female students than male students agreed with each statement. The greatest gender difference related to 'I worry that I will get poor grades in mathematics' (55.0% of males and 69.4% of females) (Perkins et al., 2013a).

Correlations between mathematics self-beliefs and mathematics performance (print mathematics) in Ireland were in the moderate to strong range, and were positive for mathematics self-efficacy (.55) and for mathematics self-concept (.40), and negative for mathematics anxiety (-.38).

3.3.3. Mathematics Self-Responsibility and Openness to Problem Solving

In PISA 2012, students in Ireland scored below the OCED average on a measure of self-responsibility for failure in mathematics. This means students in Ireland were less likely to attribute mathematics failure to themselves and more likely to attribute it to others (Perkins et al., 2013b). Male students in Ireland scored significantly lower on mathematics self-responsibility than female students, indicating

[20] Students in non-Initial Project Maths schools in PISA 2012 in Ireland had a significantly lower anxiety about mathematic score than students in Initial schools. This may reflect greater stress among students in Initial schools because they were among the first to take Junior and Leaving Certificate examinations based on the revised syllabus introduced as part of Project Maths (Merriman et al., 2014).

that female students were more likely than male students to blame themselves for their failures in mathematics.

PISA 2012 also included a measure of openness to problem solving. Students in Ireland did not differ from students across OECD countries on average on openness to problem solving, and male students in Ireland scored significantly higher than females on this scale. Five items comprised the scale; among students in Ireland, 52.4% agreed or strongly agreed that they can handle a lot of information; 55.3% that they are quick to understand things; 66.2% that they seek explanations for things; 56.8% that they can easily link facts together; and 29.8% that they like to solve complex problems (Perkins et al., 2013a).

In PISA 2012 in Ireland, mathematics self-responsibility and performance in mathematics were negatively correlated (-0.19 for print mathematics), which means that students who are strong performers in mathematics tend to blame others for their mathematics failures rather than themselves (Perkins et al., 2013a). Openness to problem solving had a moderate positive correlation with performance in mathematics (0.40).

3.4. Summary

This chapter explored the attitudes to mathematics of students who studied the revised Junior Certificate syllabus introduced as part of Project Maths using data drawn from the TIMSS 2015 study in Ireland. Findings relating to students' attitudes to mathematics from PISA 2012 were also summarised.

TIMSS 2015 data revealed that students in Ireland who had studied the revised syllabus under Project Maths scored significantly above the OECD average on indices assessing their liking of learning mathematics and their valuing of mathematics. Male students in Ireland scored higher on valuing mathematics than female students, but did not differ from female students in their overall liking of mathematics learning. On the TIMSS 2015 Students Like Learning Mathematics index, a minority of students in Ireland were classified as liking learning mathematics very much (13.6%) and around half (51.7%) as not liking learning mathematics. Students who did not like learning mathematics scored significantly lower on TIMSS overall mathematics scale than students who liked learning mathematics.

The TIMSS 2015 data revealed high levels of agreement from students in Ireland with statements about the instrumental value in learning mathematics; for example, 91.2% of students agreed with the statement that 'learning maths will give me more job opportunities when I am an adult'. However, most students in Ireland (59.3%) (and more females than males) disagreed that they would like a job that involves using mathematics.

Students in Ireland did not differ from their OECD counterparts in TIMSS in terms of mathematics confidence, and female students in Ireland had less mathematical confidence than male students. Students who were confident or very confident in mathematics scored significantly higher on TIMSS mathematics than those who were not confident in mathematics.

Students in Ireland did not differ from students across the OECD 16 countries on average in terms of their scores on a scale assessing their views on engaging teaching in mathematics classes. Also, no gender differences were found in the extent to which students reported experiencing engaging teaching in their mathematics classes. Among items addressing engaging teaching, the highest level of disagreement among students in Ireland was with the statement 'my teacher gives us interesting things to do', with which 45.5% of students disagreed a little or a lot. Overall, 37.1% of students in

Ireland were classified by TIMSS as experiencing very engaging teaching in their maths classes, 40.6% as experiencing engaging teaching, and 22.4% as experiencing less than engaging teaching.

Students from lower socioeconomic backgrounds (based on their parents' highest education level) scored lower than other students on liking learning mathematics, valuing mathematics and mathematics confidence. Students in SSP schools under DEIS liked learning mathematics less and had lower mathematics confidence than their counterparts in non-SSP schools.

Attitudinal measures from PISA 2012, when mathematics was a major assessment domain in PISA, but when a majority of students taking PISA in Ireland had not studied the revised syllabus, were broadly consistent with those reported for TIMSS 2015. Students in Ireland did not differ from students across OECD countries on average in terms of their mathematics self-concept or self-efficacy, but their mathematics anxiety was significantly above the OECD average (see Perkins et al., 2013). Moreover, female students in Ireland had significantly higher levels of mathematics anxiety than male students.

Taken together, the findings suggest a need for measures to further build on students' experience in mathematics class, to increase their liking of learning mathematics, and to enhance their mathematics confidence. This is especially important for students from lower socioeconomic backgrounds who tend on average to like learning mathematics less and to have lower mathematics confidence than other students. It is also important for female students as on average they have lower mathematics confidence than male students and higher levels of mathematical anxiety.

Chapter 4: The Junior Certificate Mathematics Examination

Along with changes to the curriculum under Project Maths, the Junior Certificate mathematics examination has changed in recent years. This chapter looks at the evolution of the exam between 2003 (the year in which mathematics was a major assessment domain in PISA for the first time) and 2016. First, changes to the Junior Certificate mathematics are described; second, the distributions of exam grades by year, overall and by gender, are described; third, the Chief Examiner's Reports on Junior Certificate Mathematics in 2015 is reviewed; fourth, comparisons are drawn between performance on the Junior Certificate mathematics examination and PISA; fifth, the content and processes assessed by the Junior Certificate examination are considered with reference to the assessment frameworks for the PISA and TIMSS, and the readability of the examination papers is considered.

4.1. The Junior Certificate Mathematics Examination Paper

2013 marked the first year in which aspects of the curriculum changes introduced under Project Maths were assessed at Junior Certificate level (though, from 2011, aspects of the syllabi were assessed on separate papers for students in initial or pilot schools). This section describes some key features of the Junior Certificate exam papers prior to Project Maths (e.g., 2010) and at the end of the phasing in of Project Maths (e.g., 2015).

Some features of the pre-Project Maths and the Project Maths papers are similar:

- There are three exam levels, with students taking two papers at Higher and Ordinary levels, and one at Foundation level[21]
- The time limits for papers have not changed – 2 X 2.5 hours at Higher level, 2 X 2 hours at Ordinary level, and 1 X 2 hours at Foundation level
- Some content areas, which cut across the 2000 syllabus and the syllabus introduced under Project Maths are assessed on the same papers – for example, much of Number and almost all of Algebra continue to be assessed on Paper 1 and Geometry, Trigonometry and Statistics on Paper 2.
- The overall number of marks assigned to each paper has not changed. A maximum score of 600 is still possible at Higher and Ordinary levels, and 300 at Foundation level.
- Students have access to a Formulae and Tables booklet and may bring a calculator, ruler and mathematical drawing instruments to the exam.

There are also some key differences between pre-Project Maths and Project Maths exam papers:

- Pre-Project Maths, information was provided on the overall number of marks available for each question. This information is no longer available to candidates; instead, they are given a suggested maximum time for each question.
- Pre-Project Maths, students were alerted (via a 'pen-in-hand' symbol) where they were expected to show their work. This symbol no longer appears. However, students are informed that may lose marks if 'all necessary work is not clearly shown'. On occasion, students are also cued as to when to show work, with terms such as 'calculation' and 'justification' appearing in the spaces provided for answers.

[21] There is a Junior Certificate Mathematics Syllabus for Higher Level and Ordinary Level, but the final examinations are offered at Higher, Ordinary, and Foundation levels.

- Whereas previously, students at Ordinary and Foundation levels were expected to record their answers on the exam booklet, now all students are expected to do so. This arrangement provides a level of scaffolding to students, as the size of the available space provides an indication of the amount of work they are expected to show.
- As described in a later section of this chapter, Project Maths questions contain more text than questions on the Pre-Project Maths papers. Where items are embedded in context, the contexts tend to be somewhat more realistic (see later section on readability). There are fewer pseudo or minimal contexts than in pre-Project Maths papers (e.g., calculating the volume of a cylindrical vitamin tablet, cutting discs of a given radius from a rectangular piece of silver, or finding the volume of a block of wood).
- Some items in Project Maths papers draw on a number of mathematics content areas (for example, item 11 in 2015 Higher paper 1 drew on knowledge of Geometry & Trigonometry, Algebra and Functions). Prior to Project Maths, most items were viewed as drawing on one content area only.

The examination papers were changed over a number of years (see Table 4.1), based on when new strands were introduced, with 2015 being the first year in which all aspects of the Project Maths syllabi were assessed in both initial and non-initial schools on the Junior Certificate exam.

Table 4.1. Schedule for the introduction of new Junior Certificate exam incorporating Project Maths

Year	Initial Schools	Non-Initial Schools
2010	No change	No change
2011	New Paper 2 (partial change to paper); Paper 1 (no change)	No change
2012	New Papers 1 (partial change to paper) & 2 (full change to paper)	No change
2013	New Papers 1 (full change to paper) & 2 (full change to paper)	New paper 2 (partial change); Paper 1 (no change)
2014	New papers 1 & 2 (as for 2013)	Paper 1 (partial change) and Paper 2 (full change)
2015	All schools get new Project Maths papers	All schools get new Project Maths papers

Source: Adapted from State Examinations Commission (2014, p. 17)

The State Examinations Commission (2013) described changes to the format of the papers as follows:

> The format of the Junior Certificate Project Maths examination papers represents a significant departure from the presentation of the traditional Mathematics examination papers. The number of questions on the examination papers is not pre-specified and may vary from year to year and from level to level. The type, length and difficulty of the questions on the paper are not pre-specified and can vary within each paper. Marks are not specifically pre-allocated to questions or sections of questions. In order that students can manage their time in the examination each question carries a "suggested maximum time" with a few minutes left over for a review of work. (p. 17)

4.2. Participation and Performance of Students on the Junior Certificate Mathematics Examination

Figure 4.1 shows the percentages of students at Higher, Ordinary and Foundation levels taking Junior Certificate mathematics exam from 2003 to 2016. In 2016, for example, 55.3% took the exam at Higher level, 39.9% at Ordinary level, and 5.0% at Foundation level. The figure shows that, whereas more students took Ordinary than Higher level between 2003 and 2011, the situation has now been reversed, with a majority of students taking Higher level from 2013 onwards. During the period 2003-2016, the proportion taking Foundation level has dropped from 12.5% to 5.0%, and continues to fall year-on-year. It is interesting that the proportions taking Higher level had been rising gradually between 2003 and 2010 (i.e., before the advent of Project Maths). After that point, perhaps linked to the availability of bonus points for mathematics at Leaving Certificate level, the proportions taking Higher level increased more quickly.

Figure 4.1: Percentages of students taking Junior Certificate mathematics at Higher, Ordinary and Foundation levels, 2003-2016

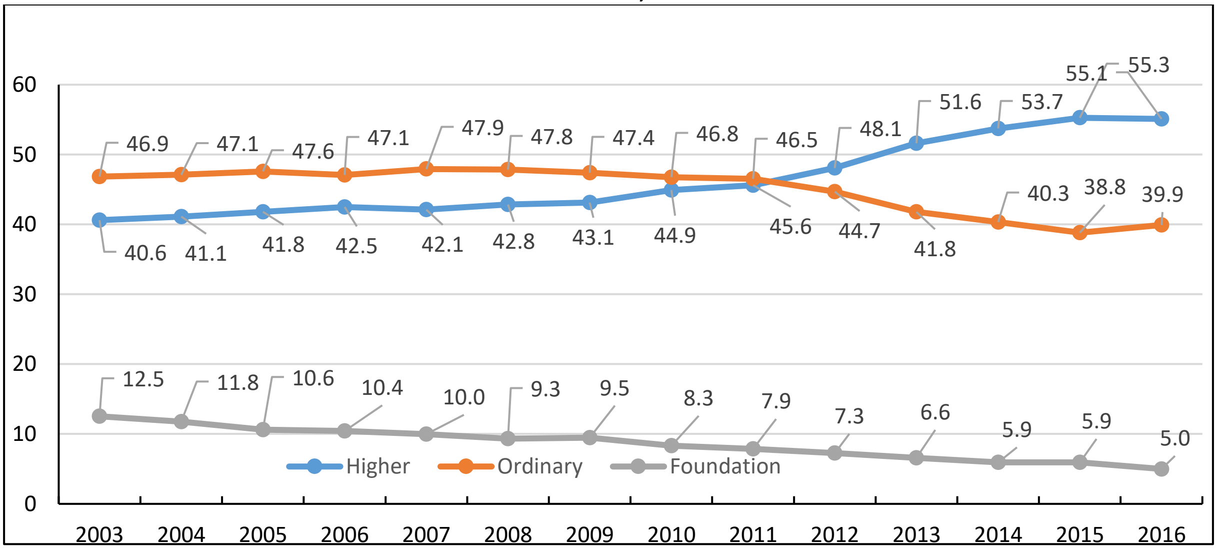
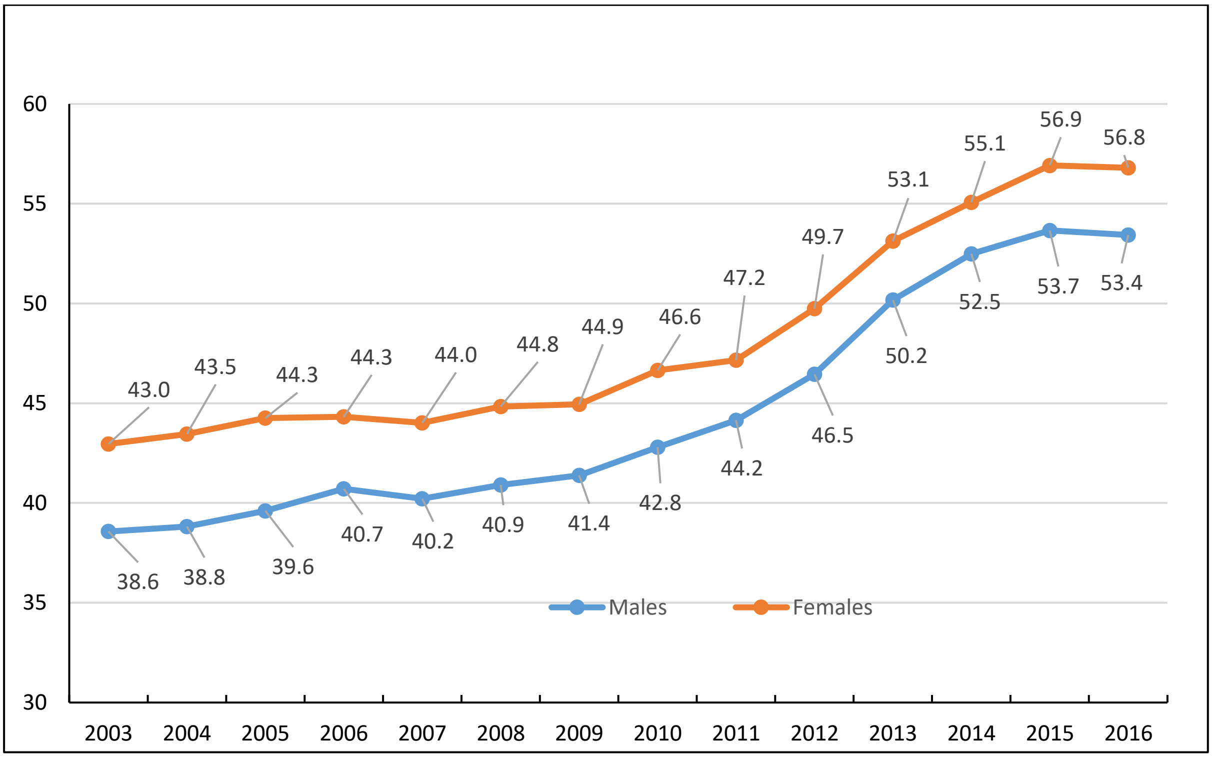


Figure 4.2 provides breakdown of the percentages of male and female students taking the Higher level mathematics paper between 2003 and 2016. In 2016, a greater proportion of female (56.8%) than male students (53.4%) took the Higher level mathematics paper. The gap was 3.4%. In the early-to-mid 2000s, the gap was marginally greater – for example, it was 4.6% in both 2004 and 2005. It was at its lowest level (2.6%) in 2014. These data are noteworthy, given the gender difference in favour of male students on overall mathematics in recent PISA cycles (Chapter 2), lack of a significant gender difference in overall performance on TIMSS 2015 mathematics (even though male students had a mean score that was 5.2 score points higher than females on the overall mathematics scale) and higher levels of anxiety about mathematics and lower levels of confidence among female students in PISA and TIMSS (Chapter 3).

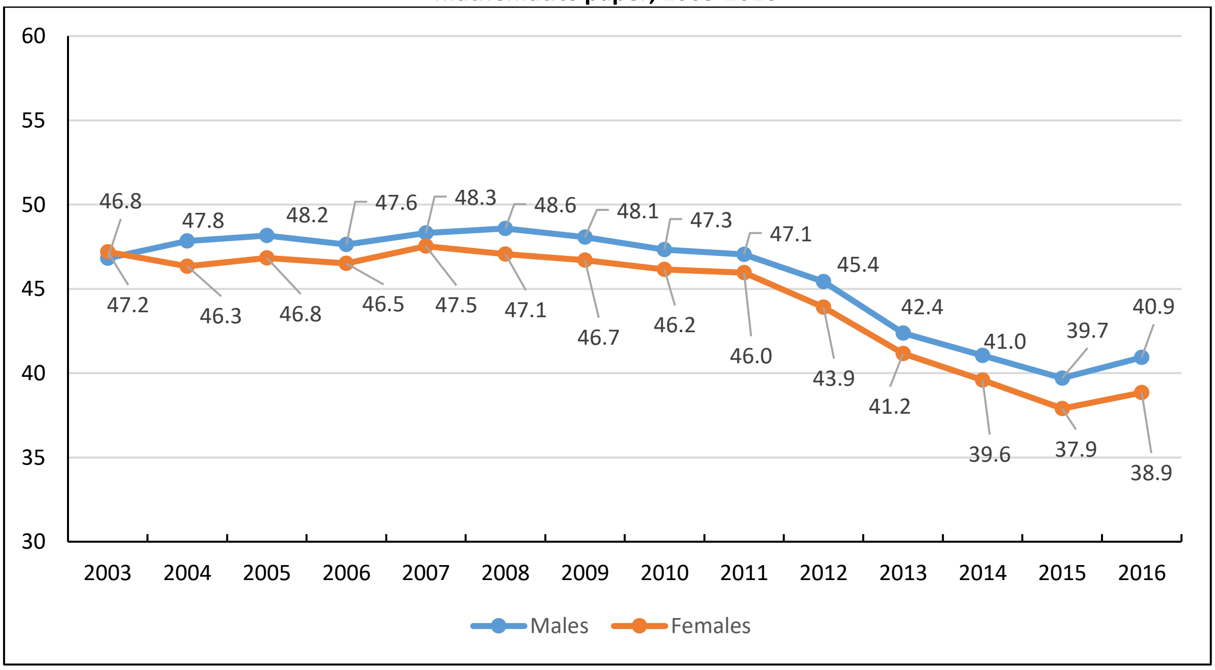
Figure 4.3 provides a similar breakdown for Ordinary level. For example, in 2016, 40.9% of males and 38.9% of females took the Ordinary level paper. Moreover, a greater proportion of males than females took Ordinary level in each year between 2004 and 2016. However, there is a relatively narrow gap in each year under consideration, with the greatest gap (2.1% in favour of males) in 2016. Finally, after reductions in the percentages of males and females taking Ordinary level between 2007 and 2015, both genders saw a small increase in 2016, compared with 2015. It is clear from Figures 4.2 and 4.3 that the proportions taking Higher and Ordinary levels changed over a number of years, but that the changes accelerated from 2010 onwards.

Figure 4.2: Percentages of male and female students taking the Junior Certificate Higher level mathematics paper, 2003-2016



Source: SEC, Junior Certificate results by gender, 2003-2016. www.examinations.ie

Figure 4.3: Percentages of male and female students taking the Junior Certificate Ordinary level mathematics paper, 2003-2016



Source: SEC, Junior Certificate results by gender, 2003-2016. www.examinations.ie

Figure 4.4 shows the percentages of students taking Foundation-level mathematics in each year since 2003. In 2016, 5.6% of males and 4.3% of females took Foundation level. It is also clear from Figure 4.4 that that the proportion taking Foundation level has been falling for some time.

Figure 4.4: Percentages of male and female students taking the Junior Certificate Foundation level mathematics paper, 2003-2016

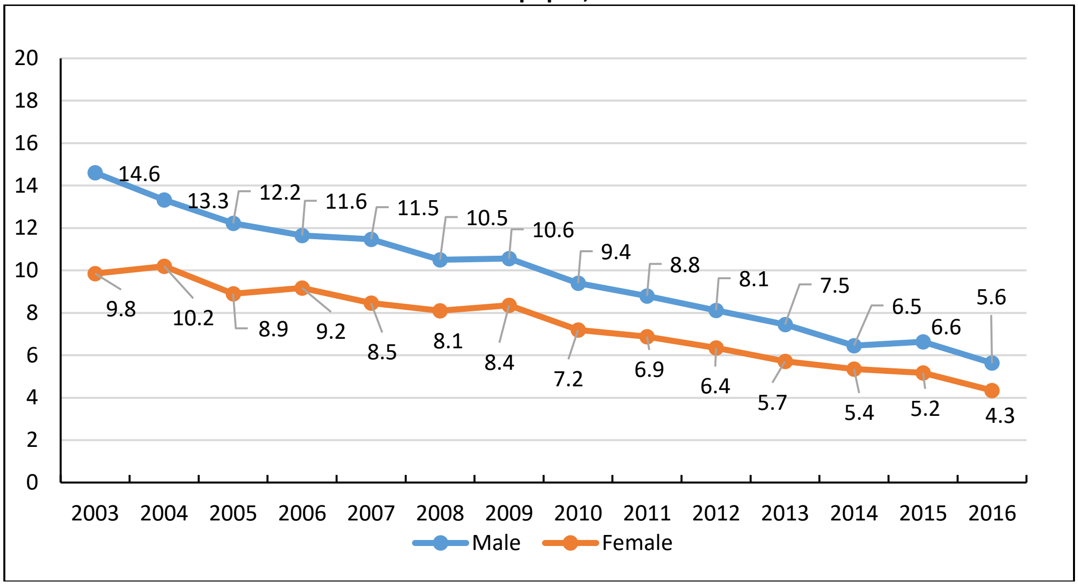


Table 4.2 shows the distribution of grades by year on Higher level Junior Certificate mathematics between 2003 and 2016. In 2016, 76.3% achieved grades A-C ('honours'), while 3.5% achieved an E, F or no grade.

Table 4.2: Percentages of students achieving each grade on the Junior Certificate examination 2003-2016, Higher level

Higher level		A	B	C	D	E	F	NG	A-C	E-F-NG	JCPS Points Score
Year	No. of Candidates	%	%	%	%	%	%	%	%	%	
2003	23734	17.2	33.6	28.6	17.0	3.1	0.5	0.0	79.4	3.6	10.4
2004	23006	16.1	28.4	28.9	20.3	5.2	1.1	0.1	73.4	6.4	10.3
2005	23288	14.5	30.3	30.8	20.0	3.7	0.6	0.0	75.6	4.3	10.3
2006	24205	18.0	32.0	28.7	17.2	3.3	0.6	0.0	78.7	3.9	10.4
2007	23804	17.6	29.9	28.1	19.2	4.4	0.8	0.0	75.6	5.2	10.3
2008	23634	16.6	31.6	31.6	17.4	2.3	0.4	0.0	79.8	2.7	10.4
2009	23592	16.7	31.7	29.2	18.6	3.4	0.5	0.0	77.6	3.9	10.4
2010	24840	15.5	31.5	30.7	18.2	3.5	0.6	0.1	77.7	4.2	10.4
2011	25554	17.5	31.8	30.4	16.8	3.0	0.5	0.1	79.7	3.6	10.4
2012	27913	15.1	31.4	32.7	18.0	2.5	0.3	0.0	79.2	2.8	10.4
2013	30500	12.0	32.3	34.1	18.4	2.8	0.5	0.0	78.4	3.3	10.3
2014	32041	10.7	28.4	33.9	22.5	3.9	0.5	0.0	73.0	4.4	10.2
2015	32535	11.3	31.2	32.2	21.1	3.7	0.5	0.0	74.7	4.2	10.2
2016	32830	11.7	31.6	33.0	20.2	3.1	0.4	0.0	76.3	3.5	10.3

The distribution of grades from year to year is fairly stable, with generally small fluctuations. However, there were drops in the proportions of A grades awarded in 2012 (the year before aspects of the curriculum changes introduced by Project Maths were assessed in non-initial schools for the first time), in 2013 (the first year in which aspects of the curriculum introduced under Project Maths were assessed in non-initial schools) and in subsequent years.

As part of the analysis of performance, we computed points scores (last column on Table 4.3). These arise from allocating points to grades based on a Junior Certificate Performance Scale, where 12 points is allocated to a higher-level A, 11 points to a higher-level B and so on (see Inset 4.1). The JCSP-M points totals across grades move within a very narrow range (10.2 to 10.4) and are relatively stable from year to year. In both 2014 and 2015, the points total was 10.2. It moved up to 10.3 in 2016. However, across the 14 years for which data are presented, the points total has remained fairly stable, even though actual numbers taking Higher level have increased. This suggests that, overall, standards have remained constant, and that the awarding of fewer A grades was compensated for elsewhere in the distribution of grades.

Insert 4.1: The Junior Certificate Performance Scale – Mathematics

The Junior Certificate Performance Scale – Mathematics (JCPS-M) involves the allocation of point scores to examination grades, as per the scheme shown below, using the formula (sum of %*Pts for each column)/100. It has been used in a number of studies to examine changes in grade distributions and their associations with other measures (e.g., Cosgrove et al., 2005). The scale includes some overlap between examination levels. For example, a Grade D at Higher level is deemed to have the same points value (9 points) as a Grade A at Ordinary level.

Higher level: A (12 points), B (11), C (10), D (9), E (8) and F (7)

Ordinary level: A (9), B (8), C (7), D (6), E (5) and F (4)

Foundation level: A (6), B (5), C (4), D (3), E (2) and F (1).

Table 4.3. Percentages and numbers of students achieving Grade A on Higher level Junior Certificate mathematics

Year	Number of Students Taking Higher Level	% Achieving Grade A	Number of Students Achieving Grade A*
2003	23734	17.2	4088
2004	23006	16.1	3204
2005	23288	14.5	3387
2006	24205	18.0	4354
2007	23804	17.6	4199
2008	23634	16.6	3933
2009	23592	16.7	3939
2010	24840	15.5	3871
2011	25554	17.5	4450
2012	27913	15.1	4218
2013	30500	12.0	3652
2014	32041	10.7	3420
2015	32535	11.3	3667
2016	32830	11.7	3855

*Numbers were arrived at by multiplying the number taking Higher level by the percentage achieving Grade A.

Table 4.3 shows that the absolute number of students achieving Grade A at Higher level has fluctuated over the years, but has dropped since 2013, when Project Maths was examined in non-initial schools for the first time. In 2016, 3885 students achieved Grade A, which is fewer than in 2006 (4354), when 8625 fewer students sat the Higher level exam.

A tendency for fewer students to achieve A grades at Higher level in papers based on the syllabu8s under Project Maths can be observed from data published by the SEC in 2013 relating to the performance of students on mathematics in 2012. In 2012, 12.3% of students taking Higher level mathematics based on Project Maths (i.e., those in initial schools) achieved an A grade, compared with 15.1% across all schools (both initial and non-initial) (SEC, 2013).

Table 4.4 shows the distributions of grades by year for students taking the Ordinary level mathematics examination. Seven percent fewer students achieved A grades in 2016, compared with 2012, while 4% fewer achieved A-C grades. The JCPS-M points total is also down, from 7.3 in 2012 to 7.1 in 2016.

Table 4.5 provides equivalent data for students taking the Foundation level mathematics exam. As at Ordinary level, there is evidence of lower performance since 2012, with 6.7% fewer achieving A grades in 2016, compared with four years earlier. However, the proportion of A-C grades is down by just 0.4% between 2012 and 2016, suggesting that increases in the proportions achieving B and C grades offset the drop in the proportion achieving A grades. However, the proportion achieving D grades is up by 5.1% between 2012 and 2015 and the proportion achieving below Grade D is up by 0.5%. Hence, the JCPS-M points total is lower by 0.2 points between 2012 and 2016.

Across examination levels, fewer students achieved Grade A in 2016 compared with 2012 and earlier, while, in general, similar proportions achieved Grades B-C. The larger proportion of students taking Higher level does not appear to have led to a large decline in examination performance at Higher level, though there is a clear effect at Ordinary and Foundation levels, where there are now fewer students than in the past. There is some evidence of a small improvement in performance at Higher level between 2015 and 2016, though fewer A grades were achieved.

Table 4.4: Percentages of students achieving each grade on the Junior Certificate examination 2003-2016, Ordinary level

Ordinary level		A	B	C	D	E	F	NG	A-C	E-F-NG	JCPS-M Points Score
Year	No. of Candidates	%	%	%	%	%	%	%	%	%	
2003	27383	9.2	31.0	31.3	20.8	5.8	1.8	0.1	71.5	7.7	7.1
2004	26347	10.1	34.3	30.8	17.7	5.2	1.9	0.2	75.2	7.3	7.2
2005	26518	11.8	32.5	28.7	18.8	5.9	2.1	0.2	73.0	8.2	7.2
2006	26820	13.3	36.7	27.9	16.1	4.4	1.5	0.1	77.9	6.0	7.3
2007	27095	9.3	32.5	31.4	20.2	5.1	1.4	0.1	73.2	6.6	7.2
2008	26384	12.3	36.0	28.5	16.4	4.9	1.8	0.2	76.8	6.9	7.3
2009	25930	11.7	33.4	29.6	17.9	5.4	1.8	0.1	74.7	7.3	7.2
2010	25853	12.0	32.2	29.9	18.5	5.3	1.9	0.2	74.1	7.4	7.2
2011	26064	12.4	33.4	29.2	17.9	5.2	1.6	0.2	75.0	7.0	7.2
2012	25945	14.3	33.9	28.0	17.0	5.0	1.5	0.2	76.2	6.7	7.3
2013	24687	9.9	35.5	31.8	17.7	4.1	1.0	0.1	77.2	5.2	7.3
2014	24047	6.3	33.2	35.3	20.6	3.6	0.9	0.1	74.8	4.6	7.3
2015	22856	7.4	28.3	34.5	23.8	4.9	0.9	0.1	70.2	5.9	7.1
2016	23781	7.6	30.8	33.4	22.1	4.9	1.1	0.1	71.8	6.1	7.1

Table 4.5: Percentages of students achieving each grade on the Junior Certificate examination 2003-2016, Foundation level

Foundation Level		A	B	C	D	E	F	NG	A-C	E-F-NG	JCPS-M Points Score
Year	No. of Candidates	%	%	%	%	%	%	%	%	%	
2003	7324	15.4	37.8	29.5	13.6	3.2	0.4	0.0	82.7	3.6	4.5
2004	6584	16.4	40.4	29.1	12.0	1.8	0.3	0.0	85.9	2.1	4.6
2005	5908	18.0	31.5	27.4	17.7	4.4	1.0	0.0	76.9	5.4	4.4
2006	5941	17.1	37.8	29.0	13.7	2.1	0.4	0.0	83.9	2.5	4.5
2007	5640	16.2	32.6	30.7	16.6	3.3	0.6	0.1	79.5	4.0	4.4
2008	5140	18.4	37.5	27.7	12.9	2.9	0.6	0.0	83.6	3.5	4.5
2009	5186	19.0	32.6	28.4	15.8	3.5	0.6	0.1	80.0	4.2	4.5
2010	4597	18.2	35.1	28.6	15.0	2.5	0.5	0.0	81.9	3.0	4.5
2011	4407	19.6	37.8	26.9	12.9	2.3	0.5	0.0	84.3	2.8	4.6
2012	4211	17.1	34.2	30.1	15.6	2.3	0.7	0.0	81.4	3.0	4.5
2013	3901	12.5	35.9	35.8	13.0	2.2	0.3	0.2	84.2	2.7	4.4
2014	3532	10.9	34.5	35.0	16.5	2.4	0.5	0.1	80.4	3.0	4.3
2015	3484	14.6	35.9	30.2	16.2	2.7	0.4	0.1	80.7	3.2	4.4
2016	2978	9.5	36.6	34.8	15.6	2.6	0.8	0.1	80.9	3.5	4.3

Tables 4.6 to 4.8 provide a breakdown of performance by gender. Table 4.6 shows that, since 2008, more male students than females taking Higher level achieved Grade A in mathematics. In 2016, for example, 13.1% of males and 10.4% of females achieved A grades. However, in every year except 2009, more females than males achieved Grades A-C. In 2016, 76.7% of female students and 75.9% of males achieved Grades A-C. In all years, more males than females achieved below Grade D. In 2016, 3.9% of males and 3.1% of females did so. JCPS-M point scores are similar for males and females in recent years, with only minor variation, indicating similar overall performance among males and females.

Table 4.7 provides the same data for Ordinary level students. In all years under review, more females than males achieved Grade A and Grades A-C at this level. For example, in 2016, 8.6% of females and 6.7% of males achieved Grade A, while 74.7% of females and 69.2% of males achieved grades A-C. On the other hand, in the same year, more males (7.0%) than females (5.1%) achieved a grade that was lower than D.

According to Table 4.8, in all years between 2003 and 2016, more males than females achieved Grade A at Foundation level. In 2016, for example, 10.5% of males and 8.2% of females achieved Grade A. The proportions of males and females achieving Grades A-C were broadly similar in most years. In 2016, marginally more females (81.1) than males (80.5%) achieved Grades A-C. In the same year, more males (4.2%) than females (2.5%) achieved below Grade D. This pattern has held in each year since 2007.

The relatively large gender difference in favour of female students, especially at Higher level, contrasts with the outcomes of PISA and TIMSS (see Chapter 2), where males had higher overall mean scores than females (with a statistically significant difference in PISA in 2012 and 2015). On the other hand, the slightly smaller representation of female students among those who achieved Grade A at Higher level is consistent with the larger proportion of males (12.0%), compared with females (6.5%), who achieved at Proficiency levels 5-6 on PISA 2015 mathematics. However, the relatively stronger performance of female students at Foundation level (Table 4.10) is inconsistent with the finding reported in Chapter 2 that 14.1% of males and 15.8% of females achieved below Proficiency Level 2 on PISA mathematics. It may be the case that the strong focus on Space and Shape on PISA mathematics, on which male students do better than females, is not reflected in Junior Certificate mathematics, and hence females students are not disadvantaged by apparent shortcomings in that aspect of mathematics to the same degree as in PISA.

Finally, the lower percentages of female students achieving A grades at Higher level in 2016 and in earlier years is consistent with the lower proportion of females (5.1%) compared with males (8.3%) who performed at the Advanced Benchmark on TIMSS mathematics in 2015. However, the relatively higher proportion of male students achieving below Grade D on Foundation level in 2016 and in earlier years is inconsistent with the broadly equivalent proportions of males and female students achieving below the Low benchmark on TIMSS 2015 (6.1% of males, 5.9% of females).

Table 4.6: Percentages of students achieving each grade on the Junior Certificate examination 2003-2016, Higher level, by gender

Higher level Year	Gender	No. of Candidates	A %	B %	C %	D %	E %	F %	NG %	A-C %	E-F-NG %	JCPS Points Score
2003	Female	12382	17.7	35.2	28.3	15.7	2.6	0.4	0.0	81.3	3.0	10.5
	Male	11352	16.7	31.7	29.0	18.4	3.6	0.6	0.0	77.4	4.3	10.4
2004	Female	12098	16.9	30.2	28.9	18.8	4.3	0.9	0.0	75.9	5.3	10.3
	Male	10908	10.7	26.4	28.9	21.9	6.1	1.4	0.1	65.9	7.6	9.6
2005	Female	12209	14.1	31.8	31.4	19.0	3.3	0.4	0.0	77.3	3.7	10.3
	Male	11179	14.9	28.6	30.3	21.0	4.4	0.8	0.0	73.8	5.2	10.3
2006	Female	12443	17.9	32.6	29.1	16.7	3.0	0.5	0.0	79.6	3.5	10.4
	Male	11761	18.1	31.4	28.3	17.8	3.7	0.7	0.7	77.8	5.1	10.4
2007	Female	12361	18.1	30.9	27.8	18.4	4.1	0.6	0.0	76.8	4.7	10.4
	Male	11443	17.1	28.8	28.3	20.0	4.7	1.0	0.1	74.2	5.8	10.3
2008	Female	12191	16.2	32.9	32.1	16.5	2.0	0.2	0.0	81.2	2.2	10.4
	Male	11443	17.2	30.1	31.1	18.3	2.7	0.5	0.0	78.4	3.2	10.4
2009	Female	12032	16.5	31.9	19.1	18.8	3.2	0.4	0.0	67.5	3.6	10.4
	Male	11560	16.9	31.4	29.2	18.4	3.5	0.6	0.0	77.5	4.1	10.4
2010	Female	12677	14.9	32.9	31.5	17.4	2.9	0.3	0.0	79.3	3.3	10.4
	Male	12163	16.3	29.9	29.9	19.0	4.1	0.8	0.1	76.1	5.0	10.3
2011	Female	12853	17.0	32.9	30.9	16.4	2.5	0.3	0.0	80.8	2.8	10.4
	Male	12701	17.8	30.8	29.8	17.2	3.6	0.6	0.1	78.4	4.3	10.4
2012	Female	14198	14.4	33.8	33.1	16.5	2.1	0.2	0.0	81.3	2.3	10.4
	Male	13715	15.9	29.0	32.3	19.5	2.9	0.4	0.0	77.2	3.3	10.3
2013	Female	15371	11.1	32.8	35.3	18.1	2.3	0.3	0.0	79.2	2.6	10.3
	Male	15129	12.8	31.8	32.9	18.7	3.3	0.6	0.0	77.5	3.9	10.3
2014	Female	15933	10.6	29.9	34.4	21.6	3.2	0.4	0.0	74.9	3.6	10.2
	Male	16108	10.8	27.0	33.4	23.4	4.7	0.7	0.0	71.2	5.4	10.1
2015	Female	16456	10.6	31.8	32.6	21.4	3.3	0.4	0.0	75.0	3.7	10.2
	Male	16079	12.0	30.7	31.7	20.8	4.2	0.6	0.1	74.4	4.9	10.2
2016	Female	16691	10.4	32.4	33.9	20.2	2.8	0.3	0.0	76.7	3.1	10.3
	Male	16139	13.1	30.7	32.1	20.1	3.5	0.4	0.0	75.9	3.9	10.3

Table 4.7: Percentages of students achieving each grade on the Junior Certificate examination 2003-2016, Ordinary level, by gender

Ordinary Level Year	Gender	No. of Candidates	A %	B %	C %	D %	E %	F %	NG %	A-C %	E-F-NG %	JCPS Points Score
2003	Female	13602	10.8	33.5	30.5	19.0	4.9	1.3	0.1	74.8	6.3	7.2
	Male	13781	7.6	28.6	32.1	22.7	6.8	2.2	0.1	68.3	9.0	7.0
2004	Female	12903	11.8	36.6	29.5	16.3	4.2	1.4	0.1	77.9	5.7	7.3
	Male	13444	8.4	32.0	31.9	19.0	6.1	2.4	0.2	72.4	8.7	7.1
2005	Female	12923	14.1	35.0	28.0	16.7	4.6	1.4	0.1	77.1	6.1	7.3
	Male	13595	9.5	30.2	29.4	20.7	7.1	2.8	0.3	69.1	10.2	7.0
2006	Female	13057	14.9	38.2	27.3	14.9	3.6	1.0	0.1	80.4	4.7	7.4
	Male	13763	11.8	35.3	28.5	17.2	5.2	1.9	0.1	75.6	7.2	7.2
2007	Female	13350	10.4	34.7	30.5	18.9	4.4	1.1	0.1	75.6	5.6	7.2
	Male	13745	8.3	30.4	32.3	21.4	5.8	1.7	0.1	71.0	7.6	7.1
2008	Female	12795	13.8	37.6	27.5	15.2	4.3	1.5	0.1	78.9	5.9	7.4
	Male	13589	10.9	34.4	29.4	17.5	5.4	2.2	0.2	74.7	7.8	7.2
2009	Female	12503	13.4	35.1	28.3	17.1	4.5	1.5	0.1	76.8	6.1	7.3
	Male	13427	10.2	31.9	30.7	18.7	6.3	2.1	0.1	72.8	8.5	7.1
2010	Female	12546	13.6	34.1	29.7	16.6	4.4	1.4	0.1	77.4	6.0	7.3
	Male	13307	10.4	30.4	30.1	20.2	6.2	2.3	0.3	70.9	8.8	7.1
2011	Female	12529	14.1	35.7	27.9	16.7	4.4	1.2	0.1	77.7	5.7	7.3
	Male	13535	10.9	31.3	30.4	19.1	6.0	2.0	0.2	72.6	8.2	7.1
2012	Female	12534	16.5	35.5	27.3	15.4	4.1	1.0	0.1	79.3	5.2	7.4
	Male	13411	12.3	32.4	28.5	18.6	5.9	2.0	0.3	73.2	8.2	7.2
2013	Female	11909	11.0	37.6	31.1	16.1	3.5	0.7	0.1	79.7	4.3	7.3
	Male	12778	8.9	33.5	32.5	19.2	4.7	1.2	0.2	74.9	6.1	7.2
2014	Female	11452	7.4	35.8	34.2	18.5	3.2	0.7	0.1	77.4	4.0	7.2
	Male	12595	5.4	30.7	36.2	22.5	4.0	1.0	0.1	72.3	5.1	7.1
2015	Female	10958	8.9	29.9	33.5	22.3	4.5	0.8	0.1	72.3	5.4	7.1
	Male	11898	6.0	26.9	35.4	25.2	5.3	1.1	0.1	68.3	6.5	7.0
2016	Female	11415	8.6	33.0	33.1	20.2	4.1	1.0	0.0	74.7	5.1	7.2
	Male	12366	6.7	28.8	33.7	23.8	5.6	1.3	0.1	69.2	7.0	7.0

Table 4.8: Percentages of students achieving each grade on the Junior Certificate examination 2003-2016, Foundation level, by gender

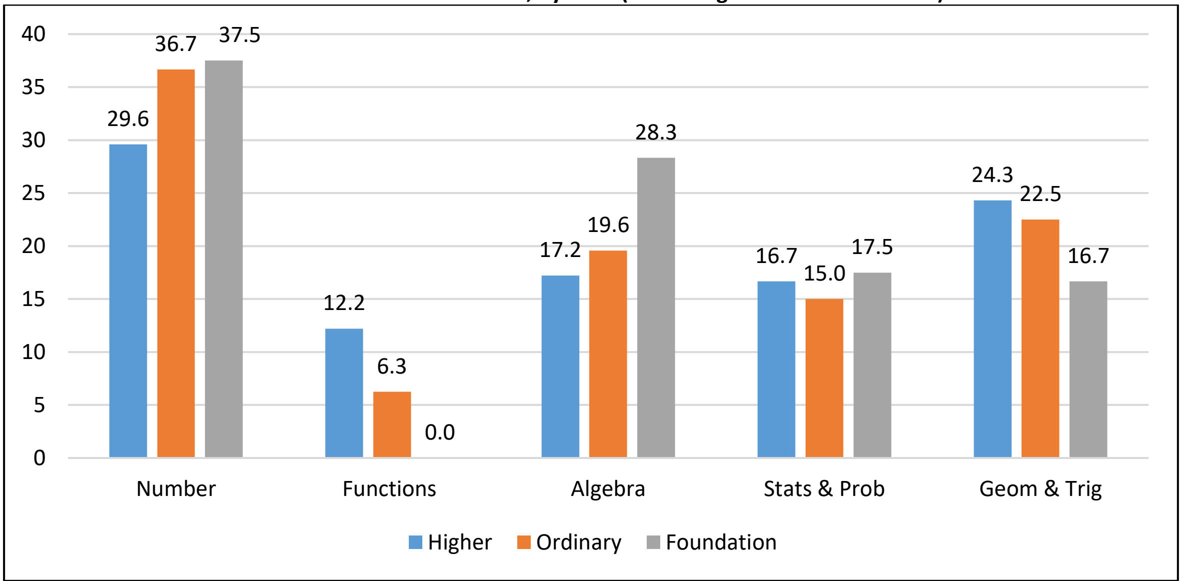
Foundation Level			A	B	C	D	E	F	NG	A-C	E-F-NG	JCPS Points Score
Year	Gender	No. of Candidates	%	%	%	%	%	%	%	%	%	
2003	Female	2839	16.2	40.6	29.0	12.4	1.7	0.1	0.0	85.8	1.8	4.6
	Male	4298	16.6	37.9	28.3	13.7	3.1	0.4	0.0	82.8	3.5	4.5
2004	Female	2839	16.2	40.6	29.0	12.4	1.7	0.1	0.0	85.8	1.8	4.6
	Male	3745	16.5	40.2	29.2	11.7	1.8	0.4	0.0	86.0	2.3	4.6
2005	Female	2457	18.6	31.9	26.9	17.3	4.2	1.1	0.0	77.4	5.3	4.4
	Male	3450	17.6	31.2	27.9	17.9	4.2	0.9	0.0	76.7	5.1	4.4
2006	Female	2576	15.6	35.6	31.1	15.1	2.0	0.5	0.0	82.3	2.5	4.5
	Male	3365	18.2	39.4	27.3	12.7	2.1	0.3	0.0	84.9	2.4	4.6
2007	Female	2376	16.0	33.4	30.7	16.4	3.1	0.4	0.0	80.1	3.5	4.4
	Male	3264	16.3	32.0	30.6	16.7	3.5	0.8	0.1	78.9	4.4	4.4
2008	Female	2202	17.0	39.0	28.8	12.3	2.4	0.5	0.0	84.8	2.9	4.5
	Male	2938	19.4	36.3	26.8	13.4	3.3	0.7	0.0	82.5	4.0	4.5
2009	Female	2237	17.4	34.2	29.1	15.6	3.2	0.4	0.0	80.7	3.6	4.5
	Male	2949	20.2	31.5	27.8	16.0	3.7	0.6	0.2	79.5	4.5	4.5
2010	Female	1956	16.3	38.2	30.3	12.8	1.8	0.5	0.0	84.8	2.4	4.5
	Male	2641	19.6	32.8	27.3	16.6	3.1	0.5	0.1	79.7	3.6	4.5
2011	Female	1876	18.2	39.6	26.9	12.8	1.9	0.5	0.1	84.7	2.5	4.6
	Male	2531	20.7	36.4	26.9	13.0	2.6	0.4	0.0	84.0	3.0	4.6
2012	Female	1813	14.7	35.0	31.4	16.7	1.6	0.7	0.0	81.1	2.3	4.4
	Male	2398	18.9	33.7	29.1	14.8	2.8	0.7	0.0	81.7	3.5	4.5
2013	Female	1653	11.9	38.7	34.9	12.1	2.1	0.3	0.1	85.5	2.5	4.4
	Male	2248	13.0	33.8	36.5	13.7	2.3	0.4	0.2	83.3	2.9	4.4
2014	Female	1549	10.3	36.7	35.1	15.0	2.5	0.4	0.1	82.1	3.0	4.4
	Male	1983	11.4	32.9	34.9	17.7	2.5	0.6	0.2	79.2	3.3	4.3
2015	Female	1497	13.2	35.1	31.3	17.4	2.7	0.3	0.0	79.6	3.0	4.4
	Male	1987	15.6	36.5	29.3	15.3	2.7	0.6	0.1	81.4	3.4	4.4
2016	Female	1276	8.2	37.9	35.0	16.3	2.2	0.3	0.0	81.1	2.5	4.3
	Male	1702	10.5	35.5	34.5	15.2	2.8	1.2	0.2	80.5	4.2	4.3

4.3. Report of the Chief Examiner for Mathematics in 2015

In addition to providing distributions of grades achieved by students on each subject in the Junior and Leaving Certificate examinations and the marking schemes on which those grades are based, on an annual basis, the State Examinations Commission issues occasional reports on the performance of students in specific examinations in specific years. The two most recent reports for Junior Certificate mathematics relate to the 2003 and 2015 examinations (SEC, 2003a, 2015a). 2003 was the first year in which the 2000 Junior Certificate mathematics syllabus was assessed, while 2015 marked the first year in which all strands of the revised syllabus introduced under Project Maths were assessed in all schools. Here, we consider key points in the 2015 report.

The 2015 report includes a breakdown of the marks assigned to each content area. Where items drew on multiple content areas, the marks were distributed evenly among the specified content areas for the purpose of developing Figure 4.5. In the case of hybrid items, each component was assigned an even proportion of the available marks. Thus, in the case of Question 11, Higher-level Paper 1, which was labelled as comprising Geometry & Trigonometry, Algebra and Functions, and worth 40 marks in all, each component was assigned 13.3 points. The figure shows the percentage of marks assigned to each content area, at each level of the examination. Thus, at Higher level, 29.6% of the available 600 marks were assigned to questions based on Number. The corresponding percentage at Ordinary level was 36.7% (of 600 marks) and at Foundation level it was 37.5% (of 300 marks). Surprisingly, perhaps, there seems to be a greater emphasis on Algebra at Foundation level (28.3% of all available marks) compared with Ordinary and Higher levels. Because Geometry and Trigonometry are combined, the relative emphasis on these areas cannot be inferred from the figure.

Figure 4.5. Relative emphasis on different mathematics content areas in the 2015 Junior Certificate mathematics examination, by level (Percentages of available marks)

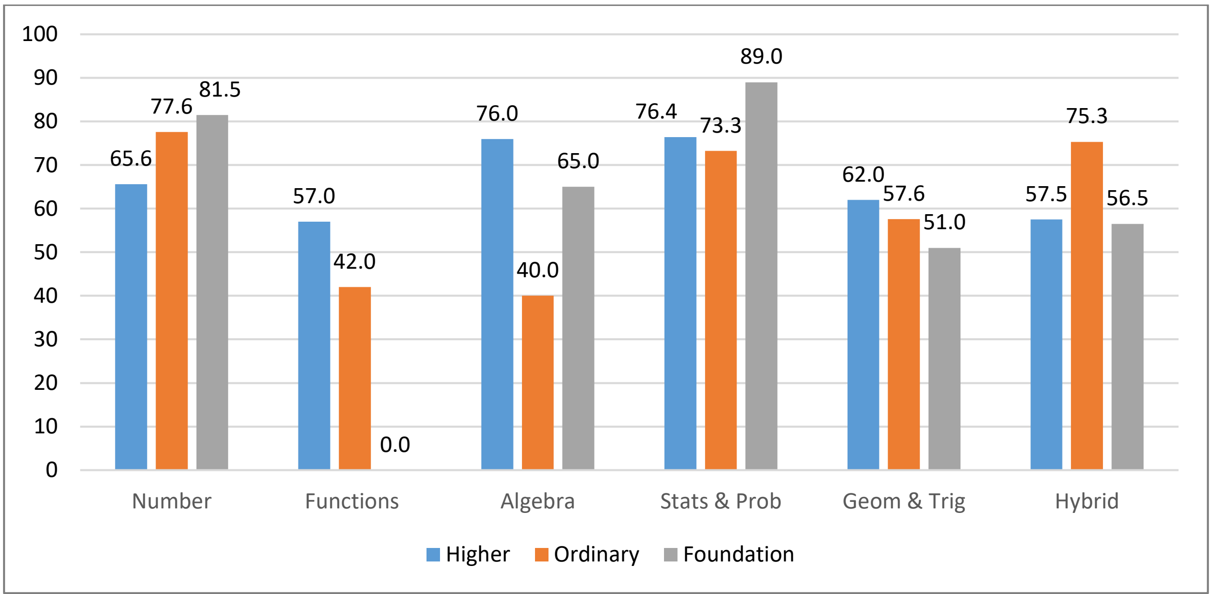


The Chief Examiner's Report includes the average mark for each item as a percentage of the total marks available for that item, based on a random sample of approximately 5% of the total cohort at each level. Figure 4.6 provides the average mark as a percentage of the total marks available for each content area, with hybrid items categorised separately in this case.

According to the figure, Statistics & Probability represents the easiest content area for students at Higher level (76.4% of available marks) and Foundation level (89.0%), while Number is easiest at Ordinary level (77.6%). At Higher level, Functions (57.0%) was the most challenging content area, followed by Hybrid questions (57.5% across two such questions) and Geometry & Trigonometry (62%). At Ordinary level, Functions (42%) and Algebra (40%) were the most difficult content areas, while at Foundation level, Geometry & Trigonometry (56.5%) was the most difficult, while Functions were not assessed at that level.

It is noteworthy that students at Higher level did quite well, on average, with Algebra questions (76.0%), while their counterparts at Ordinary level struggled on such questions (40%).

Figure 4.6. Average marks as percentage of available marks by content area, 2015 Junior Certificate mathematics examination, by level



Source: SEC (2015a), Tables 12-14.

As noted in Chapter 2, Algebra and Geometry were identified as areas of relative weakness among students in Second year in Ireland in TIMSS 2015, while Space & Shape (which includes aspects of Geometry) was an area of weakness for 15-year olds in PISA 2012. In the 2015 Junior Certificate exam, Algebra was an area of relative weakness for students in Third year taking Ordinary level (but not Higher or Foundation levels), while the combined area of Geometry & Trigonometry was an area of relative weakness (along with Functions) at Higher level, though students taking the exam at that level also found Hybrid questions (which included aspects of Geometry & Trigonometry and Functions) to be challenging as well. Question 13 on Paper 2, where students had to use aspects of Trigonometry and Applied Measures to calculate the volume of a water tank, was especially difficult.

At Ordinary level, just 2 of 24 questions were Algebra questions in their own right, while Algebra was also identified as a constituent content area in all three Hybrid questions (hence its relatively strong weighting in Figure 4.5). The most difficult question in 2015 for students taking Ordinary level was in Algebra (Candidates were awarded an average of 32% (12.7 of the 40 marks available) on Paper 1, Question 9, where they had to factorise a linear expression and two quadratic expressions). On Question 11, also for 40 marks (42% of which were achieved), students had to solve two equations, including a pair of simultaneous linear equations, and also had to show that a given value of x was a solution to a quadratic equation. Question 7 on Paper 1 (Functions) also required a knowledge of Algebra, and students were awarded 42% (8.3 of 20 available marks).

In addition to reporting on performance across questions, the Chief Examiner's report addressed students' proficiency on objectives relating to conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning. A fifth objective, productive disposition, was not considered[22]. The examples below, taken from the Chief Examiner's report, refer to specific issues in a specific year. However, they provide a broad indication of areas of strength and weakness, as well as some insights into how students address real-life problems in the examination context.

While students in general were found to have a good knowledge and comprehension of basic mathematical concepts and relations, those taking the Higher level paper struggled to complete an identity relating to the distribution of set union over intersection, and explain how one might take a simple random sample. Some students at Ordinary level had conceptual difficulties with aspects of Geometry and Trigonometry, including identifying which side in a right-angled tringle was opposite to a particular angle, and identifying whether a given triangle was isosceles, scalene or equilateral. Understanding of graphs of inequalities was also problematic. At Foundation level, many students were able to match algebraic terms with their descriptions, but confused $2x$ and x^2.

In relation to procedural fluency, the Chief Examiner observed that students taking Higher level 'employed a range of different methods to solve Algebraic problems, including the array method to multiply out brackets or perform algebraic long division, which often led to correct answers' (p. 20). However, student responses in Algebra were also observed to conflate methods for simplifying expressions with those for solving equations. Students also struggled with traditional co-ordinate geometry questions, and with identifying which side of a triangle was the hypotenuse (albeit when the triangle was rotated with the hypotenuse facing downwards). Finding the mean of a grouped frequency distribution also presented a challenge, prompting a recommendation to teachers to support students in understanding 'why the procedures they are using actually work, rather than simply learning to follow them mechanically' (p. 21).

At Ordinary level, students did well on such tasks as using a graph to identify maximum heart rate of someone, given their age, and completing a linear pattern, given the first two terms. However, they struggled to write 60 as a percentage of 80, and to split 4000 in the ratio of 3:5. In relation to Algebra, where students performed poorly, they 'had great difficulty presenting relevant work' (p. 23). In Geometry and Trigonometry, many had difficulty understanding the significance of m and c, when a line is written as $y = mx + c$. Students also struggled to find the volume of a cylinder of given dimensions. Those who obtained the correct answer were unable to round their answers accurately. At Foundation level, some students mixed up the hour and minute hands when reading the time from an analogue clock. They also struggled to find the slope of a line, and to construct the bisector of an angle. Algebraic manipulation was a problem. Some correct answers were presented without supporting work.

According to the Chief Examiner, strategic competence (defined as the ability to formulate, represent and solve mathematical problems in both familiar and unfamiliar contexts – a key goal of Project Maths as well as an objective of the new Junior Certificate Mathematics syllabus – was evident across all examination levels as students used a variety of approaches to solve non-routine problems. Many students were observed to apply 'trial and improvement' methods to solve problems, and it was acknowledged by the Chief Examiner that this was indicative of engagement with problems. However, students, particularly at Higher level, tended not to 'engage accurately and effectively with algebra' to solve non-routine problems'. Students taking Higher level also struggled 'when algebra became more central to the solution of problems', such as finding the value of n for a triangle with sides 9, n

[22] These objectives were new to the version of the syllabus issued 2013 (for first examination in 2016 onwards), but were nonetheless applied to samples of scripts from the 2015 Junior Certificate examination. They mirror the objectives of mathematics at Leaving Certificate level.

and $n + 1$. Some wrote $(n + 1)^2$ as n^2+1. This is perhaps surprising since students at Higher level achieved 76% of available marks on items dealing more directly with Algebra. It suggests that students struggle to apply existing knowledge in a flexible way, perhaps because it is too routinized.

With regard to Ordinary level, a good level of strategic competence was observed, though students were not required (and generally didn't) apply algebra-based approaches to solving problems. They struggled to find the mean number of cars sold per month, having been given quarterly data. At Foundation level, students were able to scale up an aerial photographic of the perimeter of a pitch, but were unable to find the perimeter of the scaled up pitch.

Adaptive reasoning (an objective of the Junior Certificate mathematics syllabus) is defined as the capacity for logical thought, reflection and explanation, justification and communication. While an overall improvement on questions requiring adaptive reasoning had been observed, Higher-level students were found to struggle on an item requiring them to interpret a graph detailing mobile phone rates. In particular, they struggled to explain that one phone company would be cheaper for x values to the left of the point of intersection, and others would be cheaper for x values to the right. Students also struggled to express their reasoning on aspects of geometry, including the application of a geometric proof (i.e., if the diagonal in a parallelogram bisects the angle, then the four sides of the parallelogram must be equal in length). Students were unable to give a reason for each step in their proofs, and some 'had difficulty expressing the required result in reasonably accurate language' (p. 30). At Ordinary level, explanations provided by students were viewed as being generally satisfactory, though the requirements were less onerous than at Higher level.

The Chief Examiner again referred to concerns expressed by markers about students' lack of basic competency in Algebra, and, in particular, algebraic manipulation, at Ordinary level. It was noted that students who struggled with Algebra at Junior Certificate Ordinary level might also be expected to do so at Leaving Certificate Ordinary level. It was also noted that, at Higher level, students' basic algebraic manipulation had declined, and that this could lead to problems at Leaving Certificate level. The tendency of students at Higher level to avoid using algebraic strategies to solve non-routine problems was commented on again.

4.4. Junior Certificate Mathematics Performance and PISA

How do grades and scores achieved on Junior Certificate mathematics compare with those achieved on PISA? In the context of analysing data from PISA 2015, it was possible to link students' scores on the Junior Certificate mathematics exam, taken in either 2014 (Fifth and Transition Year students) or 2015 (Third years). Table 4.9 shows the percentages of students in PISA 2015 who took mathematics at Higher and Ordinary/Foundation levels in 2014 or 2015. It was decided to combine Ordinary and Foundation levels as the numbers of students taking the Junior Certificate exam at Foundation level were small (matched data were available for 149 students). The table shows that 15% of students taking Higher level achieved Levels 5-6 (the highest proficiency levels) on PISA 2015, compared with fewer than 1% taking the examination at Ordinary/Foundation levels. On the other hand, fewer than 4% who took the Higher level paper achieved below Level 2 (the lowest proficiency levels) compared with over 30% who took Ordinary/Foundation levels.

Table 4.9: Percentages of students at each PISA 2015 mathematics proficiency level, by Junior Certificate mathematics level (all matched PISA 2015 students)

PISA Proficiency	Higher Level JC Mathematics		Ordinary/ Foundation Level JC Mathematics	
	%	SE	%	SE
Below Level 1	0.2	0.10	8.0	1.08
Level 1	3.1	0.43	23.3	1.17
Level 2	15.0	0.95	37.6	1.51
Level 3	34.3	1.19	24.3	1.37
Level 4	31.3	0.95	6.2	0.74
Level 5	13.5	0.79	0.6	0.21
Level 6	2.5	0.37	0.1	0.90

Unweighted N's: Higher: 3288 students; Ordinary/Foundation: 2205;
Note: All percentages are weighted. SE = standard error.

For comparison purposes, Table 4.10 shows the percentages of PISA 2015 students who were in Third year and took the Junior Certificate mathematics exam in 2015. The table shows that fewer than 5% taking Ordinary/Foundation levels achieved Levels 4, 5 or 6 on PISA mathematics. However, it is of interest to note that 22% of students taking Ordinary/Foundation levels achieved Level 3 on PISA, whereas 35% who took Higher level did so. This suggests that some students of broadly equivalent ability are taking the Junior Certificate examination at Higher or Ordinary/Foundation levels.

Table 4.10: Percentages of students at each PISA 2015 mathematics proficiency level, by Junior Certificate mathematics examination level (PISA 2015 students in third year)

PISA Proficiency	High Level JC Mathematics		Ordinary/Foundation Level JC Mathematics	
	%	SE	%	SE
Below Level 1	0.3	0.15	9.1	1.24
Level 1	3.7	0.62	25.6	1.44
Level 2	17.1	1.33	38.1	1.71
Level 3	34.9	1.51	22.3	1.71
Level 4	30.0	1.11	4.6	0.77
Level 5	11.9	0.90	0.3	0.21
Level 6	2.0	0.34	0.0	0.05

Unweighted N's: Higher: 2206 students; 1366 Ordinary/Foundation students.
Note: All percentages are weighted. SE = standard error.

Table 4.11 provides a breakdown of the percentages of students achieving each PISA proficiency level, for each grade obtained by students in the Junior Certificate maths exam in 2014 or 2015 at Higher and Ordinary Levels. For example, among students taking the Higher-level paper who achieved an A grade, 55% achieved Levels 5-6, while 35% achieved Level 4. Surprisingly, however, 10% achieved below Level 4. It may be the case that these students did not engage with the low-stakes PISA mathematics problems to the extent that they could have. However, in general, the distribution of scores at each Junior Certificate grade across PISA proficiency levels is along expected lines, with, for example, very few students who achieved a D grade at Higher level performing at the highest PISA proficiency levels.

One comparison that may of particular interest is that between students achieving a grade D at Higher level and those achieving a grade A at Ordinary level, given that students achieving Grade D might have taken the Ordinary level paper in the past, while those achieving A at Ordinary level might have contemplated taking Higher level. The distributions are similar in terms of the proportions achieving PISA Proficiency levels 2-4 (about 80% of those achieving each Higher D and Ordinary A). However, marginally fewer students achieving Grade A at Ordinary level performed at Proficiency levels 4-6 (15.3%) compared with those who achieved Grade D at Higher level (20.4%), and more achieving Grade A at Ordinary level achieved at lower Proficiency levels (18.8% scored below Level 2) compared with those achieving D at Higher level (8.0%).

It is also noticeable that, outside those achieving a Grade A at Ordinary level, relatively few students taking Junior Certificate maths at this level achieved at PISA Proficiency levels 4-6. However, over 30% of students achieving Grades A and B at Ordinary level, and 23% achieving Grade C performed at PISA Proficiency level 3 – a level that was achieved by large proportions achieving Grades B, C and D at Higher level.

4.5. Analysis of Content Domains, Cognitive Processes and Contexts Assessed in the Junior Certificate Mathematics Examination

In this section, we draw on a report by Cunningham, Close and Shiel (2016)[23], in which the 2003 (pre-Project Maths) and 2015 (Project Maths) Junior Certificate examination papers were examined through the lens of the PISA and TIMSS assessment frameworks (described in Chapter 2 of this report). They sought to identify any changes in emphasis in the examinations with the introduction of Project Maths. The 2003 examination was selected because it was the first based on the 2000 Junior Certificate mathematics syllabus, and it had been a focus on earlier analysis (e.g., Close & Oldham, 2005; Close, 2006). The 2015 examination was selected because it was the first in which all students in all schools were assessed on the revised syllabus introduced under Project Maths. The analyses in this section are based on items (for example, Question 1, part 1(a)), rather than on entire questions.

[23] A detailed description of the methodology involved in classifying items can be found in Cunningham et al. (2016).

Table 4.11: Percentages of students in Ireland at each Junior Certificate mathematics grade achieving each PISA 2015 mathematics proficiency level

	A		B		C		D		E		F		NG	
Higher	%	SE	%	SE	%	SE	%	SE	%	SE	%	SE	%	SE
Below Level 1	0.0	0.00	0.0	0.08	0.1	0.13	0.5	0.39	2.1	1.79	0.0	0.00	0.0	0.00
Level 1	0.1	18.00	0.7	0.39	2.3	0.60	7.5	1.45	14.2	4.20	16.3	10.13	0.0	0.00
Level 2	1.2	0.78	6.4	0.92	17.0	1.70	28.6	2.22	34.8	6.56	30.1	16.72	0.0	0.00
Level 3	8.7	2.10	29.8	1.73	41.3	2.25	42.9	2.92	35.9	6.41	39.5	16.46	80.0	44.44
Level 4	35.0	2.73	41.2	1.74	31.5	1.00	18.0	2.54	12.2	3.69	12.8	13.97	20.0	44.22
Level 5	42.6	2.82	19.0	1.76	7.2	1.80	2.4	0.74	0.8	1.10	1.4	4.59	0.0	0.00
Level 6	12.4	2.39	2.9	0.68	0.7	0.95	0.0	0.17	0.0	0.00	0.0	0.00	0.0	0.00
Ordinary	%	SE	%	SE	%	SE	%	SE	%	SE	%	SE	%	SE
Below Level 1	5.5	2.22	3.8	1.01	7.1	1.40	14.7	14.7	21.7	6.68	43.8	24.04	0.0	0.00
Level 1	13.3	3.29	15.7	1.78	23.5	1.79	36.0	36.0	42.9	9.27	46.3	24.91	0.0	0.00
Level 2	28.4	6.24	37.8	2.65	41.4	2.64	35.6	35.6	29.8	7.65	9.9	17.26	0.0	0.00
Level 3	37.6	6.11	32.4	2.30	22.7	1.89	12.2	12.2	5.4	3.32	0.0	0.00	0.0	0.00
Level 4	13.6	3.42	9.2	1.48	5.0	1.18	1.4	1.4	0.3	0.98	0.0	0.00	0.0	0.00
Level 5	1.6	1.27	1.0	0.46	0.4	0.33	0.0	0.0	0.0	0.00	0.0	0.00	0.0	0.00
Level 6	0.1	0.33	0.2	0.24	0.0	0.08	0.0	0.0	0.0	0.00	0.0	0.00	0.0	0.00

Based on students who participated in PISA 2015 and took the Junior Certificate mathematics exam in 2014 or 2015. Particular care should be exercised in interpreting the distributions of Proficiency levels for Grades E and F and for NG as the numbers of students represented at these grades bands were very small.

4.5.1. Content Domains of Exam Items

Table 4.12 presents the percentages of items by content domain for the 2003 and 2015 examinations compared to the equivalent percentages for the content dimensions of the TIMSS and PISA frameworks. The most obvious change, between 2003 and 2015, across the three examination levels, is the percentage of items dealing with Data & Chance (as defined in TIMSS) and Uncertainty (as defined in PISA). The proportion classified as Data & Chance doubled at both Ordinary level (from 10% to 20%) and at Foundation level (from 13% to 26%). This change corresponds to the greater emphasis on Statistics & Probability in syllabi under Project Maths. It also brings the proportions of items on Statistics & Probability more into line with TIMSS and PISA. There is also a substantial increase in the proportion of Algebra items in the Higher level examination in 2015, with 37% of items classified as Algebra in TIMSS (up from 29%) and 45% classified as Change & Relationships in PISA (up from 32%).

Table 4.12. Comparison of item percentages for the TIMSS and PISA frameworks, and the 2003 and 2015 Junior Certificate examinations at Higher, Ordinary and Foundation levels, by content domain

TIMSS Content Domain	TIMSS % Items	JC 2003 Items	JC 2015 %	PISA Content Domain	PISA % Items	JC 2003 % Items	JC 2015% Items
Higher Level				Higher Level			
Number	30	12	5	Quantity	25	21	8
Algebra	30	29	37	Change & R'ships	25	32	45
Geometry	20	37	26	Space & Shape	25	29	23
Data & Chance	20	12	19	Uncertainty	25	12	19
Not covered	--	10	13	Not covered	--	7	5
Ordinary Level				Ordinary Level			
Number	30	20	22	Quantity	25	23	26
Algebra	30	23	21	Change & R'ships	25	34	28
Geometry	20	29	27	Space & Shape	25	26	25
Data & Chance	20	10	20	Uncertainty	25	10	19
Not covered	--	18	10	Not covered	--	7	2
Foundation Level				Foundation Level			
Number	30	47	26	Quantity	25	53	26
Algebra	30	16	19	Change & R'ships	25	22	23
Geometry	20	19	26	Space & Shape	25	13	26
Data & Chance	20	13	26	Uncertainty	25	13	26
Not covered	--	6	5	Not covered	--	0	0

Source: Cunningham et al., 2016, Table 2

These increases at Higher level have been matched by a corresponding decrease in Geometry items according to TIMSS (from 37% in 2003 to 26% in 2015) and Space & Shape according to PISA (from 29% to 23%). At Foundation level, the proportion of items categorised as Number in TIMSS dropped from 47% to 26%, while the proportion categorised as Quantity according to PISA dropped from 53% to 26%. Small percentages of items in the 2003 and 2015 examination papers (about 11% across all examination papers) were not covered by the TIMSS framework, and to a lesser extent, by the PISA framework, including items on Sets, Trigonometry, and some theorems in Geometry. It might also be noted that TIMSS is administered to students near the end of their Second Year and PISA to 15-year-olds who are distributed across a range of grade levels. Therefore, the Junior Certificate examination allows for more time to cover additional topics compared with TIMSS. There are also some differences in the way content areas are defined in the syllabi introduced under Project Maths, compared with the international frameworks. For example, some elements of the 'Number – Applied Measure' topic in Project Maths would be considered Space and Shape items in PISA and Geometric Measurement items in TIMSS.

4.5.2. Cognitive Domains of Exam Items

In Chapter 2, the key PISA mathematics processes were identified as Formulating, Employing and Interpreting. These processes were first identified in PISA 2012. Prior to that, three competency clusters were identified:

- Reproduction – knowing facts and common problem representations, performing procedures, applying standard algorithms and manipulating expressions;[24]
- Connections – integrating, connecting and problem solving in situations that are more than routine, but placed in some familiar contexts;
- Reflection – planning and implementing solutions to more multi-faceted and original problems and some reflection on the process.

It was decided to use the earlier framework because it had been used in earlier research and because it seemed to offer greater clarity than its successor.

Table 4.13 shows that, for both the 2003 and 2015 Junior Certificate examinations at all three levels, the majority of items, from 70% at 2015 Higher level to 100% at 2003 Foundation level, were categorised as belonging to the Reproduction cluster. There are few items, from none at Foundation level in 2003 to 27% for Higher level in 2015, in the Connections cluster, and generally no items in the Reflection cluster, apart from 2% in the Higher level examination in 2015.

The percentages of the 2003 and 2015 examination items categorised with reference to the TIMSS cognitive domains are broadly similar for the Knowing domain at Higher level (32% and 35% respectively), while the percentage of Knowing items at Junior Certificate Ordinary level increased from 29% to 47%, and the percentage at Foundation level remained more or less the same (59% and 60% respectively). The percentages of Applying items dropped at all three levels between 2003 and 2015, with the largest drop occurring at Ordinary level (from 65% to 43%). The percentages at both Higher (49%) and Ordinary (43%) levels are now broadly in line with TIMSS (40%). Surprisingly, in light of Project Maths' emphasis on developing adaptive reasoning, the percentages of Reasoning items at all three examination levels lagged well behind the TIMSS figure of 25%, though increased were observed at all levels between 2003 and 2012.

[24] Cunningham et al. interpreted the Reproduction cluster in PISA as involving 'reproduction of practised ideas' and therefore was viewed as including Knowing in TIMSS, but also some of Applying, such as routine problem solving. The Reflections cluster in PISA was interpreted as involving 'advanced' reasoning, abstraction and generalisation in novel contexts, which often requires a higher cognitive demand than some TIMSS reasoning items.

Table 4.13: Comparison of item percentages for the TIMSS and PISA frameworks, and the 2003 and 2015 Junior Certificate examinations at Higher, Ordinary and Foundation levels, by cognitive domain

TIMSS Content Domain	TIMSS % Items	JC 2003 Items	JC 2015 %	PISA Content Domain	PISA % Items	JC 2003 % Items	JC 2015% Items
Higher Level				Higher Level			
Knowing	35	32	35	Reproduction	25	79	70
Applying	40	56	49	Connections	50	21	27
Reasoning	25	12	17	Reflection	25	0	2
Ordinary Level				Ordinary Level			
Knowing	35	29	47	Reproduction	25	90	84
Applying	40	65	43	Connections	50	10	14
Reasoning	25	6	10	Reflection	25	0	0
Foundation Level				Foundation Level			
Knowing	35	59	60	Reproduction	25	100	84
Applying	40	41	28	Connections	50	0	16
Reasoning	25	0	12	Reflection	25	0	0

Source: Cunningham et al., 2016, Table 3

4.5.3. Contexts of Exam Items

A key goal of Project Maths is to place more emphasis on using mathematics to solve problems set in practical and preferably realistic contexts. Most PISA items are embedded in rich contexts, along four dimensions – personal, occupational, societal and scientific. In TIMSS, about one-half of items are embedded in some sort of practical context, and, by default, the remaining half are embedded in intra-mathematical contexts. Cunningham et al. (2016) categorised questions in the 2003 and 2015 exams according to whether they mainly have an intra-mathematical context, an authentic practical context, or a minimal context. Items were classified as 'authentic' if situated in a practical context that was carried through the item (i.e., where students had to engage with real-life concepts in order to generate a solution, such as items on tax credit and net pay). Items in a 'minimal context' were defined as those characterised by 'pseudo contexts' that did not affect the solution and were effectively the same as an intra-mathematical item (e.g., find the surface area of a ball in the shape of a sphere). The results of this analysis are summarised in Table 4.14.

The data show that, in 2015, about half of Junior Certificate items at each examination level were embedded in practical, authentic contexts – up from 29% at Higher level, 41% at Ordinary level, 41% at Foundation level in 2003. The data indicate that many of the items at Ordinary and Foundation levels were set in practical-authentic contexts prior to Project Maths. A consequence of Project Maths is a reduction in the proportion of items at each level categorised as intra-mathematical, with a 12% reduction at Higher level, an 8% reduction at Ordinary level, and a 14% reduction at Foundation level. There was a decline in the proportion of items categorised as embedded in a minimal or pseudo context at Higher level (from 7% to 0%) and small increases at Ordinary level (2% to 6%) and Foundation level (0% to 7%).

Table 4.14: Comparison of item percentages for the TIMSS and PISA frameworks, and the 2003 and 2015 Junior Certificate examinations at Higher, Ordinary and Foundation levels, by cognitive domain

Level	Intra-mathematical (%)	Practical-Authentic (%)	Minimal (%)
Higher 2003	64	29	7
Higher 2015	52	48	0
Ordinary 2003	56	41	2
Ordinary 2015	48	46	6
Foundation 2003	59	41	0
Foundation 2015	44	49	7

Source: Cunningham et al., 2006, Table 4

4.5.4. Readability of Exam Items

King and Burge (2015) reported that the average readability of PISA clusters (of which students typically complete one or two, along with clusters of reading and/or science items) ranged from Grades 8-11 in PISA 2012, with word counts ranging from 821 to 1193 words per cluster. Hence, students taking one hour of PISA mathematics (2 clusters, the amount of mathematics completed by most participating students) are expected to read about 2000 words.

Based on data for three TIMSS 2015 test booklets, Cunningham et al. (2016) estimated that a student taking 45 minutes of TIMSS mathematics (all students take this amount) is expected to read about 1200 words, in texts with an average readability of Grade 2.

Table 4.15 shows that students taking 2.5 hours of items at Higher level in 2015 were expected to read around 1500 words[25], written at an average readability of Grade 4.[26] While the reading load increased (for example, from 765 words to 1335 words on Paper 1), there is no evidence that the overall difficulty of the text changed since 2003. At Ordinary level, there was a similar increase in the number of words to be read (for example, from 662 to 1240 on Paper 1). However, the average number of words per sentence, the percentage of complex words, and the readability levels for Papers 1 and 2 were unchanged. At Foundation level, the number of words increased from 530 to 1027 words, while the average readability level increased from Grade 0 (pre-reading) to Grade 2. Hence, while the number of words to be read doubled on each paper (except Ordinary Level Paper 2), the average readability stayed at about the same level. Moreover, the Higher level papers were at a higher readability level than the TIMSS test, while the Ordinary and Foundation level papers were at about the same readability level as TIMSS. The papers at all exam levels were written at a considerably lower readability level than the PISA mathematics test, and students taking each exam level were expected to read considerably fewer words, on a pro-rated basis.

[25] Note that the readability formulae here may not be well-suited to analysing the difficulty of mathematics texts, as they do not measure the complexity of equations, formulae etc.

[26] A computer programme provided data on number of words, number and percent of complex (three-syllable) words, number of sentences, average sentence length, seven readability formulae (Flesch Reading Ease, Flesch-Kincaid Grade Level, Fog Scale, Smog Index, Coleman-Liau Index, Automated Readability Index, and Linsear Write Formula), and the average readability in US grade level units across the seven formulae.

Table 4.15. Comparison of item percentages for the TIMSS and PISA frameworks, and the 2003 and 2015 Junior Certificate examinations at Higher, Ordinary and Foundation levels, by cognitive domain

Exam Paper	No. of Items	No. of Words	No of Sentences	Avg words per sentence	Number of complex words (%)	Flesch-Kincaid Grade Level	Avg. Readability Grade Level
2003 HL Paper 1	32	765	58	10	61 (8)	5.0	4
2015 HL Paper 1	45	1335	135	13	123 (9)	4.1	4
2003 HL Paper 2	41	835	78	11	73 (9)	4.3	4
2015 HL Paper 2	39	1620	160	10	131 (8)	4.4	4
2003 OL Paper 1	40	662	76	8	35 (5)	2.6	2
2015 OL Paper 1	41	1240	181	7	64 (5)	2.2	2
2003 OL Paper 2	42	935	106	9	65 (7)	3.2	3
2015 OL Paper 2	40	1051	117	9	86 (8)	3.3	3
2003 FL	32	530	73	7	23 (4)	1.6	0
2015 FL	43	1027	140	7	57 (6)	2.3	2

Source: Cunningham et al. (2016), Table 5; HL = Higher level; OL = Ordinary level; FL = Foundation Level.

4.6. Summary

This chapter examined the Junior Certificate mathematics examination from a number of perspectives, including changes to the examination since the advent of Project Maths, the participation of students at different examination levels, distributions of examination grades over time, overall and by gender, the Chief Examiner's report for 2015, links between performance on the Junior Certificate examination and PISA, changes in the content and processes assessed by the Junior Certificate exam, and changes in reading load and readability.

Key characteristics of the Junior Certificate Mathematics examination were considered. It was noted that some aspects of the examination had not changed at all (such as the numbers of papers at each level and the overall time allocated to each paper). Other aspects were found to have changed, including the number of questions presented on each paper and the specific marks allocated to each question. Each question now provides an indication of how much time students should allocate to it, rather than the number of marks it is worth.

The numbers and percentages of students taking different Junior Certificate examination papers have changed over time. In 2016, 55.3% took the Higher level exam paper, while 39.9% took Ordinary level and 5% Foundation level. It was shown that increases in the proportions taking Higher level have been gradual from 2003 onwards, as have decreases in the proportions taking Foundation level. For example, in 2003, 40.6% took Higher level, and 12.5% Foundation level. In each year from 2003 onwards, a greater proportion of females than males took the Junior Certificate mathematics examination at Higher level, while greater proportion of males than females took the examination at Ordinary and Foundation levels, with the exception of 2003 at Ordinary level. This is surprising in light of the stronger average performance of male students in Ireland on PISA mathematics in 2012 and 2015, and the slightly stronger performance of male students on TIMSS mathematics.

While the distributions of examination grades from year to year have tended to be quite stable, there has been a noticeable drop in the proportion and absolute number of students achieving Grade A at Higher level. For example, 17.5% achieved Grade A in 2011, while 11.7% did so in 2016. Moreover, 4450 students achieved Grade A in 2011, compared with 3855 in 2016. In most years over the past decade, more male than female students have achieved Grade A at Higher level. The relatively low proportion of students achieving Grade A is consistent with the low proportion of students achieving the highest Proficiency levels (levels 5-6) in PISA and the Advanced benchmark in TIMSS. At Higher level, there has also been a small drop in the proportion achieving Grades A-C (down from 80% in 2011 to 76% in 2016), with marginally more females than males achieving grades in this range in 2015. The decline in the proportion achieving Grades A-C may reflect migration from Ordinary level to Higher level in recent years. As noted earlier, the migration from Ordinary to Higher level is likely to have occurred because students at Junior Cycle want to position themselves to take Higher level at Leaving Certificate and qualify for 25 bonus points by achieving Grade D3 (now H6) or higher. However, some migration was observed even before the re-introduction of bonus points. In recent years, the Project Maths initiative may also have played a role.

Performance at Ordinary level has also declined in recent years, with fewer students achieving Grade A and Grades A-C combined. In 2016, 71.8% achieved Grades A-C, compared with 76.2% in 2012 – the year before the first full cohort of Junior Certificate students completed examination questions on some aspects of Project Maths. Again, this may reflect migration from Ordinary to Higher level, leaving fewer higher-achieving students at Ordinary level. At Foundation level, there was also a drop in the proportion of A grades since 2012, but the proportion of A-C grades remained constant between 2012 and 2016. Female students outperformed males at Ordinary level, achieving more A and A-C grades in 2016, while males and females performed at about the same level on the Foundation paper.

In 2016, relatively sizeable proportions of students achieved a grade below D – 3.5 % at Higher level, 6.1% at Ordinary level, and 3.5% at Foundation level – about 2700 students, or some 4.5% of the cohort of 59,589 students taking the exam. The proportion of students performing below Grade D is quite high at Ordinary level, and may be indicative of a lack of certainly among teachers and students on the standard required at different examination levels.

The Chief Examiner's report for 2015 indicates a relatively strong focus on Number across exam levels, with over 35% of questions at Ordinary and Foundation levels focusing on this content area. Relative to other content areas, Functions was under-represented in 2015, accounting for 12% of items at Higher level, 6% at Ordinary level, and none at Foundation level. Performance at Higher and Foundation levels was strongest on items on Statistics & Probability questions, while, at Ordinary level, it was highest on Number questions. The weakest domains for students taking Higher level were Functions and Hybrid. On average, students taking Ordinary level achieved just 40% of available marks on Algebra questions. At Foundation level, students did least well on Geometry & Trigonometry Questions. The Chief Examiner's remarks in relation to Algebra are instructive. Performance on Algebra at Higher level was deemed to have declined in recent years, and the report noted that students were often unable to apply their algebraic skills to solving more complex problems where it would be helpful to do so. Furthermore, markers were reported to have expressed concern about Ordinary level candidates' lack of basic competency in Algebra, and the difficulties this would cause for these candidates as they move on to study Leaving Certificate Mathematics.

A comparison of the performance of students who took part in PISA 2015 and for whom Junior Certificate mathematics grades were also available showed that, for the most part, students scoring at the highest Proficiency levels in PISA also performed well on the Junior Certificate exam. However, students achieving Ordinary level Grades A-C performed at the same level on PISA as some students achieving a Higher level Grade D. This indicates some overlap in performance across Higher and Ordinary levels, though more in the mid-range of proficiency rather than at the upper end.

An analysis of the content categories and cognitive processes assessed by the Junior Certificate examinations of 2003 and 2015 through the lens of the PISA and TIMSS assessment frameworks, as reported by Cunningham et al. (2016), was considered. Not unexpectedly, there was an increase in the proportion of items (defined as question parts) in the Data & Chance (TIMSS) and Uncertainty (PISA) content domains across all examination levels. An increase was also observed in the proportion of items assessing Algebra in TIMSS and the corresponding domain of Change & Relationships in PISA, at Higher level. The emphasis on Geometry (TIMSS) and Shape & Space (PISA) decreased at Higher level between 2003 and 2015, remained constant at Ordinary level and increased at Foundation level. Finally, there was a drop in the emphasis on Number (TIMSS) and Quantity (PISA), at Higher level, no change at Ordinary level, and a drop at Foundation level. The relative under-emphasis on Number/Quantity on the Junior Certificate exam is partly linked to the fact that some items categorised as Number under Project Maths, including Applied Measure are categorised as Space & Shape (PISA) and Geometry (TIMSS), or don't appear on the assessments at all (e.g., SETS, some theorems in Geometry). Overall, the distribution of content across the Junior Certificate papers more closely resembles TIMSS than PISA.

An analysis of the cognitive processes underpinning Junior Certificate mathematics indicated a strong emphasis on Knowing and Applying (as defined by TIMSS) and some emphasis on Reasoning. On the other hand, while there is a relatively strong emphasis on items in the PISA Reproduction cluster at all examination levels, and some emphasis on Connections, there is relatively little emphasis on Reflection across levels. It is worth noting that, in order for an item to be classified as Reflection, it needs to involve both solving a multi-faceted problem, and reflecting on this process. Most of the time in the JC maths exam papers these tasks are split up, and students are asked either to solve a problem or to reflect on a given solution/statement. As part of the upcoming review of the curriculum and assessment arrangements, it may not be considered desirable to have substantial numbers of question parts on a single 2-hour paper that require candidates to perform both these skills. In fact, it may be that the Classroom Based Assessments are seen as a more fitting place to ask students to do tasks that would be classified as Reflection items under the PISA framework.

An analysis of contexts in which Junior Certificate items were embedded showed declines in the proportions of 'intra-mathematical' or context-free items at Higher and Ordinary levels. This was matched by increases in the proportions of items embedded in practical-authentic contexts, though the change was greater at Higher level than at Ordinary and Foundation levels, where many of the items were already embedded in context before the introduction of Project Maths. The analysis indicated that about one-half items at each examination levels can be described as intra-mathematical. This is consistent with the TIMSS assessment, but differs from PISA, where there is almost no emphasis on intra-mathematical items.

Finally, an analysis of the readability of Junior Certificate examination papers shows that, while the volume of material to be read increased substantially between 2003 and 2015, the difficulty of the embedded vocabulary, sentence length and the readability level of the papers at Higher level (average Grade 4 readability level) and Ordinary level (Grades 2-3) did not change. An exception was at Foundation level, where there was an increase from Grade 0 (pre-reading level) to Grade 2. Overall, there is less reading in the Junior Certificate examination papers than on PISA, which is also written at a higher readability level (Grades 8-11).

Chapter 5. Teaching Mathematics – Perspectives from International Studies

5.1. Introduction

This chapter draws on two sources of data to describe instructional factors in Junior Cycle mathematics classes: the TIMSS 2015 database, which includes responses to questionnaires by students in Second year, their teachers and their school principals; and the PISA 2012 database, which includes responses to questionnaires by 15-year olds and their school principals. The main focus is on TIMSS data, since PISA data are from an earlier assessment cycle (when mathematics was a major assessment domain), and Third- and Transition-year students in the PISA 2012 sample had experienced Project Maths. Hence, PISA data are used to amplify or question findings from TIMSS.

The following topics are considered in this chapter: class size, instructional time, instructional practices, content coverage in mathematics, in-career development in mathematics, use of technology in mathematics classes, and factors limiting the teaching of mathematics.

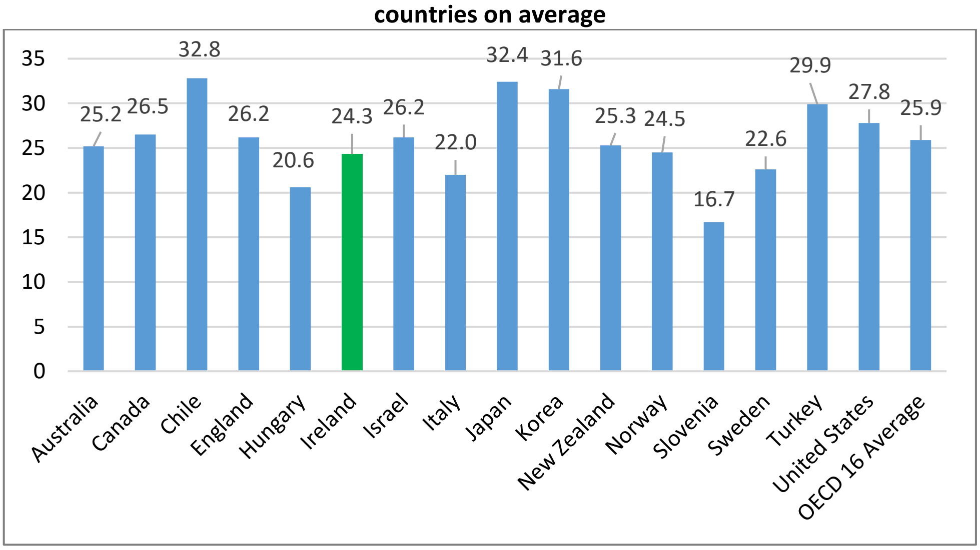
Analyses based on TIMSS refer to an OECD-16 average. This is an average score for the 16 OECD countries that participated in TIMSS 2015 at Grade 8 (see Chapter 2).

5.2. Class Size

TIMSS 2015 provides an estimate of the size of Second-year mathematics classes. In Ireland, average class size was 24.3 students (Figure 5.1). The corresponding OECD-16 average was slightly higher, at 25.9 students. Average class size in Ireland is marginally lower than in Australia (25.2), Canada (26.5) and New Zealand (25.3). Slovenia (16.7) had the smallest average class size in the OECD-16.

In Ireland, the correlation between class size and mathematics performance was moderately strong (r = 0.42; SE = 0.03) (Table A5.1). This means that, as class size increases, performance on TIMSS 2015 increases (possibly because lower-performing students tend be in smaller classes). On average across the OECD-16, the correlation between class size and mathematics performance was 0.16. After Ireland and England (0.50), New Zealand (0.25) and Australia (0.23) had the strongest correlations between class size and mathematics performance.

Figure 5.1. Average number of students in students' mathematics classes, in Ireland and across 16 OECD countries on average



It may be that, in Ireland and England, the assignment of students to classes by ability level, with lower-ability students assigned to smaller classes, accounts for the relatively stronger positive associations between class size and performance.

Neither PISA 2012 nor PISA 2015 gathered data on class size in mathematics classes.

5.3. Instructional Time

The Junior Cycle mathematics syllabus (NCCA, 2013) specifies that the mathematics course is a 240-hour course over three years (or 80 hours per school year). In Ireland, the weekly allocation of time to mathematics in Second year was 192.7 minutes (Table 5.1). On average across the OECD-16, the allocation was somewhat greater at 210.4 minutes. Chile (300.7) and Canada (270.1) had the highest allocations.

Based on data provided by teachers (number of instructional hours per year) and principals (number of school days per year; number of days per school week), TIMSS 2015 reported that, in Ireland, 109 hours were allocated to mathematics instruction in Second year (Table 5.1). The average number of hours of mathematics instruction across the OECD-16 was above this level at 131 hours, with teachers in just two countries – Sweden (99 hours) and Japan (106 hours) – reporting that they allocated less time to mathematics instruction on a weekly basis than teachers in Ireland.

In Ireland, the allocation of instructional hours to mathematics on a yearly basis represents 11.3% of total instructional hours. Again, this is on the low side relative to other countries participating in TIMSS 2015 (at Grade 8). In Canada, for example, the number of instructional hours per year in mathematics represents 18% of total instructional time, while on average across the OECD-16, it is 13.3%. Just two countries – Sweden (10.8%) and Japan (10.2%) reported allocating a smaller proportion of available instructional time to mathematics.

Interestingly, Ireland had the lowest standard deviation (16.0) for instructional minutes per week among the OECD-16. The OECD-16 standard deviation is 46.8 minutes (Table A5.2). This suggests that there is relatively little variability in Ireland with regard to weekly instructional time, compared with OECD countries in TIMSS 2015.

TIMSS did not collect data on average lesson length. Lessons that are 30-40 minutes in length may not be long enough for teachers and students to engage in meaningful ways with lessons that are presented in the style of Project Maths.

Table 5.1. Instructional time spent on mathematics as a percentage of total instruction time per year in rank order from highest to lowest for 16 OECD countries including Ireland (teachers' reports)

	Total instruction hours per year	Number of instructional days per year	Number of instructional minutes per week - maths	Hours per year for mathematics instruction	% of total instructional time
Canada	949	190.0	270.1	168	17.7
Chile	1,127	187.1	300.7	192	17.0
New Zealand	966	190.7	223.8	144	14.9
Italy	1,047	201.2	247.0	149	14.2
Australia	1,011	196.4	213.6	139	13.8
United States	1,135	178.7	258.1	155	13.7
Israel	1,133	211.8	243.4	153	13.5
Hungary	842	181.7	185.6	113	13.4
Slovenia	867	190.1	180.0	114	13.2
England	1,009	191.7	195.4	126	12.5
Korea	947	192.6	177.1	114	12.0
Turkey	983	181.0	191.7	117	11.9
Norway	895	189.6	165.4	105	11.7
Ireland	**963**	**167.0**	**192.7**	**109**	**11.3**
Sweden	921	179.2	166.2	99	10.8
Japan	1,036	204.5	155.6	106	10.2
OECD 16 Avg.	989	189.6	210.4	131.4	13.3

Data from TIMSS 2015 International Database

In Ireland, the correlation between instructional time per week and student performance is weak (-0.05) and not statistically significant (t = 1, df = 80, p. = 0.32) (Table A5.3). This may reflect the low variance in estimates of instructional time in Ireland. On average across the OECD-16, the correlation was a non-significant 0.00, again indicating no clear relationship between instructional time and performance.

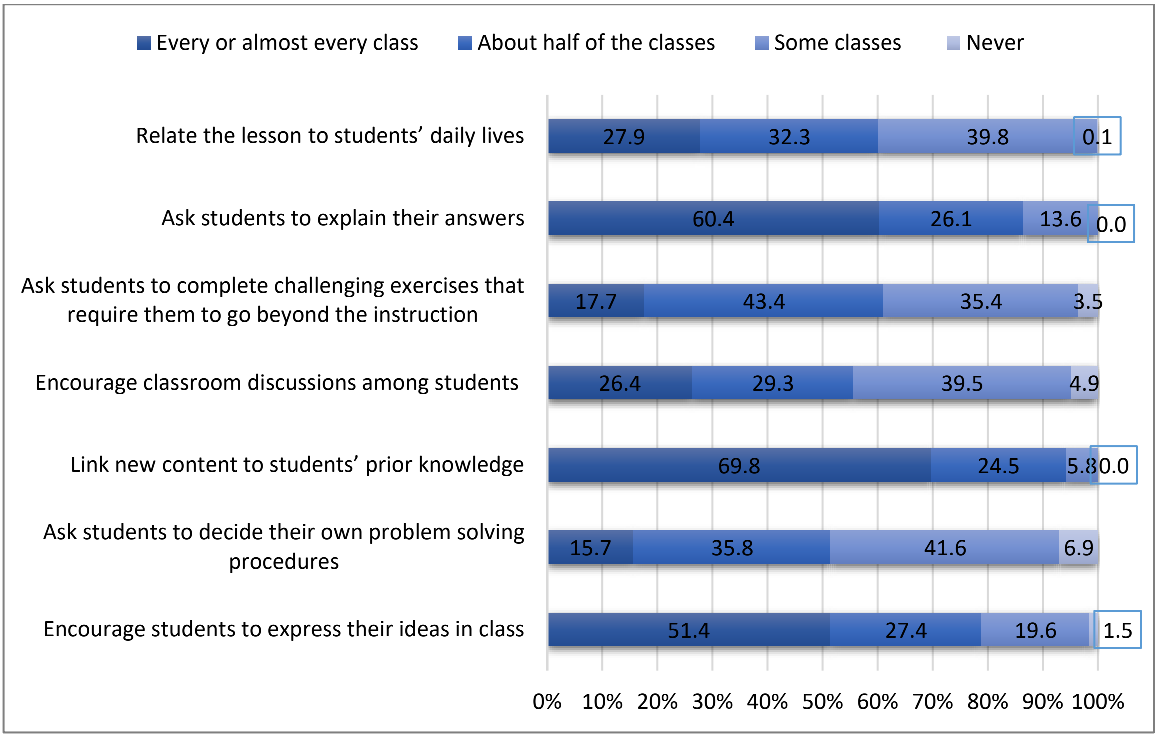
In PISA 2012, students in Ireland spent an average of 189 minutes per week in mathematics classes compared with an OECD average of 198 minutes. Students in Transition Year (25% of the sample) reported spending 160 minutes per week, while those in Second and Third years (60%) spent 195 minutes on average, and those in Fifth year spent 219 minutes. Hence the data for weekly instructional time in Second year in TIMSS 2015 (193 minutes per week) and PISA 2012 in Second and Third years (195 minutes) are broadly consistent.

While most countries in PISA increased the allocation of instructional time to mathematics between 2003 and 2012 (the OECD average increase was 13 minutes per week), there was no change to the allocation of time in Ireland (as reported by students) (OECD, 2016b).

5.4. Instructional Practices in Mathematics Classes

TIMSS asked teachers of Second years about the frequency with which they engaged students in their classes in a number of activities designed to promote learning in mathematics. Many of these practices are compatible with Project Maths, including asking students to explain their answers and relating lessons to students' daily lives. Three teaching practices were experienced by at least 50% of students in Ireland in 'every or almost every class': asking students to explain their answers (60.4%); linking new content to students' prior knowledge (69.8%); and encouraging students to express their ideas in class (51.4%) (Figure 5.2). Practices that occurred less often in 'every or almost every class' included asking students to complete challenging exercises that go beyond the instruction (17.7%), encouraging classroom discussion among students (26.4%), and asking students to decide on their own problem-solving procedures (15.7%).

Figure 5.2. Percentages of students whose teachers report engaging in various general teaching practices to promote conceptual understanding in every or almost every class, in about half of classes, in some classes, and never, in Ireland, in TIMSS 2015



Students in Ireland and on average the OECD-16 experience similar levels of engagement with most teaching practices (Table 5.2). For example, in Ireland 60.4% of students are asked by teacher to explain their answers 'in every or almost every class', while 59.2% of students on average across the OECD-16 are asked to do so. Similarly, 26.4% of students in Ireland are encouraged by their teachers to engage in classroom discussions, while 31.6% on average across the OECD-16 are encouraged to do so. On the other hand, fewer students in Ireland (15.7%) than on average across the OECD-16 (31.4%) are asked to decide on their own problem-solving procedures when solving problems.

Table 5.2. Percentages of students whose teachers report engaging in various teaching practices to promote conceptual understanding in 'every or almost every' class, in Ireland and in 16 OECD countries on average, in TIMSS 2015

	%	
	Ireland	OECD-16
Relate the lesson to students' daily lives	27.9	31.2
Ask students to explain their answers	60.4	59.2
Ask students to complete challenging exercises that require them to go beyond the instruction	17.7	19.7
Encourage classroom discussions among students	26.4	31.6
Link new content to students' prior knowledge	69.8	65.1
Ask students to decide their own problem-solving procedures	15.7	31.4
Encourage students to express their ideas in class	51.4	57.1

In a number of countries, there is a stronger emphasis on several of these teaching practices, compared with Ireland (Appendix Table A5.4). In Canada, for example, according to their teachers, 42.9% of students experience instruction that relates lesson content to their daily lives 'in every or almost every class', while in Ireland 27.9% of students experience such instruction with the same frequency. On the other hand, teachers in Southeast-Asian countries such Japan and South Korea place considerably less emphasis on these teaching practices than teachers Ireland. For example, in Japan, just 1.5% of students are taught by teachers who ask students to complete challenging exercises that go beyond the textbook 'in most or all classes', while in Korea, just 12% of students are taught by teachers who encourage classroom discussion among students with the same frequency. Nevertheless, it might be concluded that, for the most part, the frequency with which students in Ireland experience the teaching practices considered by TIMSS is similar to the frequency with which those practices are experienced by students on average across participating OECD countries.

In PISA 2012, students reported on whether they participated in various activities designed to promote cognitive activation strategies in mathematics. These are strategies students can call on when learning mathematics or solving mathematical problems. They overlap with the teaching practices considered by TIMSS. More students in Ireland than on average across OECD countries 'strongly agreed' or 'agreed' that their teacher asked students to explain how a problem has been solved (79% in Ireland, 70% on average across OECD countries), and or asked questions to make students reflect on a problem (71% vs. 59%) (Perkins & Shiel, 2016a, Table 4). On the other hand, there were no differences in the percentages of students in Ireland and on average across OECD countries 'strongly agreeing' or 'agreeing' that their teacher gives them problems that can be solved in several different ways (59% vs. 60%), and presents problems in different contexts so that students know whether they have understood the concepts (59% vs. 59%). Fewer students in Ireland (31%) reported that their teacher asks them to decide on their own procedures for solving complex problems, compared to the OECD average of 42%. This finding is consistent with the data in Table 5.2 on asking students to decide on their own problem-solving procedures.

Teachers in TIMSS 2015 in Ireland also reported on the frequency with which they engaged students in a range of additional mathematics activities, mainly focusing on presentation of content, group work and assessment (Figure 5.3; Table 5.3). Over three-in-five students (62.2%) engaged in listening to the teacher explain new mathematics content in 'every or almost every class', 53% listened to the teacher explain how to solve problems, and 50.4% worked problems individually or with peers under the guidance of the teacher. Activities that were undertaken with the same frequency by fewer students included memorising rules, procedures or facts (14.2%), solving problems (individually or with peers while the teacher is working with other students) (11.9%), working in mixed-ability groups (15.1%) and working in same-ability groups (8.7%). In Ireland, just 2% of students were asked by their teachers to work on problems for which there was no obvious method of solution.

The frequency with which students in Second year experienced various teaching practices in mathematics classes is remarkably similar in Ireland and on average across the OECD-16, with almost equivalent proportions listening to the teacher explain new concepts (61.6% in Ireland and 62.2% on average the OECD-16 experienced this in 'every or almost every class'), listening to the teacher explain how to solve problems (53.0% vs. 57.3%), solving (working) problems under the teacher's guidance (50.4% vs. 54.5%), and solving problems together with the whole class, with guidance from teachers (39.2% vs. 40.0%) (Table 5.3; e-Appendix Tables A5.5 and A5.6). Similarly, activities such as working with mixed-ability groups (15.1% in Ireland, 13.6% on average across the OECD-16) and memorising rules, facts or procedures (14.2% vs. 22.0%) were experienced relatively infrequently by students in Ireland and on average across the OECD-16. It is unclear to what extent discovery learning activities were built into lessons.

Table 5.3. Percentages of students whose teachers ask them to particpate in various instructional activities in mathematics class in 'every or almost every class', Ireland and OECD-16 average, TIMSS 2015

	%	
	Ireland	OECD-16
Listen to me explain new mathematics content	61.6	62.2
Listen to me explain how to solve problems	53.0	57.3
Memorise rules, procedures, and facts	14.2	22.0
Work problems (individually or with peers) with my guidance	50.4	54.5
Work problems together in the whole class with direct guidance from me	39.2	40.0
Work problems (individually or with peers) while I am occupied by other tasks	11.9	13.6
Work on problems for which there is no immediately obvious method of solution	1.8	6.2
Take a written test or quiz	2.6	8.0
Work in mixed ability groups	15.1	13.6
Work in same ability groups	8.7	9.1

Appendix Table A5.7 provides a breakdown by country for each instructional activity. In general, more students in Canada experience each activity 'in every or almost every class' than their counterparts in Ireland, with, for example, 75.1% of students in Australia experiencing new content with this frequency, compared with 61.6% in Ireland. England was remarkably similar to Ireland across all instructional activities.

In general, frequency of instructional activities in mathematics classes and mathematics performance as measured by TIMSS tended not to be strongly associated in Ireland or on average across the OECD-16 (Appendix Tables A5.5 and A5.6), with performance differences across categories of response (e.g., every or almost every class vs. never) generally too small to reach statistical significance.

Figure 5.3. Percentages of students whose teachers ask them to particpate in various instructional activities in mathematics class in every or almost every class, in about half of classes, in some classes and never, in Ireland, in TIMSS 2015

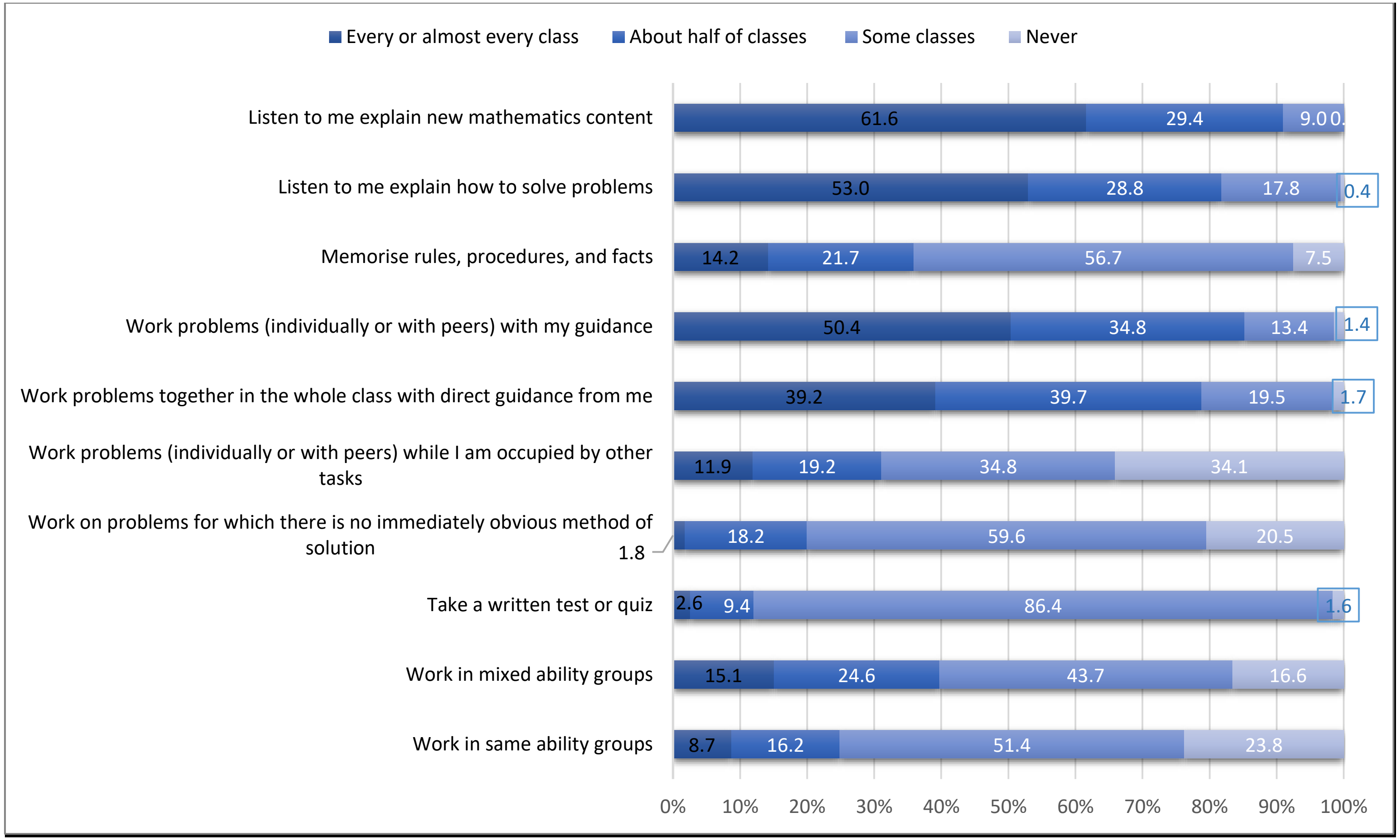
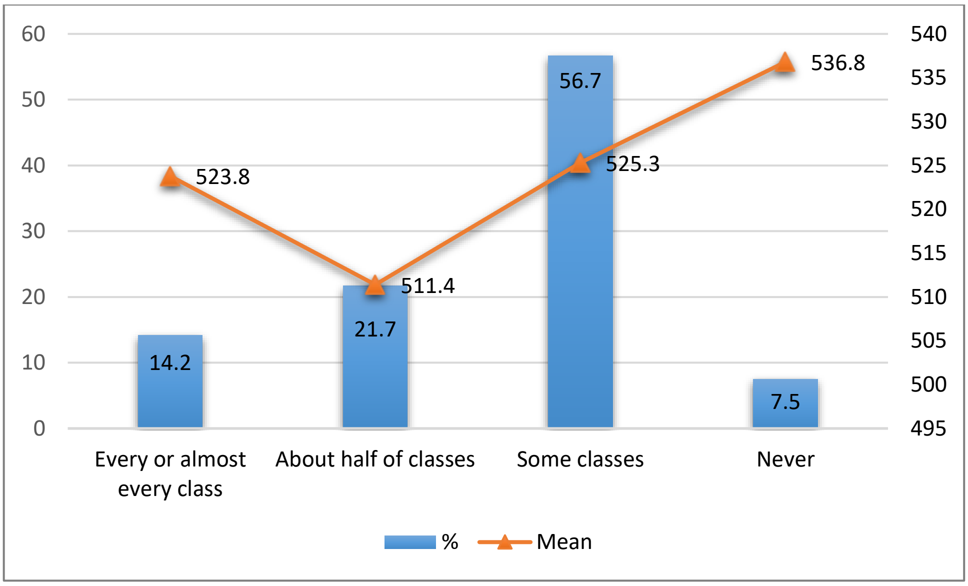


Figure 5.4 shows this for memorising rules, procedures and facts. Students in Ireland whose teachers reported that memorisation occurred 'in every or almost every class' had a higher mean score than students of teachers who reported that this occurred in 'about half of classes' (523.8 vs. 511.4). Even though there was a gradual increase in performance between those whose teachers reported that memorisation occurred in 'about half of classes', through those for whom it occurred in 'some classes' to those for whom it 'never' occurred, differences were too small to reach statistical significance, mainly due to the very large standard errors around mean scores. Similarly, although students whose teachers reported that they never ask them to memorise had a mean score (536.8) that was higher than that of students of teachers who asked them to memorise in some classes (523.8) by 13 score points, the difference was not statistically significant. This may have arisen because students in the 'Never' group are not distributed across schools in a systemic manner, and there are relatively few of them.

Figure 5.4. Percentages of students whose teachers ask them to memorise rules, procedures, and facts in 'every or almost every class', 'about half of classes', 'some classes' and 'never', in Ireland, and average achievement in mathematics, in TIMSS 2015



Reports on PISA 2012 published by the OECD (Echazarra et al., 2016; OECD, 2016b) have characterised mathematics instruction along four dimensions:

- teacher-directed instruction (where teachers tell students what they have to learn, set clear goals for students learning, ask students to present their thinking or reasoning at some length, and ask questions to check whether students have understood what was taught)
- student-orientated instruction (where teachers have students work in small groups, give different work to students who have difficulties learning, or to those who can advance faster, assign projects that require at least one week to complete, and ask students to help plan classroom activities or topics)
- formative assessment instruction (where teachers tell students what is expected of them when they get a test, quiz or assignment, give them feedback on strengths and weaknesses in mathematics, tell them about how well they are doing in their mathematics class, and tell them what they need to do to become better in mathematics)

- cognitive activation instruction, where teachers ask students to explain how they solved a problem, ask students to decide on their own procedures for solving complex problems, ask questions that make them reflect on the problem, give problems that require students to think for an extended time, problems for which there is no immediately obvious solution, and problems that can be solved in several different ways).

Among participating countries in PISA 2012, Ireland was identified by Echazarra et al. as having the strongest relative focus on cognitive activation instruction, relatively strong emphases on formative-assessment instruction and teacher-directed instruction, and the weakest relative emphasis on student-orientated instruction.

Although these findings are based on student reports of teaching, and were gathered prior to the implementation of Project Maths in all classrooms (though many teachers of Third year students in 2012 would have been teaching Project Maths at other class levels), there are broadly satisfactory. As noted earlier, many elements of cognitive activation instruction are consistent with Project Maths, and therefore, Ireland's strong standing on this element of mathematics instruction is to be welcomed. It might be argued that elements of teacher-directed instruction, such as asking students to present their thinking or reasoning at some length are also important, though the implication here may be that mathematics instruction is too teacher-directed at times. Ireland's relatively high position on formative-assessment instruction, in which, for example, students are given feedback on their strengths and weaknesses in mathematics, can also be viewed positive. However, Ireland's low positon on student-orientated instruction is a matter of concern.

5.5. Content Coverage in Mathematics Classes

Given that the TIMSS mathematics test is content based, TIMSS asked mathematics teachers of students in Grade 8 (Second year) whether or not that had covered specific content on the TIMSS test. In all, teachers were asked to indicate, in respect of 20 topics across the areas of Number, Algebra, Geometry and Data & Chance, whether each topic had been mostly taught before the current school year (presumably as part of the Common Introductory Course), mostly taught in the current (2014-15) school year, or not yet taught or just introduced. Table 5.4 provides a breakdown of the results, in terms of percentages of students in each response category. The far-right column (not yet taught or just introduced) provides an indication of which TIMSS topics had not been covered by the end of Second year.

Most topics in Number had been covered by the vast majority of students, with the exception of irrational numbers. Teachers of about one-third (32.2%) of students reported had this topic had not yet been covered. Ratings were more mixed for Algebra (including Functions). Almost all students had been taught by teachers who had covered simplifying and evaluating algebraic expressions and solving linear equations and inequalities. However, just over 50% had been taught by teachers who had not covered representation of functions as ordered pairs, tables, graphs, words or equations, or properties of functions (slope, intercepts). Over 35% were taught by teachers who reported that numeric, algebraic, and geometric patterns or sequences (extension, missing terms, generalisation of patterns) had not been covered. One quarter of students were taught by teachers who had not covered simultaneous (two variable) equations.

In Geometry, a number of topics had not been taught to a majority of students, including translation, reflection and rotation (63.1%), relationship between three-dimensional shapes and their two-dimensional equivalents (59%), and congruent figures and similar triangles (55.1%). Over one-fifth had not been taught to use appropriate measurement formulas for perimeters, circumference, surface areas and volumes. Between one-fifth and one-quarter had not been taught the geometric properties of angles and geometric shapes, or points on the Cartesian plane. The latter is surprising since locating points on the plane using co-ordinates is part of the Common Introductory Course for First-year students, while central symmetry, axial symmetry and rotation are also on the CIC.

Finally, in relation to Data & Chance, just under one third of students were taught by teachers who indicated that interpreting data sets had not been covered while one-quarter were taught by teachers who reported that judging, predicting and interpreting the chances of possible outcomes had not been addressed. Just under one-fifth of students (19.8%) had not been taught the characteristics of data sets (mean, median, mode and shape of distribution).

Table 5.4. Percentages of students in TIMSS 2015 whose teachers report teaching various mathematics topics mostly before second year, mostly in second year, and not yet or just introduced, in Ireland

	Mathematics topics taught		
	Mostly taught before this year	Mostly taught this year	Not yet taught or just introduced
	%	%	%
Number			
Computing with whole numbers	95.2	4.4	0.4
Comparing and ordering rational numbers	83.6	14.5	1.9
Computing with rational numbers (fractions, decimals, and integers)	82.9	16.0	1.1
Concepts of irrational numbers	33.2	34.6	32.2
Problem solving involving percentagess or proportions	56.2	38.8	5.1
Algebra			
Simplifying and evaluating algebraic expressions	37.6	61.9	0.47
Simple linear equations and inequalities	19.3	75.1	5.6
Simultaneous (two variable) equations	2.2	72.3	25.6
Numeric, algebraic, and geometric patterns or sequences (extension, missing terms, generalisation of patterns)	15.7	46.8	37.5
Representation of functions as ordered pairs, tables, graphs, words, or equations	2.8	45.7	51.5
Properties of functions (slopes, intercepts, etc)	2.19	47.8	50.1
Geometry			
Geometric properties of angles and geometric shapes (triangles, quadrilaterals, and other common polygons)	35.0	42.2	22.8
Congruent figures and similar triangles	5.6	39.3	55.1
Relationship between three-dimensional shapes and their two-dimensional representations	5.4	35.5	59.0
Using appropriate measurement formulas for perimeters, circumferences, areas, surface areas, and volumes	6.6	63.0	34.4
Points on the Cartesian Plane	33.0	45.4	21.6
Translation, reflection, and rotation	11.7	25.2	63.1
Data and Chance			
Characteristics of data sets (mean, median, mode, and shape of distributions)	25.0	55.2	19.8
Interpreting data sets (e.g., draw conclusions, make predictions, and estimate values between and beyond given data points)	15.8	52.2	32.0
Judging, predicting and determining the chances of possible outcomes	30.4	45.2	24.4

Overall, coverage of TIMSS mathematics topics in Ireland (73%) was just below the OECD-16 average of 76.6% (Table 5.5). Ireland was well above the OECD-16 average on Data & Chance (75% vs. 66%), and marginally above it on Number (92% vs. 90%) and Algebra (72% vs. 70%). In Geometry, however, Ireland was well below the corresponding OECD-16 average (58% vs. 78%). However, care should be exercised in interpreting these findings. If a topic included two are three elements (such as mean, median, mode and shape of distribution), teachers may have decided a topic had not been taught if one of these elements had not been covered. Examples of items in each content area might have provided a better indication of content coverage than one based on verbal descriptions.

It is interesting that, in Korea and Japan, the highest-performing OECD countries in TIMSS Mathematics at Grade 8, average coverage scores for Algebra and Geometry were at or exceeded 90%. This is also evident in the actual performance of countries by content domain. In Ireland, students achieved average percent correct scores of 39% on Algebra and 40% on Geometry (Mullis, Martin, Foy & Hooper, 2016). In Korea, students achieved scores of 67% on Algebra and 67% on Geometry, while in Japan, students achieved 62% on Algebra and 64% on Geometry.

Table 5.5. Percentages of students taught various TIMSS mathematics topics, by country and OECD-16 average

	Students taught the TIMSS mathematics topics*				
	All mathematics topics (20 topics)	Number (5 topics)	Algebra (6 topics)	Geometry (6 topics)	Data & Chance (3 topics)
Australia	76 (0.9)	90 (0.9)	65 (1.5)	77 (1.4)	71 (2.2)
Canada	76 (0.8)	89 (0.8)	61 (1.2)	85 (1.3)	69 (1.9)
Chile	80 (1.5)	90 (1.3)	65 (2.4)	87 (1.6)	78 (3.4)
Hungary	85 (0.7)	98 (0.4)	78 (1.0)	90 (0.9)	67 (2.7)
Ireland	**73 (1.0)**	**92 (0.8)**	**72 (1.5)**	**58 (1.8)**	**75 (2.3)**
Israel	82 (0.8)	90 (0.7)	90 (0.8)	78 (1.1)	59 (2.4)
Italy	75 (0.8)	99 (0.3)	49 (1.4)	92 (0.9)	56 (2.9)
Japan	88 (0.6)	81 (1.2)	91 (0.8)	95 (0.6)	79 (2.3)
Korea	80 (0.6)	81 (0.7)	94 (0.6)	90 (0.9)	34 (3.1)
New Zealand	75 (1.1)	87 (1.0)	69 (1.5)	69 (1.8)	76 (2.2)
Norway	65 (0.8)	87 (1.0)	51 (1.7)	58 (1.6)	65 (2.5)
Slovenia	60 (0.7)	97 (0.6)	49 (1.1)	63 (1.1)	14 (1.2)
Sweden	61 (1.3)	78 (1.4)	55 (2.7)	59 (1.7)	49 (2.6)
Turkey	82 (0.7)	100 (0.2)	62 (1.7)	79 (1.0)	99 (0.6)
United States	90 (0.7)	98 (0.4)	92 (0.8)	84 (1.3)**	83 (2.0)**
England	77 (1.3)	82 (1.0)	72 (1.9)	77 (2.2)	76 (2.2)
OECD-16	76.6	89.9	69.7	77.6	65.6

Adapted from IEA (2015). OECD-16 = 16 participating OECD countries in TIMSS 2015.*Percentage mostly taught before or in the assessment year averaged across topics. *Data are available for at least 70% but fewer than 85% of the students.

It should be noted that a somewhat different perspective was adopted in the test-curriculum matching analysis conducted in each country participating in TIMSS 2015. In that analysis, curriculum experts in each participating country indicated, for each TIMSS item, whether or not the item was likely to have been covered by most students in a country. Since there is no set curriculum for students in Second year in Ireland (the curriculum covers Grades 7-9, or Grades 8-9 if the CIC is considered separately), those engaging in the matching analysis had to provide their professional opinion on whether the topic of the item was likely to have been covered by most students by the end of Second year. In Ireland, the topics underpinning 202 of the 209 TIMSS Grade 8 mathematics items (97%) were judged to have been covered. Interestingly, however, despite variation in curriculum coverage across TIMSS countries, students in Ireland tended to perform at about the same level on, for example, the TIMSS item sets covered in the curricula of Korea (50% correct), Japan (51%), the United States (49%) and England (49%). While the average percent correct score for students in Ireland on all TIMSS 2015 items was 49%, it improved marginally to 50% on the 202 items for which the underlying topics were judged to have been covered by most students in Second year.

PISA mathematics does not examine coverage of traditional mathematics content areas. However, in 2012, information was provided by students on the frequency with which they encountered procedural mathematics tasks, contextualised mathematics problems, and pure mathematics problems:

- Procedural mathematics problems (e.g., solving $2x + 3 = 7$; finding the volume of a box with sides of 3 cm, 4 cm and 5cm).
- Contextualised mathematics problems (e.g., compare two formulas for recommended maximum heart rate based age ($200 - age$; $208 - 0.7 * age$) and identify the point at which the second one leads to an increase in the maximum recommended rate).
- Pure mathematics problems (e.g., determine the height of a pyramid using a geometrical theorem; if n is any number, investigate if $(n + 1)^2$ is a prime number).

PISA 2012 students were asked to indicate how often they encountered different types of mathematics tasks in class or on tests (see above). The data pointed to a relative over-emphasis on procedural tasks in Ireland (77% reported frequent experience with these problems, compared with an OECD average of 68%), though this may have changed since the introduction of Project Maths at all class levels (Perkins & Shiel, 2016b). Overemphasis on procedural tasks is potentially problematic if students do not understand the concepts underlying the procedures they perform, but, instead, routinely apply computational routines, with limited flexibility. Fewer students in Ireland (26%) than on average across OECD countries (34%) encountered pure mathematics problems frequently. This is a concern to the extent that large proportions of students in high-scoring countries such as Korea (36%) and Japan (48%) report that they encountered such problems more frequently. More students in Ireland (27%) encountered contextualised problems frequently, compared with the corresponding OECD average (21%).

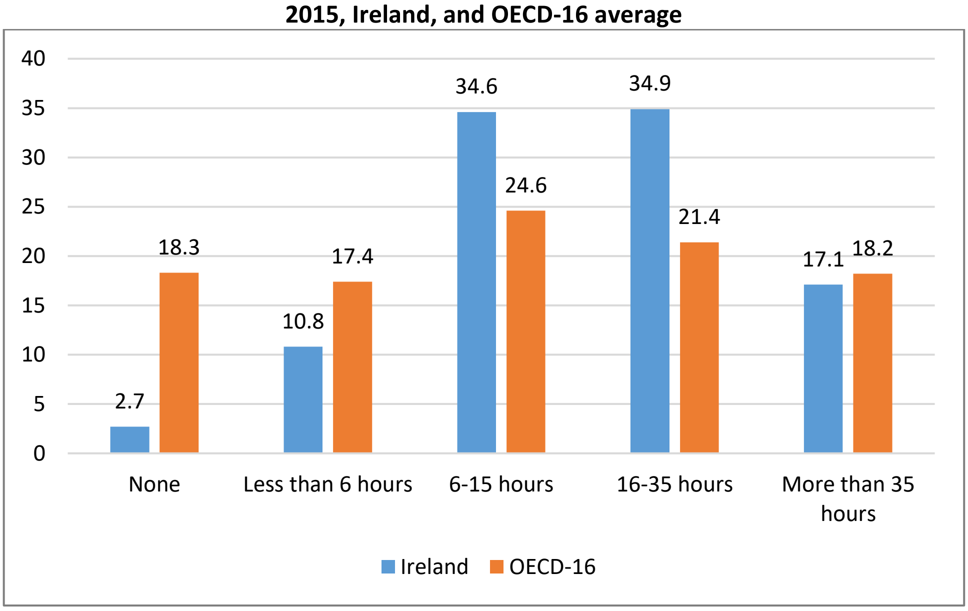
In general, across OECD countries, more frequent exposure to particular problem types was associated with higher overall performance on PISA mathematics. However, in Ireland, students who solved contextualised problems frequently performed significantly less well than students who rarely or never did so. This suggests that, prior to full implementation of Project Maths, lower-achieving students were more likely to be asked to solve contextualised problems, compared with higher-achieving students. This, in turn, may reflect the types of questions asked of students at different levels of the Junior Certificate mathematics examination.

Students in PISA 2012 indicated their familiarity with each of 13 mathematical terms (referred to as concepts by the OECD) on a scale ranging from ‘know and understand the concept well’ to ‘never heard of the concept’. The list included exponential functions, vectors, polygons, congruent figures, arithmetic means, divisors, complex numbers, and probability. Fewer students in Ireland than on average across OECD countries indicated that they were familiar with each of these mathematical terms, with the exception of quadratic functions. Across most terms (with the exception of vectors), students in Ireland with higher levels of socioeconomic status were more familiar with target term than students of lower socioeconomic status (OECD, 2016b). The OECD describes familiarity with mathematics concepts as a measure that captures the cumulative opportunity to learn mathematical content over a student's career. However, it also recognises that, while familiarity with mathematical concepts is important, additional exposure to mathematical concepts (terms) in and of itself is not enough. Students also need extensive exposure to problems that 'stimulate their reasoning abilities and promote conceptual understanding, creativity, and problem-solving skills' (OECD, 2016b, p. 3).

5.6. In-career Development in Mathematics

TIMSS 2015 asked teachers of students in Second year about involvement in formal and informal professional development. TIMSS defined formal professional development as attendance at seminars and workshops, and informal professional development as involving, for example, mathematics teachers working together to try out new ideas. As per Figure 5.5, in Ireland, 13.5% of students were taught by teachers who had attended fewer than 6 hours of formal professional development, compared with 35.7% on average across the OECD-16. On the other hand, more students in Ireland (69.5%) than on average across the OECD-16 (45.0%) were taught by teachers who had attended between 16 and 35 hours of professional development. Equal proportions of students in Ireland and in the OECD-16 were taught by teachers who spent more than 35 hours on professional development, and these may have included teachers taking post-graduate courses and degrees. It is likely that the stronger participation in professional development overall by teachers in Ireland is linked to participation in professional development around the implementation of Project Maths, as well as the Professional Diploma in Mathematics for Teaching (see Chapter 1).

Figure 5.5. Percentages of students whose teachers participated in formal professional development activities (e.g., workshops, seminars) for mathematics of various durations in in the two years prior to TIMSS 2015, Ireland, and OECD-16 average



Teachers were also asked about whether or not they had covered a range of topics in the course of professional development in the two years prior to TIMSS 2015. Teachers were asked to indicate 'yes' or 'no' in respect of each topic. No reference was made as to whether professional development was formal or informal. In Ireland, 94% of students were taught by teachers who had participated in professional development relating to mathematics content, compared to an average of 54.8% across the OECD-16. Teachers in Ireland reported more engagement than their counterparts on average across the OECD-16 on a number of additional topics including mathematics pedagogy/instruction, mathematics curriculum, integrating information technology into mathematics and improving students' critical thinking and problem solving skills. The proportions of students in Ireland and on average across the OECD-16 were similar for two topics: assessment of mathematics (40.5% in Ireland

and 37.9% across the OECD-16), and addressing individual students' needs (35.1% and 38.1% respectively). These were also topics least often covered in professional development availed of by teachers in Ireland.

Table 5.6. Percentages of students whose mathematics teachers reported engaging with specified topics across professional development activities in the two years prior to TIMSS 2015 – Ireland and OECD-16 average

Topic	Ireland	OECD-16
Mathematics content	94.1	54.8
Mathematics pedagogy/instruction	78.3	56.9
Mathematics curriculum	91.1	46.0
Integrating information technology into mathematics	65.2	45.1
Improving students' critical thinking or problem solving skills	70.6	39.5
Mathematics assessment	40.5	37.9
Addressing individual students' needs	35.1	38.1

OECD-16 = 16 participating OECD countries in TIMSS 2015

Teachers in TIMSS 2015 were also asked about the frequency with which they engaged in various professional development activities with other teachers. In Ireland, 56.2% of students were taught by teachers who had 'often' or 'very often' discussed how to teach a particular (mathematics) topic with other teachers, which is marginally lower than the corresponding OECD-16 average of 62.1% Table 5.7). More students in Ireland (63.0%) than on average across the OECD-16 (51%) were taught by teachers who worked with other teachers in a group to implement curriculum – an activity consistent with lesson study (see Chapter 1). On the other hand, fewer students in Ireland (25.7%) than on average across the OECD-16 (40.1%) worked together to try out new ideas. One activity in which relatively few teachers in Ireland engaged was visiting another teacher's classroom to learn more about teaching (mathematics) (5%). On average across the OECD-16, 16.9% of students were taught by teachers who reported doing this.

It is noteworthy that, in Ireland, 53.8% of students are taught by teachers who frequently share what they have learned about their teaching experiences. Although a little lower than the corresponding OECD-16 average (62.0%), it is something that can be built on going forward.

Table 5.7. Percentages of students whose mathematics teachers report various interactions with other teachers 'often' or 'very often', in 16 OECD countries including Ireland, in TIMSS 2015

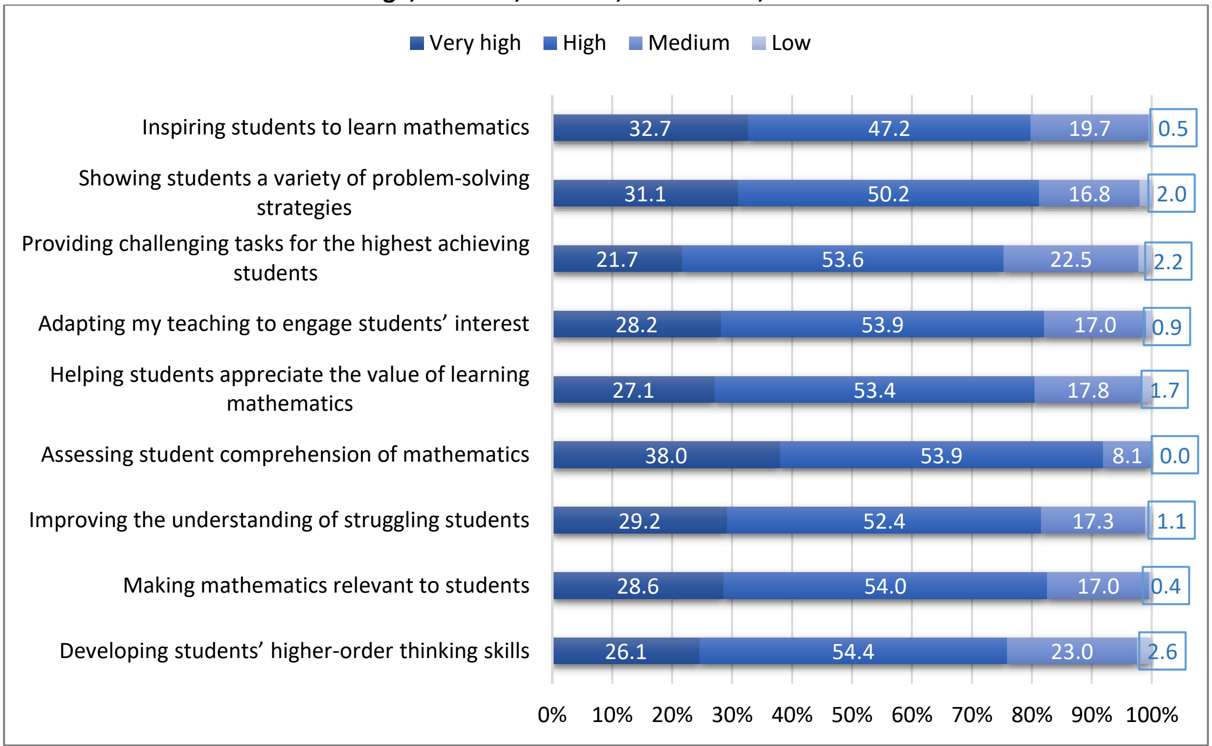
	Ireland	OECD -16
Discuss how to teach a particular topic	56.2	62.1
Collaborate in planning and preparing instructional materials	50.9	54.3
Share what I have learned about my teaching experiences	53.8	62.0
Visit another classroom to learn more about teaching	5.0	16.9
Work together to try out new ideas	27.5	40.1
Work as a group on implementing the curriculum	63.0	51.0
Work with teachers from other grades to ensure continuity of learning	42.8	38.6

OECD-16 (16 participating OECD countries). See Table A5.8 for data for individual countries.

Teachers in TIMSS 2015 were asked to indicate their level of confidence with regard to various instructional and assessment activities related to mathematics. The resulting data may provide some insights into areas in which professional development may be needed. In Ireland, 38% of students were taught by teachers who expressed themselves as 'very confident' in assessing student comprehension of mathematics, while 32.7% were taught by teachers who held a similar level of confidence in relation to inspiring students to learn mathematics (Figure 5.6). Relatively fewer students (21.7%) were taught by teachers who were 'very confident' in providing challenging tasks for higher-achieving students in mathematics.

It is noteworthy that about one-in-five students were taught by teachers who expressed themselves to have 'medium' or 'low' confidence in teaching various aspects of mathematics. For example, 18.8% of students were taught by teachers who expressed 'medium' or 'low' confidence in showing students a variety of problem-solving strategies. In general, students of teachers with 'medium' or 'low' confidence for activities such as showing students a variety of problem solving strategies have lower mean scores than students of teachers with higher levels of confidence. However, differences tend not to be statistically significant (due to large standard errors around mean scores). Moreover, some teachers' confidence may be low as a result of working with lower-achieving students rather than the other way around.

Figure 5.6. Percentages of students whose teachers report aspects of their (teacher) confidence as very high, high, medium, and low, TIMSS 2015, Ireland



See Appendix Table A5.9 for full details and student performance linked to teacher confidence.

Table 5.8 compares the percentages of students in Ireland and on average across the OECD-16 whose teachers are 'very confident' or 'confident' in relation to each aspect of teaching. In general, the proportions are broadly similar, though more students in Ireland (82.6%) than on average across the OECD-16 (76.2%) are taught by teachers who are 'very confident' or 'confident' in making mathematics relevant to students.

PISA does not gather information on teachers' confidence in their ability to teach various aspects of mathematics.

Table 5.8. Percentages of students whose mathematics teachers report aspects of their (teacher) confidence is 'very high' or 'high', TIMSS 2015, Ireland and OECD-16

	Ireland	OECD-16
	% Very High or High	% Very High or High
Inspiring students to learn mathematics	79.8	78.1
Showing students a variety of problem-solving strategies	81.3	85.0
Providing challenging tasks for the highest achieving students	75.3	74.4
Adapting my teaching to engage students' interest	82.0	78.0
Helping students appreciate the value of learning mathematics	80.5	78.8
Assessing student comprehension of mathematics	91.9	86.1
Improving the understanding of struggling students	82.6	76.2
Making mathematics relevant to students	82.6	73.2
Developing students' higher-order thinking skills	74.5	68.3

5.7. Use of Technology in Mathematics Classes

This section considers the use of calculators and computing devices in mathematics classes.

Teachers of students in TIMSS 2015 were asked whether their students in Second year were allowed to use calculators in mathematics classes without restriction or use calculators with restricted use, or were not allowed to use calculators at all. Almost three-quarters of students in Ireland (72%) were allowed to use calculators without restriction, compared to just 30% on average across the OECD-16 (Table 5.9). Over one-quarter of students in Ireland were allowed to use calculators with restricted use – about half of the proportion on average across the OECD-16 who could use calculators in this way. Only 1.3% of students in Ireland were not allowed to use calculators, compared to 15.6% on average across the OECD-16. Performance on TIMSS 2015 mathematics was marginally (though not significantly) higher in Ireland among students who could use calculators with restricted use (529.0), compared with those who could use them without restriction (520.3). On average across OECD countries, the mean scores of students with different levels of access to calculators did not differ significantly on mathematics performance, though those who used calculators with unrestricted use had a marginally higher score than the other groupings.

Table 5.9. Percentages of students whose teachers report that students are permitted calculators in mathematics classes with 'unrestricted use', with 'restricted use', and not at all, and average achievement in mathematics, in Ireland, and across 16 OECD countries on average, in TIMSS 2015

	Ireland			OECD-16		
	%	Mean	SE	%	Mean	SE
Yes, with unrestricted use	72.0	520.3	(3.37)	29.7	516.7	(2.52)
Yes, with restricted use	26.7	529.0	(5.95)	54.7	510.9	(1.49)
No, calculators are not permitted	1.3	514.1	(37.50)	15.6	505.4	(4.57)

OECD-16 = 16 participating OECD countries in TIMSS 2015.

Teachers of students in TIMSS 2015 were also asked to indicate the frequency with which students in their classes used calculators for a range of purposes. In Ireland, 51% of students were in classes whose teachers reported that they used calculators to do routine operations in 'every or almost every' class

(Table 5.10). Activities for which students in Ireland used calculators less frequently included solving complex problems (40.2% did so in 'every or almost every' class) and exploring number concepts (23.0%).

Table 5.10. Percentages of students who use calculators in mathematics class whose teachers report students use calculators for various activities, TIMSS 2015, Ireland

	Students' use of calculators for various activities			
	Check answers	Do routine computations	Solve complex problems	Explore number concepts
	%	%	%	%
Every or almost every class	48.2	51.0	40.2	23.0
About half of classes	22.4	25.6	25.4	18.6
Some classes	26.3	20.8	28.0	41.7
Never	3.1	2.5	6.4	16.7

However, relative to other OECD countries in TIMSS 2015, calculator usage in Ireland is high. For example, where checking answers is concerned, more students in Ireland (70.6%) than in any other country except Israel (71.4%) are taught by teachers who reported that calculators were used in 'about one half of lessons or more often' (Table 5.11). Similarly, in the case of computation, more students in Ireland use calculators in 'about half of lessons or more often' than in any other country, with the exception of Canada (78.6%). Ireland also fares well in terms of using calculators to explore number concepts (41.7%) compared with the corresponding OECD-16 average (29%), though, again, Canada is in the lead (64.4%).

Table 5.11. Percentages of students who use calculators for various activities (teachers' reports, 'every or almost every class', 'about half of classes'), in 16 OECD countries including Ireland, in TIMSS 2015

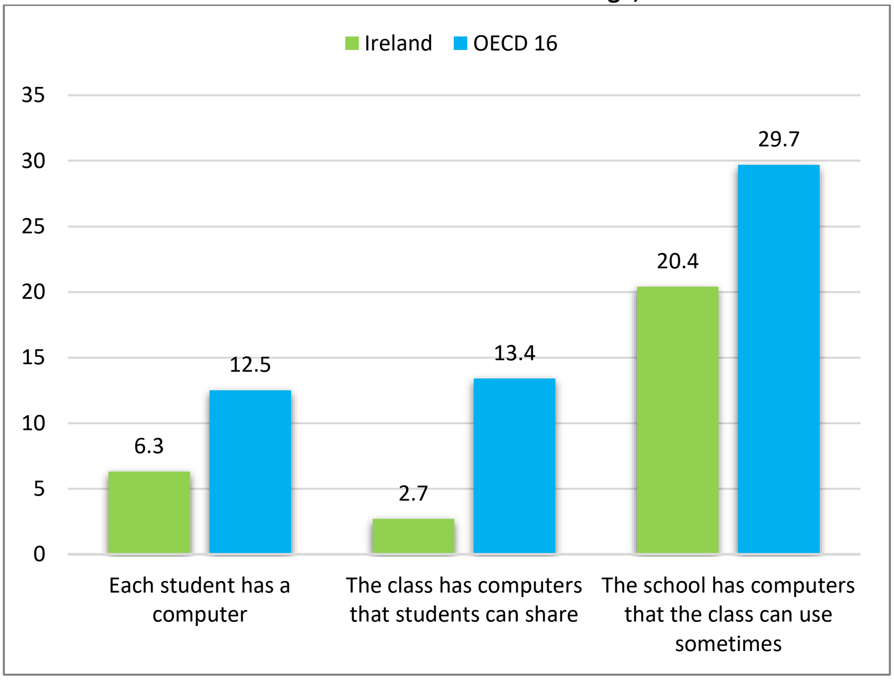
	Percentages of students			
Country	Check answers	Doing Computations	Solve Complex Problems	Explore Number Concepts
Australia	70.4	72.5	71.4	43.7
Canada	85.5	78.6	92.0	64.4
Chile	34.5	13.3	30.0	23.4
Hungary	33.9	32.5	45.2	15.5
Ireland	**70.6**	**76.6**	**65.6**	**41.7**
Israel	71.4	71.1	71.6	49.3
Italy	52.6	40.1	70.5	22.9
Japan	13.1	10.9	5.4	7.7
Korea	0.7	6.7	7.6	4.9
New Zealand	59.9	64.2	66.4	43.1
Norway	70.7	68.4	73.4	28.0
Slovenia	8.7	7.2	14.2	9.8
Sweden	45.4	61.7	57.2	20.2
Turkey	8.5	8.1	14.1	9.8
United States	67.7	63.3	74.6	55.4
England	36.8	35.3	52.9	23.9
OECD-16 Average	44.9	44.4	50.8	29.0

OECD-16 = 16 participating OECD countries. Check = check answers; Computations = do routine computations; Solve = solve complex problems; Explore = explore number concepts.

The average across OECD countries is lower than in Ireland for each activity. This arises in part because teachers in a number of countries, including Japan, Korea, Slovenia and Turkey, report low usage of calculators across all activities. For example, just 7.7% of students in Japan and 4.9% in Korea use calculators to explore number concepts 'in about half of classes or more often', compared with 41.7% in Ireland.

Teachers in TIMSS 2015 were also asked about the availability of computers to pupils in their Second year class. In Ireland, 6.3% of students were in classes whose teachers reported that individual computing devices were available to students during mathematics classes (Figure 5.7). On average across the OECD-16, about twice as many students (12.5%) had access to individual devices. In Ireland, 2.7% of students had access to shared computing devices in their mathematics classrooms, compared with 13.4% on average across the OECD-16. Finally, 20.4% of students in Ireland were in schools where computers could be accessed for mathematics classes, compared to 29.7% on average across the OECD-16.

Figure 5.7. Percentages of students with access to computing devices for use during mathematics classes in Ireland and across 16 OECD countries on average, in TIMSS 2015



OECD-16 = 16 participating OECD countries.

In Ireland, the most common use to which computers were put on a monthly basis was to practice skills and procedures (12% were directed by their teacher to do so at least once or twice a month). (Table 5.12). Marginally fewer explored mathematics principles and concepts (11%), looked up ideas and information (10%), or processed and analysed data (10%).

Relative to other OECD countries in TIMSS 2015, access to ICT infrastructure (mainly computing devices) is low in Ireland, with just 25% having access to a computing device for mathematics lessons, either via individual devices, sharing of devices or access to devices elsewhere in the school (Table 5.12). Only in Turkey (16%), Israel (17%) and Slovenia (19%) is access lower than in Ireland. In Japan, where actual computer-usage in mathematics classes is lower than in Ireland, 43% of students have access to computing devices for mathematics lessons.

In general, differences in mathematics performance within countries between students with access to computing devices and those without access tend to be small. In Ireland, for example, there was a difference of 10 score points in favour of those who did not have access to devices, in comparison with those who did (Table 5.12). Because of large standard errors around mean scores, the difference was not statistically significant. On average across the OECD-16, there was a difference of just one point between those with access to devices and those without access.

Table 5.12. Computer activities during mathematics classes, in Ireland and across 16 OECD countries on average, in TIMSS 2015

Country	Computers available for student to use in mathematics classes			Percentage of students whose teachers have them use computers at least monthly			
	Percentage of students	Average achievement		Explore	Practice	Look up	Process
	Yes	Yes	No				
Australia	62 (3.4)	512 (3.5)	506 (5.4)	51 (3.5)	52 (3.6)	48 (3.6)	44 (3.2)
Canada	50 (3.3)	528 (3.7)	533 (3.2)	35 (2.8)	36 (3.1)	33 (3.0)	31 (3.1)
Chile	49 (4.6)	423 (5.5)	437 (5.8)	29 (4.3)	36 (4.4)	32 (4.5)	36 (4.3)
Hungary	30 (3.8)	509 (8.0)	516 (4.6)	20 (3.3	27 (3.6)	22 (3.2)	18 (3.0)
Ireland	**25 (2.8)**	**515 (6.2)**	**525 (3.4)**	**11 (1.9)**	**12 (2.0)**	**10 (1.7)**	**10 (1.8)**
Israel	17 (2.4)	536 (11.8)	508 (4.3)	11 (2.0)	13 (2.2)	12 (2.1)	11 (1.9)
Italy	43 (3.7)	493 (4.3)	495 (4.1)	28 (3.2)	29 (3.3)	31 (3.5)	26 (2.9)
Japan	43 (3.7)	585 (4.1)	588 (3.4)	3 (1.0)	6 (1.8)	4 (1.3)	5 (1.5)
Korea	39 (3.6)	604 (4.3)	607 (3.6)	25 (3.3)	22 (3.1)	24 (3.2)	19 (2.6)
N Zealand	47 (3.5)	501 (4.8)	488 (5.7)	36 (3.3)	35 (3.3)	35 (3.3)	33 (3.5)
Norway	40 (3.9)	513 (3.5)	513 (3.2)	27 (3.9)	35 (4.1)	27 (4.0)	29 (3.8)
Slovenia	19 (2.5)	517 (6.7)	516 (2.1)	12 (2.2)	14 (2.1)	13 (1.9)	13 (1.9)
Sweden	65 (3.6)	499 (4.0)	502 (4.0)	25 (3.7)	38 (4.0)	32 (4.2)	26 (3.9)
Turkey	16 (2.3)	471 (13.2)	456 (5.0)	13 (2.1)	11 (2.2)	15 (2.2)	12 (2.1)
United States	39 (2.9)	519 (5.0)	518 (4.3)	27 (2.8)	31 (2.9	29 (2.8)	26 (2.8
England	29 (4.1)	511 (9.7)	520 (6.0)	17 (3.6)	23 (3.7	17 (3.3)	13 (2.9)
OECD 16 Average	38	515	514	23	26	25	22

Adapted from Mullis et al. (2016). Explore = to explore mathematical concepts and principles; Practice = to practice skills and procedures; Look up = to look up ideas and information; Process = to process and analyse data

In PISA 2012, students in Ireland achieved a score of -0.15 on an index of computer use in mathematics lessons. This score, which was based on student reports of computer usage, was considerably lower than in countries such as Denmark (0.70), Norway (0.69) and Turkey (0.26). However, students in a number of countries with very high achievement on PISA mathematics also had reported low usage of computers, including Finland (-0.33), Korea (-0.36) and Japan (-0.62) (see OECD, 2015).

In Ireland, about one-in-five students in PISA 2012 reported that their teachers demonstrated various mathematical procedures for them on computer, including drawing a graph of a function (19%), constructing geometric shapes (19%), observing changes in a function as a constant changes (16%), and entering data on a spreadsheet (13%). However, about one-in-ten students reported engaging in these activities themselves, indicating a greater prevalence of teacher demonstration than student usage.

5.8. Factors Impacting on Teachers' Efforts to Teach Mathematics

TIMSS 2015 asked teachers to indicate the extent to which a range of factors limited how they teach mathematics. In Ireland, 28.6% of students were taught by teachers who reported that 'students lacking prerequisite knowledge or skills' impacted 'a lot' on how they teach in class. Other issues had a relatively smaller impact in terms of the proportions of students involved, including 'disinterested students' (13.6%) and 'disruptive students' (11.8%).

Table 5.13. Percentages of students whose teachers report various issues as limiting how they teach in class 'some' or 'a lot', Ireland

Issue	A lot	Some	Not at all
Students lacking prerequisite knowledge or skills	28.6	62.3	9.1
Students suffering from lack of basic nutrition	3.8	17.2	79.0
Students suffering from not enough sleep	6.9	58.8	34.3
Disruptive students	11.8	34.3	53.8
Disinterested students	13.6	64.3	22.1
Students with physical disabilities	0.5	6.8	92.8
Students with mental, emotional, or psychological impairments	3.6	43.1	53.3

See Table A5.12 for standard errors.

Table 5.14 shows that the percentages of students in Ireland and on average across the OECD-16 are broadly similar for most issues, when percentages for 'a lot' and 'some' are combined. Hence, 90.9% of students in Ireland and 86.1% on average across the OECD-16 are taught by teachers who regard 'students lacking prerequisite knowledge or skills' as limiting how they teach mathematics 'to some extent or a lot'. Fewer students in Ireland (46.2%) than on average across the OECD-16 (67.1%) are taught by teachers who consider disruptive students as limiting how they teach mathematics. Almost half of students in Ireland and just over one-half on average across the OECD-16 are taught by teachers who regard 'students with mental, emotional, or psychological impairments' as limiting how they teach.

Table 5.14. Percentages of students whose teachers report various issues as limiting how they teach in class 'some' or 'a lot', in Ireland, and in 16 OECD countries on average, in TIMSS 2015

Issue	Ireland	OECD-16
Students lacking prerequisite knowledge or skills	90.9	86.1
Students suffering from lack of basic nutrition	21.0	30.1
Students suffering from not enough sleep	65.7	65.3
Disruptive students	46.2	67.1
Disinterested students	77.9	80.5
Students with physical disabilities	7.2	9.3
Students with mental, emotional, or psychological impairments	46.7	52.0

OECD-16 = 16 participating OECD countries. See Table A5.13 for breakdown by country.

An overall measure of the extent to which teaching is perceived to be limited by students' needs was constructed by TIMSS. The mean score for Ireland (10.7, SD = 1.93)) was significantly lower than the average for the OECD-16 (10.3, SD = 1.95), indicating fewer perceived limiting needs in Ireland (Table 5.15).

Table 5.15. Mean scores on the Teaching Limited by Students' Needs scale in Ireland and across 16 OECD countries on average, in TIMSS 2015

	Teaching limited by students' needs		
	Mean	SE	SD
Ireland	**10.7**	(0.11)	1.93
OECD 16 (ref)	10.3	(0.03)	1.95

Significantly different mean scores are in bold. OECD 16 = 16 participating OECD countries. See Mullis et al. (2016), Exhibit 9.10.

In Ireland, performance was significantly lower among students of teachers reporting the highest aggregated average level ('very limited') on the scale of teaching limited by students' needs (449 score points), compared with those whose teachers who reported an average level ('somewhat limited') (514), and those who reported a low level ('not limited') (546). It is likely that scores on the index correlate in a positive way with other measures such as student socio-economic status.

5.9. Summary

This chapter looked at the responses of teachers in Ireland to the most recently-available national questionnaire completed by mathematics teachers at Junior Cycle – the questionnaire that was administered to teachers of Second-year students in TIMSS 2015. While the questionnaire responses provide useful internationally-comparable data, they are limited to the extent that they were not designed to specifically examine issues directly related to the implementation of Project Maths. Furthermore, they represent data at one point in time; no similar data prior to the initial implementation Project Maths are available. Where relevant, data from PISA 2012 (when mathematics was a major assessment domain) were referred to, though these were obtained from principal teachers and students rather than from teachers, and they were collected before Project Maths had been fully implemented in all schools.

According to TIMSS 2015, average size in mathematics classes in Ireland is 24.3 students. This is marginally below the average of 25.9 students in OECD countries in TIMSS (OECD-16). In Ireland, class size was significantly and positively associated with performance, indicating that lower-achieving students tend to be in smaller classes and higher-achieving students in larger ones.

Average instructional time for Second-year mathematics classes in Ireland, 192.7 minutes per week, is below the average of 201.4 minutes per week for the OECD-16 in TIMSS 2015. The average annual allocation of time to mathematics in Ireland was 109 hours, while on average across the OECD-16, it was 131 hours. Among OECD countries, just Sweden (99 hours) and Japan (106 hours) had a lower allocation than Ireland. While allocation of time to mathematics in Ireland is comfortably in excess of the average of 80 hours per year (240 hours in total) referenced in the Junior Certificate mathematics syllabus, it may need to be reviewed to bring it into line with other countries, and to address concerns expressed by teachers in the current study (see Chapter 6).

A number of instructional practices consistent with Project Maths (and designed to promote students' conceptual understanding) were experienced 'in every or almost every class' by at least one-half of students in Ireland. These included asking students to explain their answers, linking new content to students' prior knowledge, and encouraging students to express their ideas in class. Surprisingly, activities such as asking students to complete challenging exercises that go beyond instruction, encouraging classroom discussion among students, and asking students to decide on their own problem-solving strategies – all of which might be viewed as consistent with Project Maths – were practiced less often.

Overall, the data on instructional practices gathered in TIMSS 2015 seem to suggest broad similarities in the instructional experiences of students in Ireland and on average across the OECD-16. This might be interpreted as suggesting that the introduction of new instructional practices in Project Maths has, at this time, brought the use of such practices up to the level experienced by students in other OECD countries, and that there is now further scope to implement practices that are compatible with Project Maths.

In PISA 2012, fewer students in Ireland (31%) than on average across OECD countries (42%) 'agreed' or 'strongly agreed' that their teacher asked them to decide on their own procedures for solving problems. This is broadly consistent with the TIMSS finding that relatively few students in Ireland are asked to decide on their own problem solving procedures, compared with the corresponding OECD-16 average.

In TIMSS 2015, teachers also responded to questions about the frequency with which they asked students to implement a number of additional teaching practices, including those relating to presentation of new mathematics content and grouping of students in mathematics classes. Again, activities such as explaining to students how to solve problems, working problems under the guidance of the teacher, and working in same- or mixed-ability groups in mathematics classes are implemented with the same frequency in Ireland and on average across the OECD-16. It is noteworthy that marginally fewer students in Ireland than on average across the OECD-16 were taught by teachers who reported that students memorised rules, procedures and facts in 'every or almost every class' (14.2% vs. 22%). It is perhaps surprising, given the range of abilities that teachers encounter in mathematics classes (see Chapter 6), that relatively few students in Ireland are in classes in which they work with same- or mixed-ability students in groups (8.7% and 15.1% of students respectively).

Data on content coverage in mathematics classes gathered from teachers in TIMSS 2015 indicate that topics in Number that appear on the TIMSS mathematics test have been covered for the most part in Second year or in preceding years, with the exception of irrational numbers. While teachers reported that several topics in Algebra had been covered, one-quarter or more of students were in classes where simultaneous (two-variable) equations, representations of functions as ordered pairs, tables, graphs or equations, and properties of functions (slopes, intercepts) had not been covered. In Geometry, three-in-five students were in classes where translation, reflection and rotation, and relationships between 3-D shapes and their 2-D representations, had not been covered. Care needs to be exercised in interpreting these findings. Teachers in Ireland may have been overly-conservative in interpreting some of the topics, while some may have planned to cover specific topics in Third year, or those topics may not be on the syllabus for the class group being taught. Moreover, with the exception of Geometry, coverage of content in Ireland was similar to coverage on average across the OECD-16. Geometry stands out as a problem area, as TIMSS geometry topics had been covered by 58% of students in Ireland compared with an OECD-16 average of 77%. It is also worth noting that coverage of Algebra topics was considerably higher countries such as Japan (91%) and Korea (94%) than in Ireland (72%).

Ireland compares favourably with the OECD-16 in terms of the frequency with which teachers attended formal professional development in mathematics in the two years prior to TIMSS 2015. However, this is likely to have arisen because of professional development provided in conjunction with the introduction of Project Maths in schools and there is a need for more up-to-date data, as well as a distinction between voluntary and required attendance. Although more students in Ireland than on average across the OECD-16 were taught by teachers who had availed of professional development on integrating information technology into mathematics, data presented in this chapter suggests an under-usage of ICTs in mathematics classes. There is also evidence from TIMSS 2015 that, compared with their OECD-16 counterparts, teachers in Ireland engage less often in activities such as working together with other teachers to try out new ideas, or visiting another classroom to learn more about teaching. Furthermore, between 20% and 25% of students in Ireland are taught by teachers who lack confidence in their ability to engage in such activities as providing challenging tasks for higher-achieving students, developing students' higher-order thinking skills, and showing students a variety of problem-solving strategies.

The proportion of students in Ireland who are allowed to use calculators with unrestricted use is considerably greater than on average across the OECD-16, perhaps reflecting current policy on calculator usage in state examinations. Yet, the relatively-strong performance of students in Ireland on Number in TIMSS 2015 seems to suggest that this may not represent a problem.

Fewer students in Ireland than on average across the OECD-16 have access to individual or shared computing devices during mathematics classes, or access to computing devices that can be used sometimes. Indeed, only in Turkey, Israel and Slovenia among the OECD counties in TIMSS 2015 did fewer students have access to computing devices than in Ireland. Lack of access is reflected in the relatively low proportions of students in Ireland whose teachers report that they use computers to explore mathematics concepts and principles, practice skills and procedures, look up ideas and information, and process and analyse data (about 10% of students in Ireland engage in each activity at least monthly in their mathematics classes). Corresponding OECD averages range between 22% and 26%.

The TIMSS 2015 data are consistent with data based on PISA 2012, which show that one-in-five students in Ireland reported observing their teachers demonstrating various mathematical procedures on computer, while one-in-ten reported engaging in such activities themselves.

The data point to a need to review the role of ICTs in teaching and learning mathematics as the Junior Certificate mathematics syllabus is revised, to identify evidence-based practices that are effective, and to familiarise teachers with these. Such a review might need to take into account any future linkages between ICTs and assessment at Junior Cycle, such as the Classroom-Based Assessments that are being introduced in other subject areas.

Chapter 6. Teachers' Perspectives on Project Maths

6.1. Introduction

Focus Group interviews were conducted with teachers of mathematics at Junior Cycle. The views of 76 teachers (from 75 schools)[27] were elicited through seven Focus Group meetings that took place in various locations nationally in March –April 2017.[28] While the interviews were conducted using a semi-structured interview protocol, in practice, participants expressed their views on a broad range of issues relating to the teaching, learning and assessment of mathematics at Junior Cycle. This chapter presents the findings, drawing on Focus Group transcripts, facilitators' observations and field notes.

The Focus Group interviews were audio recorded and later transcribed. The data were analysed using Nvivo (2015) software. Themes were identified on the basis of patterns in responses across the dataset and on the basis of theoretical importance. The themes are presented below under broad headings. Links between themes are made explicit, and differing views are highlighted as appropriate.

Firstly, findings relating to the curriculum as intended and as implemented are presented under two broad headings: aspects of the curriculum that are working well, and aspects of the curriculum that are deemed problematic. Secondly, teachers' views in relation to the different content strands of Project Maths are presented. This is followed by sections on teachers' views on students' attitudes towards and engagement with mathematics, teachers' use of ICTs to teach mathematics, and teachers' views on issues related to the assessment of mathematics at Junior Cycle.

6.2. The Curriculum as Intended and as Implemented

This section addresses issues around the implementation of the curriculum as intended, focusing on aspects participants felt are working well and aspects deemed problematic.

6.2.1. Aspects of the Curriculum that Are Working Well

Five themes were identified: interconnections between Project Maths strands; the practicality of Project Maths; Statistics & Probability; changes to examinations; and the Common Introductory Course.

6.2.1.1. Interconnections between Project Maths strands

Across Focus Groups, the integration of strands was very much viewed as a positive aspect of Project Maths. It was felt that both students and teachers have a greater appreciation of the connections between the five strands. These interconnections are *'more explicit'* than in the previous curriculum. The curriculum introduced under Project Maths is seen as a whole, and in order to have a full understanding of the curriculum, the links between strands must be appreciated.

The emphasis on patterns in Project Maths was viewed as helping to link the strands and *'tie things together'*; *'it gives students a greater understanding of different areas and links between them',* and facilitates the application of knowledge to different problems. Skills developed in one strand can be transferred when studying other strands such as Linear Patterns in Strand 4 and Co-ordinate Geometry

[27] Prior to sampling, schools were categorised by school type and size, and selected at random within cells. Principal teachers nominated mathematics teachers (one per school) to attend. The sample comprised teachers from 17 community/comprehensive schools (24%), 36 secondary schools (48%) and 22 vocational schools (29%). This equates to the percentages of students in the population who attend these school types.

[28] All Focus Group interviews were held in Education Centres – two in Dublin and Cork, and one each in Athlone, Galway and Monaghan.

of the Line in Strand 2. The links students are establishing between patterns and Algebra in particular were highlighted as being positive.

However, the point was made that students can find other aspects of the curriculum more difficult to link, such as Algebra and Functions. It was further observed that making connections between topics is easier for some students than others, with *'weak students'* (students who have the most difficulty with mathematics) needing more time, as one participant noted:

> *In general I think the emphasis on the connection between topics is a positive thing, although I do think that some weak students find that very difficult. So traditionally, we would have had trigonometry separate completely from coordinate geometry, separate completely from geometry. But now there's kind of a lot of link over, even patterns comes into it. So I think that's positive. Especially for stronger students, because I think it sets them up better for problem solving down the line.*

6.2.1.2. The Practicality of Project Maths

Participants identified the practicality and *'real-world application'* associated with Project Maths as aspects of the curriculum that are working well. Students respond well to active teaching and learning methodologies and to solving problems that are related to real-life situations. The Statistics & Probability strand in particular lends itself to the use of these methodologies.

In one example, a participant described how Trigonometry is *'going better'* as a result of the practical approach. He observed that, *'when you do it, they remember it and understand it'*. The point was also made that students can have difficulty with visualising.

Project Maths is considered *'less abstract'* than the previous curriculum and this is viewed as a positive. A teacher commented: '*I think maths is more meaningful to them now than it was in the old course'.* Another teacher commented:

> *The abstractness that was there in the old course, which was very far removed from their everyday life. Because the question you were always asked continually for 25 years, I was asked – why are we doing this? And what good is it? You actually don't get asked that anymore.*

Linked to this, some participants noted that teachers and students are discussing topics more than before. Some participants made links between the teaching methodologies associated with Project Maths and enhanced understanding of mathematical concepts among students.

Due to the teaching and learning methodologies encouraged by Project Maths, teachers find the curriculum enjoyable to teach and this can create an active, busy, and engaging classroom atmosphere.

Strongly related to the practicality of Project Maths is the issue of time. There was consensus among participants that not enough time is allocated to Mathematics to ensure that the practical aspects of Project Maths are utilised to their full potential. They noted that active teaching learning and methodologies (e.g., discussion and reflection) need more time to implement in comparison to more traditional instructional methods.

With Project Maths, teachers felt that the emphasis had shifted away from rote learning; this was seen as a positive in some respects and a negative in other respects. For example, the view was expressed by some that increased emphasis on mathematical applications in real life situations has meant that

'Pure Maths' skills are not being developed. Some participants argue that *'drill and practice'* still has a place in the classroom.

Problem solving

From the perspective of participants, a positive aspect of Project Maths is that mathematics is being used as a problem solving tool. It was recognised that the curriculum encourages students to develop their problem solving skills. For example, a teacher commented:

> *It is a bit of fun, I mean some of the questions you can say they are like a puzzle that they can figure out. Whereas before it would be just you know no thinking involved just learning how to do it and that's that.*

Another teacher added:

> *I think what's great sometimes is when you get questions up, if you give a test and there might be three or four different ways that they've tried, even if they didn't work it out. But they have way more skills I suppose in that way of trying different methods to work something out.*

Another teacher replied: '*Yeah problem solving, which is great I think. Because before they had the one way and that would be it. So, they can approach a question from several different aspects, which I think is good'.*

Participants also deemed it a positive that Project Maths facilitates the application of knowledge across different problems. The view was expressed by some participants that *'weaker students'* (students who have the most difficulty with mathematics) are now more likely to tackle unfamiliar problems. Some participants also believe that students' problem solving skills have improved as a result of Project Maths.

Some participants felt that students need to learn procedures before they can apply them. Across Focus Groups, participants expressed the opinion that students were coming from primary school without the basic mathematical skills needed to engage with the Junior Cycle curriculum. These participants felt that students coming from primary school are weak numerically and a lot of time is spent in First year developing basic numeracy skills at the expense of other areas such as Algebra.

The issue of literacy was frequently raised in relation to students' problem solving abilities. There was a strong feeling among participants that literacy levels required for some problems were disadvantaging some students. An example was given about questions that require a written response. This activity was viewed as requiring a *'high level of understanding'*, which takes time to achieve. It was also noted that some students do better with more *'wordy'* questions.

Key issues identified in relation to practicality and problem solving are instructional time and literacy. These are discussed further in Section 6.2.2.

6.2.1.3. Statistics & Probability

The increased emphasis on Statistics & Probability and the increased content in the Statistics & Probability strand were viewed as positives by participants. Statistics & Probability was described as the strand to which students respond most favourably and with which students are most engaged. Participants felt students respond well to this strand because, compared to the other strands, it is *'more relatable to everyday life'*. Participants noted that Statistics & Probability has more relevance to students' future lives and work than some of the other strands, and that it sets students up for Leaving

Certificate mathematics. There was a feeling among some participants, however, that the increased emphasis on Statistics & Probability is at the expense of other areas of the curriculum. Also, a participant raised the issue that students still have difficulty with the higher-order aspects of this strand.[29]

6.2.1.4. Changes to Examinations

Aspects of Junior Cycle examinations that changed alongside Project Maths were identified as a positive. Specifically, Junior Certificate examination questions that are not specific to one strand were identified as beneficial to students.

It was also identified that the questions are *'less procedural'* than before; now the student must *'read it, analyse it, and decide what to do'*. While this was identified as a positive, it was also cited as an area where some students who are studying Higher level mathematics (i.e., those who perform at less well at this level) are '*struggling'*.

6.2.1.5. The Common Introductory Course

There were differing opinions with regard to the CIC and both positive aspects (this section) and negative aspects (see Section 6.2.2.7) were identified.

In terms of what is working well, some participants highlighted that with the CIC, it is possible to start at the basics with all students in First year. It is also considered a positive that the CIC allows participants to *'touch on all topics'* or *'sample'* the whole Junior Cycle curriculum in one year. Further to this, it was noted there is scope within the CIC to add in additional topics as required in First year.

Another positive aspect identified is that the CIC enables teachers to see where individual students' strengths lie. This means that those who need extra support can be easily identified.

6.2.2. Aspects of the Curriculum that are Considered Problematic

Seven main themes were identified in relation to aspects of the implementation of the curriculum considered problematic by participants: instructional time; literacy and Junior Cycle mathematics; the curriculum and the spectrum of students' abilities; the increased percentage of students taking Higher-level mathematics; students' preparedness for Leaving Certificate mathematics; standards at Junior Certificate; and the Common Introductory Course.

6.2.2.1. Instructional Time

A recurring theme across Focus Groups was that to implement Project Maths *'in the spirit'* that is envisaged requires more instructional time than is currently available for mathematics. A number of teachers indicated that their comments on this issue are based on their ability to take an overall view of the curriculum, now that all the elements are more firmly in place.

Participants stressed that active teaching and learning methodologies require more time than do more traditional methods. The lack of instructional time means that there is not enough opportunity for students to do practical *'hands-on'* work and to *'practice'*, activities which are deemed *'necessary for Project Maths to work'*. Participants reported teaching some topics (particularly those within Statistics & Probability) *'in the spirit'* of Project Maths and others (especially those within Algebra) using more traditional methods. As a participant stated: *'You either get the course completely done or you do it in the spirit of Project Maths, in which case you run out of time'.*

[29] Additional discussion on the five strands, including Statistics & Probability, can be found in Section 6.3.

According to a majority of participants, increased instructional time for Mathematics would enable the potential of the practical and problem solving aspects of the curriculum to be maximised. It takes time to develop skills such as spatial awareness and it takes time to develop a deep understanding of mathematics. One teacher stated: *'We're not getting time to go into it the way we'd like to go into it in any of the strands really'.*

Additionally, teachers reported trying to teach procedures and problem solving simultaneously, but that the lack of instructional time means that students are *'not getting it'* (i.e., not grasping the material), as teachers must move on to finish the course. The lack of instructional time means there is insufficient time for revision. Across Focus Groups, it was noted that students can have difficulty with retention of material and that classroom revision is a necessary part of the learning process. A participant commented:

> *The syllabus I think was designed for a three-year course without taking into account that a lot of students in my class … are great while we are doing them [topics]. You give them a test on statistics 100%, 96% whatever, I can guarantee you give them the same question in the summer without going back over it and that 100% will drop to 50, 40, 30, 20% … Time is the thing.*

Another participant commented:

> *We have 5 periods a week. If we had 6 periods a week then by the time you come around to say this time of the year you'd have everything done and then you'd have time to revise and then that would sink in. It would sink in through revision, but we never quite get to that compared to a few years ago where you would have the time to revise and the students would, you know it would come together for them over those few months. Whereas now you are still you know doing new stuff as you are coming towards the end of the year.*

Participants feel under pressure to finish the Junior Cycle course on time. As well as the methodologies involved in Project Maths, participants cited the volume of content in the curriculum as an issue in this regard. However, the central issue expressed by participants is the tension between the volume of the content and the time available to cover it, rather than the volume itself. In general participants were happy with the curriculum content, with no major additions or subtractions to same suggested (see below). This was put by one participant as follows:

> *We're not getting time. It's not even that we're leaving out stuff or putting in stuff, it's that we're not getting enough time to spend at the stuff.*

Participants expressed feeling frustrated and stressed because of time constraints:

> *It's always the sense that you are not doing enough. You are never quite getting it all done.*

> *Well if I didn't have to cover everything I would do things much slower and I think I would teach them better … But I'm just under so much pressure.*

Teachers teaching Ordinary- Higher-level mathematics reported experiencing time constraints in implementing the curriculum. Some participants who are teaching Higher level feel the content of the curriculum is especially difficult to cover within the current instructional time available for mathematics and that it is difficult to develop problem solving skills among students. Consequently, some participants are providing extra classes during lunch breaks in order to get the course covered. Teachers of Ordinary level mathematics reported needing additional time to help students to grasp mathematical concepts.

For some participants, the tension between curriculum content volume and instructional time is compounded by the lack of choice on Junior Certificate examination papers. Large classes also compound the difficulties with time.

A further issue of concern among several participants is the possibility of a reduction in instructional time for mathematics in the context of the introduction of one-hour classes in some schools (with fewer class periods allocated to mathematics), and the addition of new curricular areas to the timetable (e.g., Wellbeing).

Some specified the amount of additional time needed as two-to-three classes per week, due to the challenge in getting mathematical concepts to take hold properly.

6.2.2.2. Literacy and Junior Cycle Mathematics

A recurrent theme across Focus Groups was the issue of literacy in relation to Junior Cycle mathematics. As stated earlier, concern was expressed about the literacy levels required of students to engage in Project Maths. In particular, participants mentioned the impact of literacy on problem solving ability, where students are required to present written explanations.

Participants felt the level of mathematical literacy required by students to access the examination paper in the State examinations has dramatically increased compared to the 2000 syllabus. Long wordy questions can present a problem for students at all levels of ability. With *'big word problems'* some students can *'fixate on the text and get a mental block'* and as a result can *'give up'*. It was recognised, however, that some students do well at the more *'wordy questions'* (associated with Statistics & Probability) but less well at the more abstract ones (associated with Algebra), and vice versa. EAL students and students with specific learning difficulties (e.g., dyslexia) were reported to be particularly disadvantaged by the language used in examination papers. However, it was noted that the students who perform at the highest levels in mathematics also encounter difficulties.

The issue was also raised that students, and particularly EAL students, can have a difficulty with topic recognition on examination papers.

Cross-curricular linkages, which are promoted by Project Maths, are considered a positive in some regards (e.g., fostering complementary skills). However, the issue was raised that there is a lack of consistency in terminology used across subjects and this can be challenging for students. There was a feeling among some participants that this might be affecting proficiency in problem solving.

A majority of teachers did not see much evidence in their classrooms that the National Literacy and Numeracy Strategy was assisting them in implementing the curriculum introduced under Project Maths.

6.2.2.3. The Curriculum and the Spectrum of Students' Abilities

Across Focus Groups, the theme arose that Project Maths at Junior Cycle is not adequately catering for the spectrum of students' abilities, but rather is trying *'to be everything to everyone'.* From this perspective, the curriculum is *'too broad'*, and as a result, is having a negative impact on standards. As one participant described:

> *It's trying to cast a really broad net so you are catering to the people who are going to do maths, science in college and you are catering for the people who need skills for maths for life. You are trying to gather all these people in … to teach them to the appropriate standard, and … it's, it's too broad.*

In particular, the view was put forward that Project Maths is not adequately addressing the needs of students who are studying Foundation level mathematics (students who have the greatest difficulty with mathematics) (*'I don't think it's serving the weak student [sic] at all'),* for which there is no specific Project Maths syllabus. As a result, teachers are trying to *'make one up'* and neither teachers nor students know what to include in terms of content. A participant commented: *'I find it hard teaching Foundation level Junior Cert because there's nothing really, there's no syllabus'.*

The experience of teachers working with less-able students was further described as follows:

> *… their [teachers'] problem is the syllabus, they don't know how far to go, there's no books out there, they're completely stuck, they say well how far will I go with the algebra, what will I do? And they're left in a vacuum.*

The difficulty with having no designated syllabus is seen as being compounded by the lack of specific text books and by the lack of predictability in the Junior Certificate examination. For students with specific learning difficulties and students with autistic spectrum disorders, the lack of structure on the Foundation level examination paper can create difficulties. A teacher from a DEIS school described her experience:

> *Some of them [students in Learning Support who were taking the Foundation level examination], some of them went in and just didn't comprehend the paper at all, between the language and because I had no structure to go by of how the paper might work. I mean if you have a child with autism taking any exam paper at all, there has to be, you have to be able to, you know they work on structure a lot and you have to be able to say this is how it's going to be structured and this is how it's going to look, I had none of that at my disposal. Because I didn't know how the paper was going to look or what questions were going to come up …The classes were fine but the exam at the end of the day was near impossible for some of them.*

Across Focus Groups, the view was expressed that there is still a place for Foundation level at Junior Certificate and that what is needed is a specific syllabus to cater for students who study mathematics at this level. The reasoning was that if Foundation level is discontinued, there will be a group of students who are denied the opportunity to achieve in mathematics: *'If the Foundation goes you'll have that small group of students who will never achieve, never succeed, never, you know?'* This perspective was put forward both by teachers from DEIS schools and teachers from non-DEIS schools.

A further issue identified by participants is that the Junior Cycle curriculum at present is not adequately challenging the students who perform at the highest levels in mathematics and hence is '*not allowing the better students to reach their full potential'*. Across Focus Groups, teachers attributed this problem to mixed-ability classes, which were referred to as *'the elephant in the room*'. From the perspective of teachers, the mixing of students with differing mathematical abilities is detrimental to the progress of students who perform at the highest levels, as it slows down the pace and reduces the time available for important activities such as reflection. Participants explained:

> *But I think that's where you're losing out for your kids taking Higher level, you don't give them the same challenges because its diluted immediately, you go to the middle ground more than likely when you have a mixed-ability class. So, they're not being challenged enough, whereas if you had all of them together you could go at a much faster pace. And bring them up to that standard, you can reflect … you can't reflect, you haven't time to reflect, you haven't time to keep moving, let alone reflect. And reflection is a huge part of*

mathematics. You know it's so necessary, especially for brighter kids, it's so, so necessary. And, therefore I just, I have a think about the mixed-ability, I just can't see the purpose of it.

I would agree with that and in terms of political correctness it's not acceptable at the moment, but mixed-ability classes are preventing the Higher level students from reaching their full potential.

In my class with the best classroom management skills your weaker students absorb more of your time, attention and work.

In First year, mixing occurs because of the Common Introductory Course, for which students are not streamed. For Second and Thirds years, teachers described an increase in mixed-ability classes, which they attributed to the increased percentage of students taking Higher level mathematics for the Junior Certificate (see Section 6.6.6.4).

Across Focus Groups, the possibility of a *'two-tiered'* approach was suggested as an alternative to mixed-ability classes.

And if you can get them with functioning skills leaving school or going into senior cycle that's great. If we can get percentages and fractions and you know basic sums as a tool kit going into senior cycle. For another cohort of students, you need them to be able to problem solve and they should be challenged and so on. I wonder if there's room for two different courses to cater for the two levels of needs. I would hate to see that the better-able students we'll say would have problem solving or some such removed from them because others can't do it. I wonder is there some differentiation that could happen in terms of the courses being presented to the students. I would like to see everybody being able to come into senior cycle with a skill set.

A *'two-tiered'* approach was further suggested to cater for the spectrum of students' abilities. For example:

For me personally, I think there should be a two-tiered maths. Maths for life, and there should be maths for people who are interested or who need maths for subsequent study, and they should have some kind of introduction course where they do a base and then they should split.

I know this will never happen but I think there's an argument to be made for having two different maths curricula … Where you have let's say a functional LCA [Leaving Certificate Applied] sort of maths for everyday use, every day, versus let's say abstract maths for those who are maybe going to go on and study mathematics subjects … like nearly applied maths sort of just pure maths for the sake of maths as opposed to, doesn't necessarily always have to be functional and useful.

A further solution offered is that a *'Pure Maths short course'* be developed at Junior Cycle level, with a view to developing and challenging students with the greatest abilities in mathematics.

6.2.2.4. Increased Percentage of Students Taking Higher Level Mathematics

There was much discussion across Focus Groups around the increase in the percentage of students taking Higher level mathematics at Junior Cycle and at Senior Cycle and the impact this has had on the teaching and learning of mathematics at Junior Cycle. The increase has been seen both in DEIS schools and in non-DEIS schools. Teachers described the increase as being due to the bonus points and grading system for Leaving Certificate Higher level, with a knock-on effect at Junior Cycle. As a result, Higher level classes are now more mixed-ability than before, and this is seen as presenting difficulties.

One impact identified by participants is that it *'slows down the pace'*; participants are spending time teaching students with moderate abilities and students with the most difficulty in mathematics at Junior Cycle, while students who achieve at a higher level are being neglected and/or not challenged, and as a result, do not have some of the skills needed for Leaving Certificate Higher level topics such as Algebra.

A further impact reported by teachers is that students with moderate ability in mathematics are going into Higher level mathematics at Leaving Certificate without the basic skills required. As a result, the transition is more problematic for them (see below). The issue centres on students *'doing Higher level maths at Leaving Cert who shouldn't be doing it'*, and is considered *'widespread'* and *'driven by points'*. The point was made that strong Ordinary level Leaving Certificate classes almost do not exist anymore, that Ordinary level Leaving Certificate classes tend to perform at a lower level than before, because students with moderate ability are now doing Higher level, even though they do not have the basic skills: *'They are having a shot of the Higher whether they are able or not'.*

Overall, there appeared to be support for the idea of rewarding students for the extra work required for Higher-level Leaving Certificate mathematics. A solution proposed was that bonus points for Higher level mathematics at Leaving Certificate be awarded using a sliding scale. Participants were supportive of this suggestion, though one teacher commented *'I don't want to get rid of the 25 points either because when you didn't have them, you lost some of your good students'*.

Participants also noted that the 30% 'pass mark' at Higher level Leaving Certificate (the new H7) will entice students who might otherwise drop down to Ordinary level to continue at Higher level. A participant commented:

> *It's going to get worse because of the 25 points, but now you've a thirty percent pass mark as well, so a score of 30% will now get a pass at Higher level Leaving Cert. So you are pushing more people in there because they may not have got the 40, before but they'll definitely chance a 30.*

Importantly, however, it was also noted that achieving a D in Higher level mathematics at Junior Certificate was better preparation for Leaving Certificate Ordinary level than achieving an A at Junior Certificate Ordinary level.

A number of teachers felt that having common learning outcomes on the Higher and Ordinary level courses meant that transition between levels was now easier for students in the Junior Cycle.

6.2.2.5. Students Preparedness for Leaving Certificate Mathematics / Transition to Leaving Certificate

Related to greater participation in Higher level mathematics is the view that many students are not well prepared for Leaving Certificate mathematics following Junior Certificate mathematics. This issue was identified for students moving from Higher level at Junior Certificate to Higher level at Leaving Certificate, but was highlighted as particularly problematic for students transitioning from Ordinary level at Junior Certificate to Ordinary Level at Leaving Certificate. Participants referred to the *'gap'* and the *'jump'* for students at Ordinary level and noted '*they nearly need to be doing Junior Cert Higher level to have a good chance to do Ordinary Leaving Cert'.* The need to better bridge the gap was highlighted as a key issue. There was a view among participants across Focus Groups that the transition to Leaving Certificate for students at Ordinary level was easier with the old curriculum.

Participants felt that the Junior Cycle curriculum does not prepare students for the higher-order thinking and mathematical literacy that is required at Leaving Certificate Higher and Ordinary levels. As a result, participants reported using Transition year to *'bridge the gap'* between the Junior Certificate and the Leaving Certificate. As such, many teachers of Transition year reported going over basics in Algebra and Geometry and trying to get *'a head start'* on the Leaving Certificate syllabus. Some reported developing Transition year modules using the handbooks produced by the Project Maths Development Team.

A number of teachers argued that the Leaving Certificate Ordinary level syllabus was too difficult for some students, especially those coming in with a D in Ordinary level in the Junior Certificate, who were deemed to be unable to deal well with simultaneous, linear or quadratic equations.

A number of teachers referred to an under-emphasis on Geometry in Junior Cycle, and how that impacted in a negative way on students' understanding of Geometry in Senior Cycle. One referred specifically to Trigonometry when she stated that *'they don't know how to manipulate the formulas for trigonometry; if they haven't got the basics of balancing an equation, they can't do half the course, basically'*.

One teacher referred to the *'strong focus'* on Statistics & Probability at Leaving Certificate level, and argued that such a focus was fine if the goal was to enter a course where such knowledge was deemed relevant. However, she argued that, nowadays, universities had to organise courses for Frist year students on topics like matrices, calculus and integration. The point being made was that one should look first at Leaving Certificate mathematics, and what is needed there, and then look back at the Junior Certificate syllabus through a backward-mapping process.

A small number of teachers linked what they perceived as a high *'failure'* rate at Leaving Certificate Ordinary level to a weak link between Junior and Leaving Certificate syllabi, with a significant increase in difficulty and expectations at Leaving Certificate level.

One proposed solution was a link back from the Leaving Certificate to the Junior Certificate syllabus to ensure that the two are harmonised. Others suggested that the Leaving Certificate Ordinary level syllabus be reviewed with a view to ensuring that it better reflects what students carry over from Junior Cycle.

6.2.2.6. Standards at Junior Certificate

An issue identified by participants was that while greater percentages of students are sitting Higher level mathematics for the Junior Certificate, it does not necessarily follow that mathematical abilities have improved. Rather, it was proposed across Focus Groups that both the examinations and marking schemes may have contributed to falling standards, and that the grades which students achieve in their Junior Certificate are not always a reflection of their mathematical ability. Nonetheless, increased 'pass' rates at Junior Certificate were considered positive in some respects.

Specific to marking schemes, it was noted on several occasions that the schemes often allowed students who perform at the lowest levels in mathematics to achieve a pass grade, with one teacher observing that students will do as little as they need to get by. Another commented on the unfairness of a situation in which *'a Higher level A student forgets to add cm after an answer and loses a point, whereas a B or C student can gain 3 points for simply writing a formula down'*. Hence, relative distance between scores was viewed as being too close.

Several teachers (including teachers who marked Junior Certificate maths papers) noted that it was possible for students to do moderately well on the Junior Certificate exam by doing well on Statistics

& Probability and doing poorly on Algebra. This, in turn, was perceived to give rise to problems when students worked on Algebra at Leaving Certificate level.

Several teachers also noted that, in a context where upwards of 80% of students in a cohort (in their schools) are now taking Higher level mathematics, it is not possible to attend properly to the needs of students who perform at the highest levels in mathematics. In this context, one teacher said of these students, '*they've got special educational needs as well which are not being met'.* Another commented: *'And I think if they are not challenged enough unfortunately maybe their standard falls'*.

Teachers also expressed awareness of what they perceived as the purpose of Project Maths. As one noted:

> *And I also kind of feel that the whole maths curriculum has shifted away from preparing students to study mathematics to catering for the wider community and the wider world and the wider business and I don't know necessarily whether that's a good thing or a bad thing. I don't know where that is going or whether there's scope like in other countries for something like having pure maths as a subject by itself.*

Another expressed concern at the strong focus on statistics, and linked this to the standards question:

> *I think it depends in those work areas, there are areas of work that are very statistic heavy but I don't think it's a reason for to shift the whole emphasis that way and I would feel, personally I feel standards will continue to slip. Like that academic rigour is gone. And it's also worrying for generations to come that if that's gone from our entire population's learning experience we're not going to, you know the pure mathematicians aren't going to be able to pick that up.*

A few teachers also claimed that questions in a national mathematics competition in which their schools participated had also been *'dumbed down'* in recent years.

6.2.2.7. The Common Introductory Course (CIC)

As stated above, there were differing opinions among participants in relation to the CIC. Some of the less-positive views expressed include that the CIC is to some extent a *'waste of time'* and is *'robbing a year from Junior Cycle'*; that the CIC limits the time and depth with which topics can be addressed (e.g., the emphasis that can be placed on Algebra); and that students can do well in CIC and take up Higher level mathematics in Second year even though they may not have the ability. One teacher claimed that *'the CIC is equivalent to the Fifth and Sixth class curricula at primary level'* and hence is of questionable value. However, across Focus Groups, participants expressed the opinion that students are often coming from primary school without the basic numeracy skills necessary to engage in Junior Cycle mathematics, and hence the CIC is needed.

It was felt that the CIC is working well with First-year students, but that it may not be challenging students with the greatest abilities in mathematics who may become bored and become *'lost'* (i.e., they disengage) in First year as a result. One teacher expressed this view as follows:

> *They're saying I'm bored, maths is boring and they actually almost fall out of love with maths I think in First year. And I've seen that very close to myself and I just think what happened. You know they shouldn't be like that, kids that are really good and like maths shouldn't be saying that about maths in First year.*

According to participants, differentiation of instruction is required in class to keep these students engaged, but that this is often *'is not practical',* because of time constraints.

A related issue identified by participants is that students who achieve at the highest level on the CIC may have difficulty transitioning to Second year following the CIC, because of a *'huge gap'* between the CIC and the Higher level course: *'I think the higher achievers when they go to Second year get a shock and the parents come in and want to know why are they only getting 80 now instead of 100 because they're not doing sums anymore'.* This issue relates to the lack of challenge for these students in First year: '*I think there's a huge gap between Second year Higher level and the Common Introductory Course, especially when classes are not streamed at First year, which they're not in most schools now'.*

Overall, the issue among participants is that students with differing abilities in mathematics are in the same First year class (as there is no streaming in First year) and classes are often very large. Hence, the CIC may not be challenging students with greater ability who, as a result, can sometimes disengage in First year. While more differentiation of instruction in class is necessary to keep these students engaged, participants noted that lack of instructional time and large class sizes act as barriers against this.

Teachers also referred to variation in what is covered in the CIC from class to class within schools, and that such variation could lead to *'unfairness'* when decisions are made regarding access to Higher level mathematics in Second year.

A few teachers noted that, although teachers have control over the order in which topics are presented in the CIC, students may well have forgotten what they have learned by Second year. One cited co-ordinate geometry as an example of a topic that they postpone until Second year for this reason.

A number of teachers suggested that the CIC should emphasise fractions, integers and Algebra to a greater extent. It was also suggested that there may be a place for assessing students' basic numeracy skills without the aid of calculators.

6.3. Teachers' Views on Project Maths Content Strands

6.3.1. Statistics & Probability

A number of teachers expressed the view that there is a strong focus on Statistics & Probability at Junior Cycle level, and that this enables less-able students to achieve a pass at Higher level mathematics, as they can engage with the Statistics questions, and do not have to develop extensive knowledge in other strands (Algebra in particular). Across Focus Groups, teachers were divided in terms of, on the one hand, endorsing the current content across strands, subject to the provision of additional instructional time, and, on the other, reducing the emphasis on Statistics & Probability, and increasing the focus on Algebra and/or Trigonometry. Sometimes, this was prefaced with a perceived need to provide more *'drill and practice'*.

Those endorsing no change in existing content in Statistics & Probability at Junior Cycle focused on the importance of Statistics across a range of areas after students had left school, with some recognising that the relatively strong emphasis in this area may be at the expense of topics in Algebra, Co-ordinate Geometry and Trigonometry. They also noted that the heavy emphasis on Statistics in the CIC may be at the expense of work in Number, and area in which some students struggled.

Among those proposing to strengthen the role of Algebra and Geometry, possibly at the expense of Statistics & Probability, were some who proposed a separate course at Leaving Certificate level for students who wanted to specialise in mathematics, where Statistics, in particular, would not be emphasised so heavily. Some also proposed introducing additional modules (half-year courses) at

Junior cycle level that would provide students interested in taking courses that specialise in mathematics at a later stage with additional learning opportunities.

6.3.2. Algebra

Participants identified a number of positive aspects around teaching and learning Algebra at Junior Cycle including a good emphasis on patterns as an introduction to Algebra, good links with Co-ordinate Geometry, some evidence that Algebra was transferring to problem solving, and a view that an increased focus on visualisation in applied measures could contribute to improved understanding in Algebra. Most participants were largely in agreement that Algebra is a critically-important element of the Junior Certificate mathematics syllabus (though a few questioned its value for students likely to sit the Junior Certificate exam at Foundation level). A small number believed that students' understanding of Algebra had improved under Project Maths.

Participants were almost unanimous in their view that lack of instructional time in mathematics classes hindered efforts to teach Algebra, referring to, for example, *'lack of time and space to explore Algebra at Junior Cycle'*. As noted earlier, there was a strong perception among participants that students compensated for weak understanding of Algebra by doing well on Statistics & Probability on the Junior Certificate exam, with one commenting that *'one can obtain a moderate mark on Junior Cert Higher maths without being adept at Algebra'*. This, in turn, was viewed as impacting in a negative way on the preparedness of students to engage with Algebra at Leaving Certificate level, especially at Ordinary level.

Several teachers noted that students from some primary schools lacked key prerequisite skills for Algebra, and that this held them back, with students' lack of understanding of topics such as fractions and negative numbers often raised. This, in the view of a number of participants, necessitated a stronger emphasis on basic skills in First year, with insufficient time remaining to introduce important aspects of Algebra.

In contrast to participants who viewed understanding of Algebra as improving under Project Maths, a small number tended to view Algebra as a stand-alone aspect of mathematics that was difficult to teach and was particularly challenging for students. One commented that *'it's very hard to get excited about it'*. A small number of participants pointed to an over-emphasis on Algebra in context, and argued that this could result in stronger students not acquiring the skills they need to enjoy it. Others noted that Algebra required considerable *'drill and practice'* if students were to become proficient at it, but that the time for this simply was not available.

Teachers were generally in agreement that the content of Algebra at Junior Certificate Level was appropriate and should not be changed in significant ways. A majority of teachers felt that the first four rules for indices (as per Section 3.2 in the syllabus), was sufficient for Junior Certificate students.

6.3.3. Geometry & Trigonometry

Teachers identified several positive aspects of the Geometry syllabus. These included an awareness that geometry lends itself to real-life applications or practical questions, though only after students have been *'drilled'* in a particular topic first (e.g., co-ordinate geometry). Teachers were generally in agreement that the teaching of Geometry had improved, and that it was linked in effective ways with other content areas. A number of teachers noted the links between Geometry and Trigonometry, and expressed the view that time invested in Geometry at Junior Cycle yielded rich dividends at Leaving Certificate level. Others referred to links between Co-ordinate Geometry and Algebra, including enhanced understanding of Functions, though lack of time was seen as militating against students' efforts to acquire the relevant skills to capitalise on such insights. Still others referred to links between other subjects and Geometry – for example, art and technical graphics – though a few were critical of the differences in terminology for the same concepts across subject areas, and differences in the units of measurement used in technical graphics, woodwork and mathematics

Many teachers referred to the time required to teach Geometry in depth and contrasted this with a situation in which students were viewed as over-relying on their textbooks. It was argued that access to the book of Tables and Formulae provided with too much scaffolding in examinations. The book of Tables and Formulae was also seen as contributing to too much emphasis on procedural learning as students simply plugged numbers into the formulae. In Trigonometry, shallow learning was viewed as contributing to over-use of the sine rule. Several teachers called for more time to allow students to explore and engage with the content of Geometry and Trigonometry. They also called for a clearer focus on problem solving in Geometry, which, they felt, was weak in the current syllabus and exams. A minority of teachers felt that the scope of Geometry in the syllabus was not deep enough for students at Higher level, and many teachers identified Trigonometry as an area that was not emphasised enough.

There was criticism by some teachers of the large volume of synthetic geometry on the syllabus, including a relatively large number of constructions. This was viewed by one teacher as constraining students' problem solving in Geometry, as new ideas do not fit into the current synthetic framework. Moreover, it was noted that, although fewer theorems appeared on the course, students were still required to engage with the same number of underlying concepts.

A majority of teachers felt the five formal proofs of theorems (4, 6, 9, 14, and 19) still on the course could be removed and replaced with a greater emphasis on problem solving constructions, diagrams and applications of the theorems. Teachers felt the appearance of these formal proofs in examinations did not examine the problem-solving skills they were trying to develop in Junior Cycle students.

Teachers generally agreed that students lacked spatial awareness. They attributed this, in part at least, to an over-reliance on 2-D diagrams in textbooks and examination contexts. They suggested a number of informal strategies that could be used to enhance students' spatial awareness, including use of construction games like Minecraft, where students had opportunities to move back and forth between 2-D and 3-D shapes. It was noted that many students did not have opportunities to engage with construction activities in Geometry.

Most teachers were in broad agreement that the current course in Geometry could not be shortened any further, as this would have a negative impact on students' ability to cope with the Leaving Certificate course. Several teachers proposed extending Trigonometry, with specific reference being made to the inclusion of non-right angled triangles.

6.3.4. Number

A common theme running through the Focus Groups was that of students coming into First year from primary school lacking fundamental number skills. In particular, concerns were expressed about proficiency in fractions and percentages. Students' difficulties with fractions meant that they would also have difficulty with algebraic fractions. Related to this, it was pointed out that students often struggled with rational numbers, and that this detracted from time that could be allocated to other important topics, especially in the CIC. A few teachers noted that Number did not feature strongly in the Junior Certificate exam, and that, because of this, it tended to be bypassed in favour of Statistics. One questioned how students taking the Junior Certificate examination at Foundation level could possibly understand the concept of a variable in Algebra if they struggled with negative numbers.

Teachers fled that the first four rules for indices (Number, 3.2) were sufficient for students at Junior Cycle level.

Participants referred to a Maths Competency Test administered in First year, on which students often performed poorly on items dealing with fractions. Participants also referred to students' difficulties in conceptualising irrational numbers, and regretted that sufficient time was not available for this, and that such numbers did not feature sufficiently strongly on exam papers.

A few participants referred to challenges in the Applied Measure aspect of Number and how this might focus more strongly on developing students' visualisation skills and mathematical modelling abilities. A small number suggested that this element of the syllabus might be developed further as a short course for higher-achieving students.

Focusing on aspects of Number early in the CIC was reported to have a positive impact on subjects such as science and business studies.

6.3.5. Functions

In general, issues about the Functions strand were raised less frequently than for other strands. A majority of teachers were happy with the content in this Strand, with teachers in general feeling that this Strand connects with and reinforces the concepts covered in Strand 4 Algebra.

6.4. Student Attitudes towards and Engagement with Mathematics

Across Focus Groups, there was some consensus among teachers that, overall, students are more interested in and engaged with mathematics under Project Maths than they were prior to its implementation: *'I think there's more students are engaged in the subject than ever before, they're more interested than they were when I was teaching it 10 years ago, absolutely'.* An increase in student engagement is particularly noticeable with certain topics and especially with Statistics & Probability. Teachers attributed increased student engagement to the increased emphasis on solving problems and making connections with everyday life. Teachers also attributed an increase in student engagement to the teaching methodologies associated with Project Maths, which they viewed as being more interactive, thereby generating more discussion around topics in class. Teachers enjoy teaching Project Maths and are, therefore, more enthusiastic about the subject: *'I enjoy teaching it a lot more, and obviously if I enjoy teaching it I'm going to be more enthusiastic and that comes across in the classroom'.* This, in turn, creates a more pleasant classroom atmosphere: *'the atmosphere is definitely nicer in the classroom with the Project Maths'.*

There was much agreement among teachers that students tend to like mathematics in First year. This was attributed to students being insufficiently challenged by the CIC: '*the Common Introductory Course is seen as easy, they love maths in First year'.* However, there was a feeling among participants that this can lead to unrealistic expectations among students: *'I would say generally we've probably a more positive attitude towards maths, certainly in First year, I think they all maybe feel that they can, whether that's realistic or not, they can have a go'.* Additionally, and as noted earlier, teachers felt that students performing at the highest levels, in particular, are not being adequately challenged by the CIC in First year, and as a result are at risk of disengaging from mathematics.

A recurrent theme across Focus Groups was that many Junior Cycle students have unrealistic expectations about what they can achieve in mathematics. Parents were considered as extremely influential in this regard: *'like everyone wants their child to do Higher level maths for Junior Cert … whether or not they're able'.* Reference was also made to the *'stigma'* associated with dropping from Higher level to Ordinary level at Junior Certificate. As discussed previously, teachers noted that more students who find mathematics difficult are taking Higher level mathematics than before. One consequence of the increased numbers of students taking Higher level mathematics is that students who have the most difficulty with mathematics may develop a negative attitude towards mathematics in response to failure. A teacher commented:

> *I have at least 15 kids [in Higher level classes] who should be doing Ordinary level and I'm pushing them, they're failing every test. They're not able … they don't have it, I can't make them have it … they're starting to hate maths and they're failing and you know they just have a really bad attitude now … But they'll refuse to drop to Ordinary level.*

On the other hand, reference was also made to students' persistence with problem solving associated in Project Maths, with some teachers noting that, in their experience, students are more likely to *'stick with it'* (i.e., persist) than before:

> *they'll throw something down on the paper as opposed to abandoning it, which they used to, they'd read a question before and I don't know anything about that and off to the next one. I think we're teaching them that most problems can be solved eventually. So they're writing something down'.*

One teacher commented that the emphasis on trial and error in problem solving in Project Maths is associated with increased mathematics confidence among students in his school. However, others noted that students can get easily *'frustrated'* owing to the wordiness and wording of problems: '*they just get so frustrated that they leave a blank space and move on … they don't have that ability to decipher sometimes what's being asked in a question'*. It was noted that students who have the most difficulty with mathematics may not grasp what is required within a problem and hence may not persist with it.

Female students were described by participants as more likely than male students to *'get stuck'* and to *'give up'* on mathematics problems, and as less likely to try out new ways of solving problems.

Female students were also described as less confident and more anxious about mathematics than male students. This was even observed among female students who achieve at the highest levels in mathematics: *'I'm teaching in an all-girls school and girls there who were brilliant at maths and they are saying 'I'm not sure about it, I don't know''.* Another teacher noted that, *'even among female students who love mathematics, many still lack confidence and find it stressful'*. This was attributed to some extent to what was described as the more perfectionistic nature of female students as compared to male students, and greater concern among female students about making mistakes. It was noted that male students, on the other hand, could claim to be confident, while not having the ability to match their confidence: *'you've boys and they could be very weak and they are telling you they are grand … [and] they haven't a clue'.* Also, compared to male students, females students were described as being more concerned with grades in mathematics, and as having *'massive expectations and pressure'.* One teacher commented: *'I often feel like the lads if they kind of got 60 they'd be ah that's fantastic and if the girls get it they're like I've put in this amount of work and that's the grade that I'm getting'.*

There was a sense among some teachers' comments that curriculum under Project Maths is to some extent more oriented toward male students than female students. Firstly, it was pointed out that, compared to male students, female students are more inclined toward rote learning. Secondly, it was suggested that the content of examples and questions in mathematics are more oriented to male students than female students:

I'd say it [the curriculum] probably favours boys more than girls. I think certain areas of the applications are quite technical, probability maybe, I notice one of the pre-questions in the Leaving Cert was about poker you know I think it's probably more likely that the lads would play poker than the girls. Or the cogs system or technology.

6.5. ICTs and Teaching and Learning in Project Maths

Across Focus Groups, teachers reported engaging ICTs to varying extents and there were differing views on the value of ICTs as tools for teaching mathematics at Junior Cycle. Teachers also identified a number of challenges to using ICTs effectively.

Participants reported using ICTs in a number of ways, including classroom demonstrations, checking of homework, taking classroom- and home-administered tests, and as a tool for students to use in problem solving and in checking of accuracy of problems solved by hand. Most commonly, teachers reported using ICTs for classroom demonstrations. Specific applications being used included *GeoGebra, Maths Workout, Folens Build-Up, Kahn Academy, Mathematica* (mobile app), *IXL, Edmodo,* and *Census at School*. Among them, GeoGebra appears to be the most popular and most commonly-used resource.

GeoGebra was described as being useful for demonstrations, especially for generating diagrams which help students to visualise and to understand the meaning behind the numbers. As participants described:

Where you have the constant and the end changes, you can show it really quickly without having to rub out everything again and show it, like it's great for their understanding of the application of what do the actual numbers mean.

It's very easy just to throw it up and let them see, if they do it manually they have less to see.

Teachers also cited examples of students using *GeoGebra* for problem solving in class. A teacher described using it with a Transition year class:

In TY I would always bring them and spend maybe a week doing GeoGebra in getting them all to do it, like solving their Junior Cert geometry questions by hand and on GeoGebra because I think … some of the circle questions, you know when you're getting, if you can visualise, it's just trying to help them visualise, there's nothing as powerful as an accurate diagram. And it's very quick with GeoGebra, just to help them in solving and to see what's going on. I think it is a good tool.

In class, *GeoGebra* is also considered useful for building on and reinforcing regular learning; for example, with coordinate geometry: '*they'll be doing coordinates for the class and they will be plotting the points and the coordinates make up pictures. So they have to try and guess what the picture is before they do the coordinates'.*

In this way, it was noted, learning can be more enjoyable for students. Overall, however, it appears that teachers are using *GeoGebra* (and other software) in class more often than students, though exceptions were evident.

Across Focus Groups, teachers identified of the main challenge to using ICTs in Junior Cycle mathematics as lack of infrastructure. Based on teachers' accounts, there appears to be great disparity in the resources available across schools. Some schools lack computers and a computer room, while in those that have them, there can be issues in gaining access, and issues with hardware, which make their use with large classes very difficult. The use of available resources can be further hampered by shorter class periods (40 minutes versus one hour).

Closely linked to the issues of size and length of classes is the issue of time, which was cited by some as a key challenge to the effective use of ICTs for mathematics. As one teacher described: '*We wouldn't have time to teach them [students] how to use it'.* The issue for some teachers is that they feel students do not have the basic skills necessary to engage with ICTs and as a result, valuable classroom time is spent teaching students how to use ICTs. Even where ICT resources are readily available, some teachers feel that a lack of basic skills among students serves as a barrier to effective engagement with ICTs:

> *Our students have devices, but again its lack of time, you know we don't need to go to the computer lab for them to use GeoGebra, it's on their tablets, but time. And the worst thing then is it turns into an IT lesson. [Agreement]*
>
> *You know it's like they don't have even the basic skills for IT so it turns in ...*
>
> *There's very little maths learned.*
>
> *It's not a maths lesson, it's an IT lesson, so it's, you know.*
>
> *There's an assumption nowadays that kids are coming in and they're so computer literate and they're so good at this and they're so good at that and really all they can do is play games".*

In contrast to this perspective, however, other participants described less problematic use of ICTs in the classroom. For example:

> *I come from a different side altogether here because our students use devices, they each have their own I-pad. So it's just something that came into our school and they would have the GeoGebra, they'd use it all the time you know anytime we're doing that. They have access, you know to any of those Khan Academy, Maths Tutor or Study Place or anything. So ours would be proficient in using ICTs all the time.*

Participants also discussed students' use of ICTs for mathematics learning in the home. One teacher described how it is working with her students: *'set them up on the module inside the class, you can record what they have done, how long they have spent on homework, what the result was if you give them a test. It's all done online'.*

Closely linked to this topic was discussion around which students are more likely to engage with ICTs and benefit from it, both in the class and at home. Again there were differing opinions on this issue. One view put forward was that students with greater mathematics abilities are more likely than students who have difficulty with mathematics to engage with ICTs in the home: *'the good student, you know they'll see you using GeoGebra, they'll go home and they'll use it, the other students they won't. And that's just the reality, isn't it?'* Another view put forward was that that students who find mathematics especially hard are more likely to engage with ICTs in the class, because they see it as a *'doss'*, while students with greater mathematics ability may see it as a '*waste of time',* wishing to move on. Another view put forward is that students who are higher achieving in mathematics are less in need of ICTs to support their learning:

> *I actually think nearly the opposite, that the really good kids don't need it because they can work well without it … and the kids who do need it don't bother … so I think at home like,*

they're definitely not going to the effort of going into GeoGebra and doing whatever or whatever else it would be, I don't think they bother.

Though no overall position was evident across the Focus Groups, there appeared to be some consensus that ICTs are beneficial for mathematics teaching and learning when adequately resourced:

It does, I think it helps the students to understand and you say maybe that the good students don't need it but that's fair enough, but it's for people who do need it, it's there. I think for my teaching, it's made me a better teacher, there's no doubt about it, to use the GeoGebra and also have interacted with Smart Boards, there's maths software on it, I find that brilliant. I think it's class, I would use that a lot. And when you're explaining things, for making the connections between one thing and the other you can connect to your graph, you can say look at what happens when you do this or whatever. So if there's more ICTs to be given, yeah I want it, give it to me please.

However, it is worth noting another minority viewpoint, which reflects some teachers' understanding of the potential value of ICTs in Junior Cycle mathematics teaching and learning. These teachers played down the relevance of ICTs on the basis that students will not have ICTs to support them during the examination.[30] For example:

I think in theory it would be great if we could get them, say even GeoGebra, but like in reality they have to sit down and they have to know, you know how to find points, how to plot the points, I mean GeoGebra is wonderful and it does all this for you, but they still have to learn it for exam, they need to know how to do it … At the end of the day it's a written exam, do you know … It's all based on what they write on the page, not on their computer and they won't have the computer on the day.

Some teachers reported that they lack confidence in the use of ICTs and that they would have benefitted from additional training in how to integrate ICTs into their teaching. A few suggested that ICTs could be used to enhance students' visualisation and spatial reasoning skills.

Overall, findings suggest the importance of teacher factors, including teacher understanding of how ICTs can be used effectively to support the learning of mathematics, and teacher confidence in using ICTs.

6.6. Teacher Education

Overall, teachers reported being very satisfied with the initial Project Maths workshops provided to support them in the transition to the new curriculum. However, teachers expressed considerable negativity around the phased implementation of the curriculum: *'the workshops were great but the phased implementation was horrific'.* In particular, teachers found it challenging to have multiple (as many as five or six) syllabi running concurrently. One teacher commented: *'at one stage if you taught all years, you had 5 different syllabi',* to which another replied*: 'that's right. It was a nightmare. The brain was frazzled'.* For teachers, a key challenge lay in *'keeping-up'* with what was to be taught and when:

I think it was hard for the first few years when it was like this strand was in this year and then this strand in the next year, they were all mixed up between what you were actually

[30] However, ICTs do include calculators, which students have access to during examinations.

teaching for each year and then that was taken out then, I thought that was hard at the start.

Teachers further highlighted that those who had attended the 10 initial in-service workshops were consistent in their approach to teaching Project Maths: *'I think the people who attended the 10 in services were all singing off the same hymn sheet, they'd got the same ideas, they were working well'.* However, teachers raised the issue that mathematics teachers who did not attend the in-service training are not teaching Project Maths as it was designed to be taught, but rather, *'they're doing maths the way they did maths when they were taught it, which is perfectly understandable'*. This, they feel, is '*defeating the purpose of the whole idea',* and is disadvantaging students.

Teachers referred to the on-going support provided they received from the Project Maths Development Team as generally *'excellent'*, and especially commended activities that involved the collective participation of mathematics teachers:

> *We find those visits excellent just even in terms of all the maths teachers being in the room for half a day and being able to discuss what are you doing, how do you do it that way, we learned so much from the person that came out.*
>
> *And we learned so much from ourselves … it was very good.*

However, teachers were in agreement that development activities could be provided more frequently. In particular, it was argued that the 10 initial sessions should be repeated and support should be ongoing. Teachers were concerned that Colleges of Education/Universities were not training student teachers in these methods and it was suggested that greater communication needs to take place between the Maths Development Team, the Inspectorate and Education Departments in universities.

A further issue raised by participants was that, while the training is enjoyable and useful, it is often not possible to fully implement the suggestions because of the time required to do so:

> *I enjoyed the training, the Project Maths training, just to give a different opinion, but I did come away from some of the classes thinking that was great and in an ideal world I'd love to teach all my classes that way, but time is an issue.*
>
> *And I think the people giving the course would nearly know that if you were to teach every lesson that way you wouldn't get the course covered.*
>
> *I thought it was good, but I just see a disconnect in the real-world teaching in the classroom, the time and the kind of kids and the range of abilities you have in front of you. It's all well and good to say you differentiate your teaching and learning but in practice with the time constraints it's very difficult, you know.*

In this way, Lesson Study was considered quite useful though somewhat idealistic by those who had engaged in it. They described it as *'very time consuming'* and unrealistic in terms of commitments. One teacher commented: *'like the 6 evenings sitting around for 3 hours is an awful lot of time to give to develop one lesson and I know the template you fill out is a nightmare as well'.* Similarly, another commented:

> *I mean you couldn't keep going and that seems to be a model that they're sticking with. So anyone who has done it, in my mind has done it once and said thanks but we won't be doing it again, that's our experience.*

Nonetheless, it was proposed that a *'scaled down model'* of Lesson Study might be useful.

Many teachers said that they were accessing the online resources on the Maths Development Team website. Teachers praised the quality of the online resources available, but some commented that it can be difficult to find what you need.

6.7. Assessment and Examinations

A number of interview questions focused on assessment, including the possible introduction of Classroom-Based Assessments and Assessment Tasks, and potential changes to the structure of the Junior Certificate mathematics exam.

While teachers in some of the Focus Groups contributed to the discussion about Classroom-Based Assessments, others tended not to express a view or make suggestions. A few stated that they could not see any potential in implementing such assessments. Some of those making contributions saw the introduction of Classroom-Based Assessments as inevitable if they were being introduced in other subjects.

Suggestions included a focus on statistics, and the provision of problems with multiple solutions and varying levels of credit, depending on the sophistication of students' solutions. Some referred to projects conducted in Transition year, where credit was given for carrying out surveys and making formal presentations on the outcomes. One participant suggested that it might be possible to use the same Classroom-Based Assessments across a range of related subjects (in the case of mathematics, these might include science and geography) as topics often overlap. The same underlying project could then be drawn on in completing the assessment task in mathematics. One teacher noted that, in other countries, students completed project work during a designated project week, where they had access to the resources of the school to complete their projects and teachers acted as facilitators. Another teacher explained how their school had already implemented projects in mathematics:

> *Well in first year it's a statistics project, so they have to design a question, design the questionnaires, ask the question and then graphically represent it, they do an individual poster. And then in Second year it's an area and volume question. So it's like, I think it's one of the Junior Cert Higher Level questions kind of modified slightly, just about how many candles will fit in a certain box, they have to make models and you know so some of them will really embrace it.*

Other participants responded to this by querying how much support parents provided. A few teachers raised another issue – would students, especially less-able ones, bother to invest energy and effort in the Classroom-Based Assessments when they did not perceive them to be an integral part of the Junior Certificate mathematics examination. They cited evidence of lack of effort in the area of English.

A small number of teachers suggested that areas such as finding, collecting and organising data, modelling, scaling, and assignments involving ICTs could form the basis of Classroom-Based Assessments.

Teachers had mixed views on the proposed two-hour Junior Certificate mathematics examination. One the one hand, teachers acknowledged that a two-hour exam might reduce the pressure on students. On the other, concern was expressed that students might not be adequately prepared to take two mathematics papers at Leaving Certificate if they did not have the experience of two Junior Certificate papers. For some teachers, this represented a significant discontinuity between Junior and Senior cycles.

Teachers also raised the possibility that students could become frustrated if topics that had been studied in depth did not appear on the examination paper. Reference was made to the absence of a question on financial literacy on a recent Leaving Certificate paper and the ensuing disappointment. One teacher was concerned that a student who excelled in one particular strand might be placed at a significant disadvantage if no questions related to that strand appeared. A few teachers viewed the proposal for a shorter paper as being motivated by an effort to reduce the cost of implementing the Junior Certificate exam. One queried whether questions on the shorter test might cut across multiple strands.

A number of teachers across Focus Groups called for the (re-) introduction of options on the Junior Certificate mathematics exam, and argued that this would be essential in the context of a shorter examination. In support of this, they cited other subjects where a choice was available (e.g., English), and also claimed that a lack of choice in mathematics, whether on a short or full exam, significantly increases the pressure on students. A number of teachers questioned why the number of marks available for each Junior Certificate exam question was not specified, and why a suggested maximum time for completing the question was given instead. 'Borderline A' students at Higher level were viewed as being disadvantaged by this situation, as an indication of the available marks could inform them where they should make a greater effort.

Teachers felt the lack of a choice on the examination papers at Leaving Certificate increases the gap between Leaving Certificate and Junior Certificate.

The readability of Junior Certificate mathematics papers was raised by teachers throughout the Focus Groups. Teachers argued that students often displayed poor comprehension, or scanned questions without reflecting on what was being asked. Teachers also reported observing that students with special education needs struggled with the reading load in mathematics questions, and often needed readers for mock and state exams. Specific aspects of questions that caused difficulty for students included the density of ideas and the volume of text to be read, and, in some cases, the accompanying diagrams. Students with reading difficulties were viewed as *'being excluded from doing well on the Junior Certificate exam'*. Students *'at the other end of the spectrum'* – those who were good at abstract or pure mathematics – were also viewed as being excluded.

Lack of predictability from year to year was also identified as a source of difficulty as *'students couldn't internalise the structure of the questions'*. A few teachers reported observing an improvement in readability in the last couple of years, remaking that questions have become *'less wordy'*. One teacher from a Gaelcholáiste stated that the language of the Irish papers was too complex, with even native speakers having to refer to the English version for clarification. Another called for a standardised list of translated key terms to allow consistency of usage within the school.

Teachers more generally offered evidence of specific questions that challenged students, including one that involved a pulley system:

> *I think some of the questions the way they're worded, students read and go I haven't a clue what they're asking me here, like there's actually a question here about a pulley system and 2 pulleys connected by a belt and has the belt going around but it also had a dotted line going between the 2 pulleys and like unless you know what a pulley system is … students can't visualise it.*

Teachers also observed that students taking Technical Graphics (Design and Communication Graphics at Leaving Cert) enjoyed a clear advantage on such questions. A 2012 Leaving Certificate mathematics question involving a robotic arm was cited as an example to support this perspective.

Some teachers focused on the contexts of the exam questions as well as the language:

The way the questions are asked – they're completely inaccessible to a young person, they're so adult context, don't mind higher order, extremely adult context and why am I saying you should know about mortgages and they should, they don't and they couldn't give a fiddlers. And the context in general, they're scratching their head thinking what are they talking about.

Despite such concerns, however, a number of teachers acknowledged that mathematical literacy had improved. In the words of one such teacher:

you know the maths world is relatable to something in the everyday life . . . in the past it was an abstract world, this was just something that we had to learn. Whereas I do think, you know from a literacy perspective, the big term I suppose in a lot of schools, I feel that the mathematical issue has improved and the words are relatable, they do relate them to something now unlike before.

There was also acknowledgement that students were more familiar with mathematical terms:

I think the same with algebra, like before my kids used to say, you know the number over there and the letter over there whereas now they're saying the constant, the coefficient, the variable, like they're more familiar with it now as well.

Lastly, there was some discussion in relation to the possible introduction of standardised tests. Several teachers felt that a nationally-normed standardised test at the end of Second Year (as proposed in the National Literacy and Numeracy Strategy in 2011, but subsequently deferred) might be useful in assessing students' achievement accurately, and could provide guidance to teachers and parents on advising students on their proficiency in mathematics. Other teachers, however, questioned the need for such tests.

6.8. Summary

Whereas previous chapters drew on the outcomes of international assessments and national examinations to draw inferences on the impact of Project Maths on student performance and attitudes, the current chapter drew on the outcomes of Focus Group interviews. The interviews, which involved 76 teachers of Junior Cycle mathematics, were conducted across 7 sessions in 5 Education Centres in spring 2017. The interviews were based on a semi-structured questionnaire that asked about successes and challenges of Project Maths, strengths and challenges associated with each mathematics content area, and views on student attitudes and engagement, ICTs in teaching and learning, teacher education and assessment and examinations.

Aspects of the curriculum introduced under Project Maths that teachers viewed as working well included the more explicit connections students made between mathematics strands (for example, linear patterns in Co-ordinate Geometry and Algebra), the less-abstract nature of mathematics, and the use of real-life problems. Teachers also referred to enhanced understanding of mathematical concepts among students and a willingness to 'have a go' at problems. The instructional emphasis was seen as shifting away from rote learning, and a majority of participants, though not all of them, viewed this as positive. Statistics & Probability was identified as a strength of Project Maths, with several teachers mentioning the relevance to this strand to everyday life, though it was also noted that time allocated to Statistics & Probability may be at the expense of time that could have been allocated to other strands. Some questioned emphasis on Statistics & Probability, arguing that 'pure maths' should be emphasised more strongly, in the case of higher-achieving students.

Aspects of Project Maths that teachers viewed as challenges included: instructional time, which was described as inadequate if all topics were to be taught 'in the spirit' of Project Maths, and procedures and problem solving were to be taught simultaneously; the literacy demands of the Junior Certificate examination papers, which were described as unfair on students with learning difficulties and students for whom English is an additional language; and the broad spectrum of student ability, especially in classes for students taking Higher level. Subgroups of teachers also expressed concern about the lack of a specific Foundation level syllabus, and a lack of time to address the needs of the highest-achieving students.

The Common Introductory Course was viewed both positively and negatively by teachers. Positive aspects included an opportunity to sample content across the entire mathematics curriculum, and an opportunity to appraise the strengths and weaknesses of students. However, teachers were often critical of the skill levels of students arriving from primary school, especially in the Number strand, where fractions, decimals and percentages were deemed weak. Some teachers also believed that the CIC did not challenge higher-achieving students sufficiently, leading to a risk of disengagement when content became more complex in Second year. Teachers also identified the transition from Junior Cycle mathematics to Senior Cycle mathematics as problematic, especially for students at Ordinary level.

While many teachers were pleased with the emphasis on Statistics & Probability at Junior Cycle, some argued that the emphasis in this area could be reduced so that the focus on other areas such as Algebra (the most challenging strand), Co-ordinate Geometry and Trigonometry could be strengthened, with a small number of teachers calling for more 'drill and practice' in these areas. Almost all teachers agreed that Algebra is a critically-important element of Junior Certificate mathematics, though just a few agreed that performance in Algebra had improved under Project Maths, and some expressed the view that Algebra at Ordinary level did not prepare students well for Algebra at that level in Senior Cycle. Nevertheless, almost all teachers agreed that the content of Junior Certificate Algebra should not change. Teachers were largely satisfied with performance in Geometry, though some referred to an over-emphasis on textbooks and the application of formulae by students, in the absence of proper understanding. There was criticism of the emphasis on synthetic Geometry in the syllabus, and teachers argued that the five formal proofs of theorems still on the course could be removed and replaced with a greater emphasis on solving construction problems. Number was an area of concern for many teachers, especially in the context of the CIC. A majority of teachers expressed themselves happy with Functions, and, in particular, links between Functions and Algebra.

Teachers expressed the view that students were more interested in and engaged with mathematics under Project Maths. They attributed this to the methodologies associated with Project Maths, which they viewed as being more interactive e and more discussion-based. Teachers themselves also reported greater enjoyment in teaching mathematics under Project Maths. Female students were viewed as more likely to struggle on mathematics problems, compared with their male counterparts, and more likely to worry about making mistakes.

Teachers differed in their views on the value of ICTs as tools for teaching mathematics at Junior Cycle. While teachers reported using ICTs most often as a classroom demonstration tool, some also encouraged their use among students for such purposes as taking tests, solving problems and checking answers. While some teachers used Geogebra extensively, others referred to high amounts of time associated with ICT usage, and students' lack of relevant IT skills. Nevertheless, across teachers generally, there was a consensus that ICTs are beneficial for teaching and learning mathematics, when properly resourced.

Teachers spoke highly of the initial (10) Project Maths workshops, though it was noted that some teachers did not avail of these, and therefore were teaching mathematics in ways that were not consistent with Project Maths. This led to difficulties co-ordinating the teaching and learning of mathematics at school level. Teachers were also very positive about the ongoing support they received from the Project Maths. However, they acknowledged that it was not always possible to implement all the suggestions presented during inservice activities. Concern was expressed that time requirements associated with Lesson Study, and some participated suggested that a scaled model might be implemented.

While not all participates contributed to the discussion on assessment and examinations of mathematics, those who did were divided on the value of proposed Classroom-Based Assessments. A few offered specific suggestions for these, including the use of tasks with multiple entry points and tasks that cut across a number of related subject areas (e.g., science and geography). Statistics was identified as a possible topic for such assessments. Teachers also had mixed views on the proposed two-hour Junior Certificate mathematics examination, with some arguing that students planning to take Leaving Certificate mathematics need a longer examination. Some also expressed concern that the new combined Ordinary/Foundation level paper might prove too difficult for students who, in the past, would have taken the Foundation level paper.

Chapter 7. Summary, Implications and Conclusion

7.1. Introduction

In December 2016, the Educational Research Centre was commissioned by the NCCA to conduct an evaluation of the impact of Project Maths on the performance of students in Junior Cycle mathematics. The study drew on existing sources, including the literature on the implementation of Project Maths and current national (Junior Certificate) and international assessments, as well as findings from Focus Group interviews conducted with Junior Cycle mathematics teachers, to draw inferences about the effects of Project Maths on aspects of teaching and learning mathematics, including students' performance and attitudes. In this sense, the study does not represent a systematic evaluation, but rather an assessment using the best available evidence alongside the views of teachers currently implementing the revised Junior Certificate mathematics syllabus introduced under Project Maths.

This chapter draws together findings across earlier chapters in this report. It discusses overall performance in mathematics and performance on mathematics strands and processes in state examinations and international assessments, and considers the extent to which the observed patterns in performance can be attributed to the Project Maths initiative. The chapter also describes aspects of the reform that are working well and areas in need of improvement, and in doing so, considers aspects of teaching and learning under Project Maths. Implications of the findings for the review of Junior Cycle Mathematics are highlighted at the end of each subsection.

7.2. Overall Performance in Mathematics

An initial evaluation of Project Maths conducted by the NFER showed limited effects on performance, with no significant differences on Statistics & Probability, or on Geometry & Trigonometry[31] between students in schools where Project Maths had been implemented initially, and those schools in which it had just been introduced from 2010 (Jeffes et al., 2013). However, it was acknowledged at the time that Project Maths had not been in place for very long, and that performance patterns might change over time. Data from PISA (reviewed in Chapter 2) show that performance on overall mathematics among 15-year olds in Ireland has been relatively stable since 2003 (when mathematics was a major assessment domain in PISA for the first time), though a drop in performance in 2009 was made up in 2012. Indeed, Ireland's mean score in 2003 (502.8) is almost identical to its mean score in 2015 (503.7). There is a caveat here, however. PISA transitioned to computer-based assessment in most countries including Ireland in 2015, with items previously presented on paper transferred to computer. While average overall mathematics performance across OECD countries dropped by 5 score points between 2012 and 2015, performance in Ireland increased by 2.2 score points, though this increase was not statistically significant. It may be that, without Project Maths, performance in Ireland could have dropped in 2015, as it did in several OECD countries including Australia and the Netherlands. It is also noteworthy that, in PISA 2012, students in initial Project Maths schools outperformed students in non-initial schools on overall mathematics and on all four content subscales, though differences were not statistically significant. This is consistent with the view that any drop that might have occurred in 2015 because of the transition to computer-based testing was cancelled out by stronger performance arising from participation in Project Maths.

[31] This conclusion was drawn after applying multi-level modelling, where the outcome variables were Statistics & Probability and Geometry & Trigonometry, as measures of overall performance were not available for the two points in time at which tests were administered.

The lack of significant improvement in overall performance in international assessments is also evident in TIMSS, where students in Second year in Ireland had mean scores of 518.9 in 1995 and 523.5 on the same scale in 2015. Although students in Ireland performed better in 2015, the difference was not statistically significant. This outcome can be interpreted as indicating that performance in curriculum-based mathematics, assessed on paper, has remained stable over time, though there is some indication of a possible upwards movement. This can be viewed positively in that, when new curricula are implemented, there is often an initial dip in performance. Conway and Sloane (2006) predicted that curricular change could take between five and ten years to impact on scores on international tests.

Data on overall Junior Certificate mathematics performance (Chapter 4) must be interpreted with reference to an increase in the proportion of students taking Higher level in recent years (from 46.5% in 2011, the year before bonus points for taking Higher-level Leaving Certificate mathematics were introduced, to 55.3% in 2016). The proportion of students achieving grades A-C at Higher level has ranged from 79.2% in 2012 (for the last cohort who took the pre-Project Maths course) to 76.3% in 2016. Between these years, the proportion achieving Grade A has dropped from 14.3% to 7.6%, while the proportion achieving Grade B has dropped 33.9% to 30.8%. There was an increase in the proportion achieving Grade C, from 17.0% to 22.1%. It is noteworthy that the proportion achieving Grade A has also declined in absolute terms, with 4218 achieving an A in 2012, and 3855 doing so in 2016. This is broadly consistent with the finding in PISA that marginally fewer students in Ireland (9.8% in 2015) perform at Levels 5-6 (the highest levels of proficiency) than on average across OECD countries (10.7%), even though Ireland's overall mean score was significantly higher than the corresponding OECD average (503.7 vs. 490.2). A similar situation arises in TIMSS 2015, where 6.8% of students in Ireland performed at the Advanced benchmark, compared with 10.7% on average across OECD countries in the study, even though Ireland had an overall mean score that was significantly higher than the average of OECD countries in TIMSS (523.5 vs. 513.2). Three further observations are relevant here. First, the proportion of students achieving Levels 5-6 in earlier PISA cycles (11.4% in 2003, 10.2% in 2006, and 10.7% in 2012) are not very different from 2015. Second, similar proportions of students in Ireland performed at the Advanced proficiency level in TIMSS in 1995 and 2015 (8.5% vs. 6.8%). Third, although there has been a decline in the absolute number of students achieving Grade A at Higher level between 2010 and 2016, similar or lower numbers of students than in 2016 achieved Grade A at Higher level in the past (3204 in 2004, 3387 in 2005, 3871 in 2010).

The proportion of A-C grades at Junior Certificate Ordinary level also declined in recent years, from 76.2% in 2012 to 71.8% in 2016. There was also a small drop at Foundation level (from 81.4% in 2012 to 80.9% in 2016).

A majority of teachers who participated in the Focus Group interviews (see Chapter 6) felt that there had been a decline in student performance in recent years, even if this was not apparent in the distributions of grades achieved by students. Their perceptions of a drop in the performance of the highest-achieving students seems to be borne out by decline in the absolute number of students achieving Grade A in Junior Certificate mathematics at Higher level, at least since the introduction of Project Maths. However, it might be noted, as per the Chief Examiner's Report for Junior Certificate mathematics (SEC, 2015a), that changes arising from Project Maths covered not only content, but also processes, with an increased emphasis on higher-order skills. Since such skills are more difficult for teachers to teach and for students to acquire, some change in performance might have been expected if examination questions now assess these skills.

On balance, it can be concluded that average performance in Ireland has been relatively stable in recent years, based on performance in international studies. In PISA, Ireland's mean score on overall mathematics increased by less than one score point between 2003 and 2015, while performance on TIMSS mathematics improved by a non-significant 4.6 score points between 1995 and 2015. Although the proportions of A-C grades at Higher, Ordinary and Foundation levels have dropped between 2012 and 2016 (i.e., between the last year in which the pre-Project Maths syllabus was assessed and 2016), there are years prior to the introduction of Project Maths in which similar or lower proportions of students achieved Grades A-C at each examination level, compared with 2016.

Although students in Ireland achieved mean scores that were significantly higher than the corresponding OECD country averages in PISA 2015 and TIMSS 2015, Ireland still lags well behind the highest-performing countries in both studies. In the case of PISA, Ireland performed significantly less well than several EU countries, including Slovenia, Finland, Denmark, the Netherlands and Estonia, as well as Switzerland, Korea and Japan and non-OECD country Singapore. In TIMSS, where fewer European countries participated, Ireland performed significantly less well than OECD countries Japan and Korea, and non-OECD country Singapore.

Ireland's performance in mathematics in PISA is problematic to the extent that performance on reading literacy is relatively stronger, with Ireland ranking fifth overall and second among European countries in PISA 2015. In general, countries that do very well in reading literacy in PISA perform at a very high level on mathematics, but this is not the case for Ireland.

These observations raise two key questions:

- Why has overall performance in mathematics remained stable in Ireland over the past 20 years or so?
- What measures can be taken in the context of the review of Junior Cycle mathematics that could raise average performance and, in particular, the performance of higher-achieving students?

Some answers to these questions are offered in the remaining sections of this chapter. Each section will include implications for review of Junior Cycle Mathematics and the associated professional development for teachers.

7.3. Performance on Mathematics Strands and Processes

While a consideration of overall scores or grades in mathematics provides a broad picture of performance and standards, analyses by content area and process can point to relative strengths and weaknesses of students. The focus in this section is on relative performance on mathematics content areas and processes, and on specific aspects of these that may need to be modified or adjusted in the context of curriculum revision. It draws on data from a number of sources including international studies of mathematics achievement, examinations and Focus Group interviews.

7.3.1. Statistics & Probability

There was agreement among Focus Group participants that increased emphasis on Statistics & Probability in Junior Certificate mathematics was a positive development. Teachers reported strong engagement by students with this aspect of the syllabus, and some noted its obvious relevance to everyday life. Students in Ireland perform relatively well in the corresponding area in PISA mathematics (Uncertainty & Data), with mean scores that were significantly above the corresponding OECD averages in 2003 and 2012. Indeed Uncertainty & Data was an area of strength for students in Ireland on PISA, even before the implementation of Project Maths, though there was a significant drop of 8.5 score points between 2003 and 2012. Students in Ireland also performed well on Data & Chance in TIMSS 2015, with a mean score that was significantly higher than on average across OECD countries in the study. However, performance among higher-achieving students in Ireland was not significantly different from the corresponding OECD-16 average, suggesting possible scope for improvement among higher achievers. Teachers in Focus Group interviews did not propose any specific changes to the content of Junior Certificate Statistics & Probability, though some noted that, if a rebalancing across content areas was necessary, the relative emphasis on Statistics & Probability might be reduced.

Implications

The success of the Statistics & Probability Strand is clearly apparent, as methodologies consistent with Project Maths have been widely implemented. These methodologies should be extended to other strands (see below).

Teachers should be advised on how to distribute time across elements of the Common Introductory Course so that no one strand is over-emphasised to the neglect of other strands. This could include the integration of aspects Statistics & Probability into other content areas (for example, identification and development of patterns, ratio and proportionality in Number).

7.3.2. Number

Another area of relative strength for students in Ireland in international studies is Number. In PISA 2012, students in Ireland achieved a mean score on Quantity that was significantly above the corresponding OECD average (by 10.1 score points), though students in Ireland at the 90th percentile achieved a score that was not significantly different from the OECD average at that marker. In TIMSS 2015, students in Ireland achieved a significantly higher mean score on Number than students on average in participating OECD countries. Furthermore, performance on Number in Ireland was higher than performance in any other content area, and significantly higher than overall performance on the TIMSS test. Performance on Number also increased significantly in Ireland between 1995 and 2005 (by some 33.2 score points), even though overall performance on TIMSS did not change significantly between those years. Performance on Number in the 2015 Junior Certificate examination was at about the same level or higher (in terms of the percentage of available points achieved) than on other content areas among students taking Ordinary and Foundation levels, and lower than on Algebra and Statistics & Probability among students taking Higher level.

Although national and international assessment data suggest that performance on Number in Ireland is strong relative to other mathematics content areas, except, perhaps, among the highest performers (for whom expectations would be higher), teachers in the Focus Group interviews expressed concerns about performance in this area, especially among lower-achieving students. These included a lack of proficiency in fractions and percentages among incoming First year students (in the context of having to place a strong emphasis on these topics in the Common Introductory Course), which was often verified on a Maths Competency Test administered early in First year. Difficulties on topics such as fractions were viewed as potentially impacting in a negative way on Algebra in particular. Several teachers also referred to students' difficulties in conceptualising irrational numbers. It is noteworthy that 'concepts of irrational numbers' was the only TIMSS topic in Number that had not been covered by at least 90% of students in Ireland by the end of Second year, according to their teachers.

Consideration of proficiency in Number inevitably leads to a consideration of the use of calculators in mathematics classes and examinations, with TIMSS 2015 data indicating that 72% of students in Second year in Ireland have 'unrestricted use' of calculators in mathematics classes. Moreover, more students in Ireland than on average across participating OECD countries in TIMSS are taught by teachers who report that their students use calculators in at least half of classes for such purposes as checking answers, doing calculations, solving complex problems and (to a somewhat lesser extent), exploring number concepts. Interestingly, calculator usage is very low in a number of OECD countries in TIMSS including Japan and Korea (which perform strongly on TIMSS relative to Ireland) and Slovenia and Turkey (which do less well).

Ultimately, policy on calculator usage for instructional purposes, whether at national or school level, needs to take into account the fact that pupils at primary level are allowed to use calculators from Fourth class onwards, though again some primary schools and teachers may place restrictions on usage. In terms of policy in assessment, it is noteworthy that both TIMSS (Grade 8) and PISA allow calculators to be used on their tests of mathematics (and science), though items are intended to be solved with or without calculators. In any event, the performance of students in Ireland on Number in TIMSS and, to a lesser extent, on Quantity in PISA, would suggest that performance is not affected in a negative way by access to calculators during schoolwork. As against this, a minority of Focus Group teachers called for the inclusion of calculator-free items in assessments of mathematics.

In general, Focus Group teachers are probably correct in noting that there is scope for improvement in Number at primary level, despite relatively strong performance on international assessments. In the 2014 National Assessment, the average percent correct score on Number (which also included a small number of Algebra items) at Sixth class, improved significantly to 63.4% correct, from 57.6% correct in 2009. There seems to be scope for further growth in this area. It is also noteworthy that performance on Measures[32] in the same Assessment (effectively the application of Number and related concepts to problems in areas such as time, capacity, length and weight) at Sixth class was much poorer (41.6%, up from 38.2% in 2009), indicating considerable scope for improvement in application of number in problem-solving contexts as well. The mathematics curriculum for Junior classes at primary level (3-8 years) is currently being revised.

Implications

There would be value in clarifying the emphasis that should be placed on Number, especially in the CIC, taking into account that properties of numerical operations and relationships between them are fundamental to ability to manipulate expressions and formulae, and to developing an understanding of equality.

Across the Junior Cycle mathematics, consideration should be given to increasing the emphasis on irrational numbers (as suggested by Focus Group teachers). There would be value in providing more direct teaching of the vocabulary of Number such as numerator, divisor, rational number and radical number, so that students are aware of and use the relevant terminology (though some Focus Group

[32] Whereas at primary level, Number and Measures are separate content domains, at post-primary level, Applied Measures is subcomponent of Number.

teachers reported improvement in this already, especially in Algebra). The area of Applied Measure (effectively problem solving) within Number showed be reviewed to ensure that processes such as visualisation, model building and the application of algebraic and other forms of reasoning are emphasised.

7.3.3. Geometry & Trigonometry

A consistent finding in PISA mathematics has been the low performance of students in Ireland on Space & Shape items. In the two cycles in which mathematics has been a major assessment domain in PISA (2003, 2012), students in Ireland have achieved mean scores below the corresponding OECD average scores on Space & Shape, with female students, in particular, struggling on items in this content area. In PISA 2012, students in Ireland performing at the 10th and 90th percentiles achieved scores that were significantly lower than their counterparts on average across OECD countries, confirming that Space & Shape represents an area of particular weakness for both higher- and lower-achieving students (the score of students in Ireland at the 10th percentile was significantly higher than the corresponding OECD average for the three remaining PISA content areas of Change & Relationships, Quantity and Uncertainty & Data).

Percent correct scores indicate that, with the transition to computer-based testing in PISA in 2015, performance on Space & Shape items declined further in Ireland (from 34.4% in 2012 to 32.4%), though there was a larger decline on average across OECD countries (from 37.3% to 32.0%). PISA Space & Shape includes interpreting views of 3-D scenes from different perspectives and constructing representations of shapes. Also included are transforming shapes with and without technology. The content draws on geometry, spatial visualisation, measurement and algebra.

Further evidence for lack of attention to aspects of spatial reasoning is found in TIMSS, where 60% of students in Ireland were taught by teachers who had not introduced relationships between three-dimensional shapes and their two-dimensional representations by the end of Second year, while 63% were taught by teachers who had not introduced translation, reflection and rotation (though some students may cover these topics in Third year). These data contributed to a situation in which coverage of Geometry topics in TIMSS was considerably lower in Ireland (58% of students, based on an average across 6 Geometry topics), compared with the average among OECD countries in TIMSS 2015 (78%) and the averages for Canada (85%), Korea (90%) and Japan (95%).

In moving towards a revised mathematics curriculum, there may be value in considering what elements of PISA Space & Shape are important, and whether students would benefit from more intensive engagement with these elements, with possible carry-over to other areas of mathematics, as well as other subject areas such as history, geography, technical graphics, science and literature. Within Geometry & Trigonometry, consideration might be given to strengthening the focus on visualisation and spatial reasoning.

Students in Ireland achieved a mean score on TIMSS Geometry that was not significantly different from the average of participating OECD countries, while higher-achieving students in Ireland (those performing at the 90th percentile) did significantly less well than on average across the OECD-16. Moreover, this was an area of relative weakness for students in Ireland as they performed at a significantly lower level on Geometry than on the TIMSS test as a whole. In the 2015 Junior Certificate examination, Geometry & Trigonometry was identified as an area of relative weakness. At Higher level, for example, on average, students achieved proportionately fewer of the available marks for Geometry & Trigonometry than for any other content area, with the exception of items described as ‘hybrid’.

A majority of teachers in the Focus Group interviews felt that the five formal Geometry proofs (Theorems 4, 6, 9, 14 and 19) could be reviewed, with a view to placing a greater emphasis on problem solving involving constructions, diagrams and applications of theorems – an approach that would be more compatible with the emphasis in Project Maths than the current one, which was seen as drawing heavily on ideas from synthetic Geometry. Related to this, several teachers noted an over-emphasis on use of the book of Formulae and Tables, which was seen as contributing to a procedural approach to learning. The view was expressed by several teachers that the content of Trigonometry could be broadened to include non-right-angle triangles, as current content in Trigonometry was viewed as being limited and a few noted over-use of the sine rule.

Implication

It would be useful to interpret for teachers how best to integrate *Geometry for Post-primary School Mathematics* in teaching and learning in ways that engage students and support them in applying their understanding of geometric concepts. The need for students to present formal proofs of theorems should be reviewed. More emphasis should be placed on analysis and informal deduction.

There would be value in examining whether some aspects of visualisation and spatial reasoning might be integrated into the Geometry strand so that it is better aligned with international frameworks in this area. Consideration might be given to strengthening aspects of Trigonometry, as proposed by Focus Group teachers.

7.3.4. Algebra

In PISA 2012, students in Ireland achieved a mean score on Change & Relationships[33] (501.1) that was significantly higher than the corresponding OECD average (492.5), though Ireland lagged behind a number of countries including Canada (525), Belgium (513), and Switzerland (530) as well as Japan (542) and Korea (559). Moreover, the score of students at the 90th percentile in Ireland was below the corresponding OECD average score, indicating lower relative performance among higher-achieving students in Ireland. In TIMSS Algebra, which is more clearly linked to school curricula than PISA and (like PISA) also includes Functions, students in Second year in Ireland achieved a mean score (501.0) that was not significantly different from the average of participating OECD countries (501.9). As in PISA Change & Relationships, the performance of students in Ireland at the 90th percentile on TIMSS Algebra was significantly below the corresponding average for participating OECD countries. Students in Ireland at the 10th percentile on TIMSS Algebra performed at a level that was not significantly different from the corresponding OECD average.

A somewhat different pattern of outcomes emerged on the Junior Certificate examination in 2015. Students taking the examination at Higher level achieved a greater percentage of the available marks on Algebra[34] (76.0%) than on any other content area. Students taking the Ordinary level exam achieved the smaller percentage of the available marks (40%) on Algebra than on any other content area, though they also performed relatively poorly on Functions (42%). Students taking Foundation level did better on Algebra (65%) than on Geometry & Trigonometry (51%) or on Hybrid questions (56.5%), but less well than on Number (81.5%) or Statistics & Probability (89%). The Chief Examiner's report in 2015 drew particular attention to difficulties experienced by some students at Higher level 'in engaging accurately and effectively with algebra'. Despite high overall performance on Algebra at

[33] Includes 'modelling . . . change and relationships with appropriate functions and equations, as well as creating interpreting and translating among symbolic and graphical representations of relationships' (OECD, 2016a, p. 71).

[34] Some Algebra was also required to answer questions categorised as 'Hybrid'.

Higher level, some students taking Higher level struggled 'when algebra became central to the solution of problems' (e.g., finding the value of n for a right-angled triangle with sides 9, n and $n + 1$).

Focus Group teachers identified Algebra as the most challenging area for students, and noted that expressions, equations and inequalities were particularly challenging. However, a majority of teachers noted that the inclusion of patterns in the Algebra strand was a positive step, in that it allowed links to be established between Algebra and other aspects of mathematics. In TIMSS 2015, approximately equal proportions of students in Ireland (72% of students, based on an average across six Algebra/Functions topics) and on average across participating countries (69.7%) were taught by teachers who reported that topics underpinning TIMSS Algebra items had been introduced by the end of Second year. Surprisingly, given its emphasis in the current syllabus at Junior Cycle, 37.5% of students in Ireland were taught by teachers who reported that they had not covered 'numeric, algebraic and geometric patterns or sequences' by the end of Second year.

Implications

In terms of syllabus revision, consideration might be given to combining Algebra and Functions. This might provide additional support to teachers seeking to teach algebraic concepts using a functions-based approach. Another issue that might be dealt with is the tension between, on the one hand, a need for students, especially those taking the Higher-level course, to apply algebraic reasoning to solving mathematics problems, and, on the other, encouragement of students to use a variety of approaches to solving problems, including those based on trial and error. Beyond this, there may be value in considering the role of Algebra at Leaving Certificate Ordinary level, as this has a knock-on effect on the Ordinary level syllabus at Junior Certificate level, and on teachers' perceptions of what should be taught and how.

Consideration might also be given to enhancing the learning environments in which Algebra is taught and learned. Concrete manipulatives (e.g., Algeblocks) or ICT-based environments containing manipulatives should be considered. These could promote better understanding of basic algebra concepts such as factorisation and solving equations rather than procedural knowledge, which is often taught using rote-learning approaches.

7.3.5. Functions

As noted above, the Functions content area is integrated into Change & Relationships in the PISA assessment, and into Algebra in TIMSS. Hence, international assessments do not provide separate information on how students perform in this area. In the 2015 Junior Certificate mathematics exam, the weighting given to Functions (in terms of available marks) was lower than for any other content area, and no items addressed Functions at Foundation level. In TIMSS 2015, just over 50% of students were taught by teachers who had not introduced the TIMSS topics of 'representations of functions as ordered pairs, tables, graphs, words or equations' and 'properties of functions' (slopes, intercepts, etc.), though again these topics may receive attention in Third year.

In general, Focus Group teachers were satisfied with the relative emphasis on Functions, with some noting that the content of this strand connected with and reinforced the concepts covered in Algebra.

Implications

There may be value in specifying in more detail how the Algebra and Functions strands are connected and examining whether these strands might be combined.

7.3.6. Links across Mathematics Strands

A key finding emerging from the Focus Group interviews was the high value that teachers placed on opportunities to support students in establishing links across mathematics strands. Several teachers cited how patterning in Number and Geometry impacted positively on students' subsequent understanding of Algebra. Others commented in a positive way on examination questions that allowed students to draw on content from different strands to arrive at their answers. Support materials provided by the Project Maths Development Team highlight how particular topics can be identified across a range of content areas, such as ratio/proportion in trigonometry, probability and geometry (scale, slope, ratio of circumference to diameter etc.). Other connections that might be established more explicitly include those between mathematics topics, other curricular areas and a variety of real-life situations. Such connections also capture the concept of mathematical literacy/numeracy, where students identify opportunities to apply and build on their mathematical knowledge outside of mathematics classes.

Implications

Consideration should be given to highlighting to a greater extent the links that can be established across mathematics strands and the importance of giving students opportunities to make such links themselves.

7.3.7. Mathematical Processes

International studies use a range of schemes to describe and report on students' mathematical processes. These essentially involve classifying questions based on the main thinking processes that students are believed to draw on. PISA 2012 based its categorisation on processes identified as underpinning problem solving.[35] In Ireland, students had a mean score on the Formulating subscale that was not significantly different from the corresponding OECD average, while mean scores on the Interpreting and Employing subscales were significantly higher. These outcomes suggest that, relative to other problem-solving processes, students in Ireland are less proficient on tasks such as translating real-world problems into a form amenable to mathematical treatment, and providing mathematical structures and representations that can then be used to solve problems.

TIMSS 2015 framed its processes (or cognitive domains) around the more traditional categories of Knowing, Applying and Reasoning (see Chapter 2 for definitions). Relative to overall performance on the TIMSS mathematics test, students in Ireland had a mean score on Knowing items that was significantly higher than on the test as a whole, and a mean score on Applying that was significantly lower. Performance on Reasoning items was not significantly different from performance on the test as a whole. Ireland's relatively strong performance on Knowing suggests that students in Ireland are most proficient on tasks in which they recall definitions, terms and properties, carry out basic algorithmic procedures, and retrieve information from graphs, tables, texts and other sources. Performance on Applying suggests that students are more challenged when confronted with tasks that require them to determine effective strategies for solving a problem, display data in tables or graphs, create equations and inequalities or geometric figures that model problem solving situations, and implement strategies to solve problems involving more familiar concepts or procedures. Surprisingly, perhaps, students in Ireland performed reasonably well on Reasoning tasks such as using intuitive or inductive reasoning based on patterns and regularities to arrive at solutions to problems

[35] The three categories, which are defined in detail in Chapter 2, are: Formulating situations mathematically; Employing mathematical concepts, facts, procedures and reasoning; and Interpreting, applying and evaluating mathematical outcomes.

set in unfamiliar situations, evaluating alternative problem-solving strategies and solutions, and providing mathematical arguments to support a strategy or solution. Worryingly, however, the performance of higher-achieving students in Ireland on TIMSS Reasoning (those scoring at the 90th percentile) was significantly below the corresponding OECD-16 average score, while their scores at this marker on Knowing and Applying were not significantly different from the OECD-16 average scores on these processes. In contrast, the scores of students in Ireland scoring at 10th percentile were significantly higher than the corresponding OECD-16 average on each of the three process scales.

The stated objectives of the most recent Junior Certificate syllabus (DES/NCCA, 2013) are linked to cognitive processes in mathematics. These are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive dispositions. Despite the fact that these objectives are clearly stated at the start of all post-primary mathematics syllabi, teachers at the Focus Group interviews were generally unfamiliar with them, though they recognised the hierarchical nature of processes in mathematics.

Implications

Curriculum development might focus more intensively on the nature of cognitive processes in mathematics. This could be done through providing more detail on the objectives referred to above, and integrating the concept of cognitive processes into protocols for assessment (for example, criteria for assessing performance on mathematics tasks, or descriptors or examples of increasingly-sophisticated mathematical processing within or across content strands). There may also be value in extending the steps in problem solving in the current syllabus document to include interpreting, evaluating and employing the outcomes of problem solving, even though there may already be some emphasis on this this in classroom practice.

7.4. Instructional Time for Mathematics

The amount of instructional time currently allocated to mathematics at Junior Cycle emerged as a key issue in the course of this review. Focus Group interviews revealed consensus among mathematics teachers on the view that it is not possible to implement the curriculum as intended within the instructional time currently available for Junior Cycle mathematics. The central issue described is not the volume of curriculum content per se (as teachers were broadly satisfied with its scope). Rather, the issue is with the time available to teach the content using the methodologies encouraged by Project Maths. Teachers emphasised that the embedding of mathematical concepts requires time, and that both time and conceptual knowledge are needed to develop a deep understanding of mathematics.

The issue with instructional time impacts on teachers of all Junior Cycle mathematics classes. At Higher level, teachers felt that the syllabus is heavier in content, while the classes are more mixed ability than before bonus points were made available at Leaving Certificate level. At Ordinary (including Foundation) level, teachers felt that the syllabus is lighter in content than before Project Maths, while the students need more time to grasp concepts, and the classes are weaker in ability than before. Teachers of students who sit the Junior Certificate examination paper at Foundation level argue that these factors are compounded by the absence of a specific Foundation level syllabus.

From the perspective of teachers, insufficient instructional time for mathematics impacts negatively on the teaching and learning of mathematics in a number of ways. Lack of instructional time means that teachers feel under pressure to cover the syllabus and are therefore teaching the course at a faster than optimal pace. To ensure that content is covered, teachers are teaching some topics *'in the spirit of Project Maths'*, and other topics using more traditional methodologies (e.g., 'drill and practice'). Topics most likely to be taught more traditionally tend to fall within the Algebra and Geometry strands, rather than within the Statistics & Probability strand, which teachers feel is particularly well suited to the active learning methodologies encouraged by Project Maths. For teachers, the negative effects of instructional time pressures are exacerbated in schools with high absenteeism, in schools with large numbers of EAL students, and in schools with larger classes, where differentiation of instruction becomes more difficult. Additionally, for teachers, the issue with instructional time is compounded by the lack of choice on examination papers. The corollary of this is that, if students had a choice on examination papers, they, or their teachers, might not cover the full syllabus.

Teachers' concerns about instructional time for Junior Cycle mathematics are intensified in the context of the introduction of one-hour classes from next September in some schools and the introduction of new curricular areas to the timetable. For example, some teachers moving to three one-hour classes for mathematics (where this occurs) will meet less often with students on a weekly basis.

In comparison with other OECD countries, Ireland already allocates considerably less time for mathematics instruction at Junior Cycle. The TIMSS 2015 data presented in Chapter 5 revealed that the average weekly allocation of instructional time for mathematics in Second year is 193 minutes in Ireland, compared to 201 minutes on average across the 16 OECD countries in TIMSS. Annually, 109 hours are allocated to mathematics in Ireland, compared to 131 across the OECD-16 on average. Instructional time for mathematics in Ireland appears particularly low when compared with OECD-16 countries with the highest allocations: Canada (270 minutes for mathematics weekly and 168 hours annually) and Chile (300.7 hours for mathematics weekly and 192 hours annually). In Ireland, mathematics instructional time is 11.3% of total instructional time, compared to 13.3% on average for the OECD-16, and highs of 18% in Canada and 17% in Chile. Among the OECD-16, Sweden and Japan are the only countries which allocate fewer hours to mathematics instruction annually than Ireland (99 in Sweden and 106 in Japan), and are the only countries in which mathematics instruction constitutes a smaller proportion of total annual instructional time compared with Ireland (10.8% for Sweden and 10.2% for Japan). Hence, if schools in Ireland were to allocate three one-hour classes for mathematics at Junior Cycle, average instructional time for mathematics would reduce to around 99 hours per year, leaving Ireland in joint position with Sweden at the bottom of the OECD-16 ranking (compared with a current ranking of 14th position).

Although there is no clear relationship between instructional time and performance, teachers feel an increase in instructional time for mathematics will create permitting circumstances for improvements in performance. Specifically, an increase in instructional time for mathematics would enhance the capacity of teachers to provide students with the opportunities to realise the objectives of the syllabus introduced as part of the Project Maths initiative. For instance, learning to 'communicate mathematics verbally and in written form' (a learning outcome in all syllabus strands), requires discussion and reflection in class – activities which require time. The desired outcomes of increased instructional time are a deeper understanding of mathematics among students, and greater numbers of students meeting the learning outcomes specific to each strand and common across all strands.

For Ireland to match the TIMSS OECD-16 average yearly hours for mathematics instruction (Second year), an additional class period of 40 minutes per week would be needed (on top of the 109 hours currently allocated annually). Some teachers proposed that two-to-three additional classes per week are required for Project Maths to be implemented as intended. An extra two 40-minute classes per week would entail an extra 44.5 hours for mathematics per year (making a total of 153.5 instructional hours annually). This would be similar to the levels in Israel (153 hours annually) and the United States (155 hours annually), but still considerably lower than in Canada (168 hours annually) and Chile (192 hours annually). The Junior Certificate Mathematics syllabus, like all other Junior Certificate syllabi, is designed as a 240 hour course of study (DES/NCCA, 2013, p. 10).

Implications

Overall, the findings of this review suggest that because of the instructional time constraints felt by teachers, the revised curriculum is not being implemented fully in the manner intended. Teachers emphasised that the solution is not to reduce the curriculum content, but rather to increase instructional time so that they can implement the curriculum using the appropriate methodologies. Ireland currently allocates a relatively low amount of time for mathematics instruction at Junior Cycle compared to the averages across OECD countries in both TIMSS and PISA. However, it is also important to ensure that available instructional time is used to maximum effect by, for example, ensuring that students establish links across mathematics strands, between mathematics and real-world contexts, and between mathematics and other subject areas.

7.5. The Curriculum and the Spectrum of Students' Abilities

This review identified an issue around the extent to which students across different levels of ability are being served by the current Junior Certificate mathematics curriculum and the teaching methodologies underpinning Project Maths. From the perspective of the teachers consulted, students on either end of the spectrum are being disadvantaged: those who have the most difficulty with mathematics, and those who perform at the highest levels in mathematics.

At present, students who have the most difficulty with mathematics have the option of taking the Foundation level examination at Junior Certificate. However, teachers reported that the teaching and learning of mathematics at this level is hampered by the absence of a specific syllabus, which means that teachers and students are unsure what is included. Teachers working with students who take the Foundation level examination view the difficulty as being intensified by a perceived lack of structure on the examination paper, the unpredictability of the paper, and the linguistic complexity of the examination questions (though it might be argued that these challenges can be addressed through instruction). Notwithstanding these perceived difficulties, teachers working with lower-achieving students emphasised that a Foundation level is needed at Junior Cycle.

The analysis of the Junior Certificate mathematics examination presented in Chapter 4 revealed that, in 2016, 5.0% of the Junior Certificate cohort opted for the Foundation level paper. This represents a decrease from 12.5% in 2003. However, the proportion of students who achieved below Grade D in Ordinary level mathematics in 2016 is relatively high (6.1% of the Ordinary level cohort or 1,450 students) and suggests that some students sitting the Ordinary level examination might be better suited to Foundation level. At Foundation level, the proportion of students achieving a grade below D has also increased, from 3.0% of that cohort in 2012 to 3.5% in 2016. Additionally, analysis of PISA 2015 data revealed that around one-third of the students who took Ordinary and Foundation level in the 2015 Junior Certificate performed below the baseline proficiency level (below Level 2) in PISA 2015 and hence demonstrated only very basic mathematics skills at best.

Teachers argued that they would like a specific syllabus to be introduced at Foundation level on the basis that, while students differ in their potential to achieve in mathematics, all students should be afforded the opportunity to maximise their potential. The issue is of particular concern in the context of upcoming reforms which will see students who would have taken Foundation level Junior Certificate examination in the past taking the Ordinary level paper. This measure could reduce the opportunity for a percentage of the cohort to achieve in mathematics (where achievement is viewed as success on a state examination). Although the proportion of students taking Foundation level mathematics has decreased over time, it is not clear why this has occurred. It is possible that the Project Maths initiative is implicated, at least in part, through, for example, increased student interest. However, other factors, such as parental influence and student perception may also play a role. It may be that, for the 5% or so of students who continue to sit the Foundation level mathematics examination, the broader range of assessments to be introduced at Junior Cycle, including Classroom-Based Assessments, will be of particular benefit. In this situation, there should be less emphasis on the final examination as the basis for a student's grade in mathematics.

A further issue for students experiencing the most difficulties with mathematics at Junior Cycle is that they are likely to find the transition to Ordinary level mathematics at Senior Cycle especially problematic. Teachers reported that the gap between the Junior and Leaving Certificate Ordinary level syllabi has increased since the introduction of revised syllabi under Project Maths and is now too large. Hence, students are not adequately prepared for the Ordinary level Leaving Certificate syllabus and those performing at the lowest levels might be expected to have the most difficulty. The gap between Ordinary level at Junior and Senior Cycles may be especially pertinent now that higher-performing students who would have taken Ordinary level at Junior Cycle in the past have moved up to Higher level. Lack of continuity in mathematics in Transition year in some contexts may also be a factor here.

Findings from this review also suggest that students who achieve at the highest levels in mathematics are not being adequately challenged at Junior Cycle at present. It is clear from the PISA and TIMSS international assessments that, compared with other countries with similar average performance, Ireland has a smaller percentage of students performing at the top level in mathematics. Additionally, the highest-achieving students in Ireland perform less well on mathematics content and processes in comparison to the highest-achieving students in countries with similar levels of average performance in PISA and TIMSS.

From the perspective of Focus Group teachers, students who are higher achievers in mathematics are not being adequately challenged across the three years of Junior Cycle mathematics. Teachers attributed much of this to an increase in mixed-ability classes (and also in some cases to large classes). Teachers pointed out that First year classes are not streamed for the CIC, and that Second and Third year classes are more mixed-ability than before due to the increase in students taking Higher level mathematics. In their view, the pace of teaching is slower in mixed ability classes and the time available for methodologies that include reflection and evaluation is restricted (especially in larger classes). Some teachers described aiming to teach to the middle level of ability in mixed-ability classes, but noted that a lot of classroom time is spent on the students with the most difficulties. In this perspective, students who are higher achievers may become bored and disengage (and students with the most difficulties may struggle to keep-up). However, there is an onus on schools and teachers to ensure that all students benefit from instructional activities that are set at an appropriate level. Moreover, as noted elsewhere, developments in areas of ICTs and assessment could be drawn on to address some of these concerns.

International studies have also provided valuable insights in the attitudes of students in Ireland towards mathematics (Chapter 3). In particular, data from TIMSS 2015 indicate that substantial percentages of students in Ireland lack confidence in mathematics, do not like learning mathematics, and do not look forward to mathematics classes. However, high performers in mathematics tend to be more confident in mathematics, and students who are more confident in mathematics also tend to like learning it. Students who like learning mathematics also tend to report experiencing engaging teaching in mathematics class.

Across Focus Groups, teachers suggested that some kind of *'two-tiered'* approach to mathematics would cater for students across the spectrum of abilities: *'Maths for Life'* for those needing basic skills and *'Pure Maths'* for those who will be engaging with mathematics in a deeper way in tertiary level education.

Implications

In the absence of the option of a specific Foundation level syllabus or examination, consideration should be given to measures which support those experiencing the most difficulty with mathematics at Junior Cycle, who, in the future, will sit the Ordinary level Junior Certificate examination. In the context of upcoming reforms, and in line with DES targets to reduce the percentage of students performing at the lowest proficiency levels in PISA (DES, 2017), interventions should target those whose abilities are at the lowest levels, even though the proportion of students at this level in Ireland is relatively low when compared with averages across OECD countries.

Given the influence of parents on students' decisions to study mathematics at Ordinary or Higher level, interventions should target parents with a view to managing their expectations and to reducing any stigma associated with studying mathematics at any level other than Higher level. To mitigate against future negative impacts on performance, there is a need to establish clear lines of continuity between the Junior and Leaving Certificate syllabi, ensuring that one flows logically to the next. The forthcoming revision of the Junior Certificate mathematics specification provides an opportunity for this.

Data from international studies on teaching and learning (see Chapter 5) suggest there is scope to enhance the use of strategies viewed as consistent with Project Maths (e.g., classroom discussion among students, and asking students to decide on their own problem-solving strategies). Of particular note is the low level of student-oriented instruction (e.g., asking students to plan classroom activities or projects) in mathematics classes reported by students in Ireland in PISA 2012. This suggests that there is scope to increase the engagement of students in activities such as small group work and project work, and to improve differentiation and student choice. Also of note is the limited availability and use of ICTs in the classroom for mathematics teaching and learning in Ireland.

Given teachers' concerns about the broader range of ability in their mathematics classes, additional support is required to develop teachers' pedagogical content knowledge to support differentiation in teaching mixed-ability classes, and to increase the engagement of students in activities such as small group work and project work, and in making choices in relation to such work.

Measures to provide more opportunities for students who are higher achievers to reach their potential in mathematics should include increased instructional time for mathematics, enhanced differentiation within class, increased use of group work, increased student-oriented instruction, and activities to extend mathematics learning outside the classroom. Additionally, consideration should be given to how best to enhance students' enjoyment of mathematics learning. Areas of specific weakness among higher-achieving students in Ireland (Change & Relationships/Algebra and Shape & Space/Geometry) also need to be targeted.

For all students, there is a need to harness the potential of ICTs for mathematics teaching and learning. In the context of developing a new specification for mathematics in Junior Cycle, the role of ICTs in the teaching and learning of mathematics needs to be strengthened, and measures outlined elsewhere in relation to ICTs in teaching and learning (e.g., DES, 2015b, 2016) need to be put into practice. Among the outcomes that this could give rise to are: a stronger focus on meeting the needs of all students, including higher-achieving students; an increase in the allocation of time to mathematics (especially if students engage in more 'self-learning' outside of classroom time); and a greater onus on students to accept responsibility for their progress in mathematics.

The implementation of newer approaches to assessment can also be expected to help in addressing the diverse needs of students studying Junior Certificate mathematics. These include use of a wide variety of formative and summative assessment methods, a stronger focus on the role of feedback and discussion in mathematics learning, and improved reporting so that students and their parents better understand their progress and ability in mathematics. Understanding of students' mathematics performance can also be enhanced by use of the Junior Certificate Profile of Achievement (JCPA), which will recognise the full spectrum of mathematics achievement through Classroom-Based Assessments, short courses and participation in co-curricular activities that involve mathematics.

7.6 . Literacy and Mathematics Learning at Junior Cycle

A further consideration centres on the literacy levels required for mathematics at Junior Cycle and the related impact on students' problem-solving abilities and on performance. Teachers expressed concern that students can be disadvantaged by the literacy levels required for some mathematics problems and examination questions. In particular, teachers described an increase in the linguistic complexity of questions that is causing frustration for some students and can result in a loss of confidence. It was noted that students who have the most difficulty with mathematics, in particular, may not grasp what is required within a problem and hence may not persevere with it. Teachers reported that EAL students are also disadvantaged with regard to topic recognition. However, teachers noted that even students who are higher achievers in mathematics can become frustrated by *'long wordy questions'*. It was also argued that some students do better with more *'wordy'* questions, and others with more abstract ones.

Teachers' concerns are borne out to some extent in findings from the analysis of Junior Certificate examination papers described in Chapter 4, which revealed that, between 2003 and 2015, the volume of material to be read increased substantially on examination papers at all levels (Higher, Ordinary and Foundation) (Cunningham et al. 2016). The study also found that, while the difficulty of the embedded vocabulary, sentence length, and readability levels of the papers at Higher level and Ordinary level did not change, at Foundation level, there was an increase in the readability level from Grade 0 (pre-reading) to Grade 2. Hence, over time, greater demands have been put on all students in terms of the volume of material to be read on the examination papers. In addition, greater demands have been put on students who have the most difficulty with mathematics (who are likely to take the Foundation level paper) in terms of the linguistic complexity of the examination questions. It is worth noting that the study also found that the Junior Certificate examination places less reading demands on students than does PISA. Nonetheless, the findings should be considered in future preparation of Junior Certificate examination papers. It is also possible that, over time, students' ability to solve problem embedded in context will improve. Such questions are relatively new in Ireland, especially at Higher level.

It is also notable that mathematics vocabulary among students in Ireland may be less extensive than among students in OECD countries on average (see Chapter 5). In PISA 2012, fewer students in Ireland than on average across OECD countries indicated that they were familiar (knew the concepts well or had heard of them often) with mathematical terms such as exponential functions, polygons, congruent figures, arithmetic means, divisors, complex numbers and probability. In addition, students with lower socioeconomic status were less familiar with the majority of terms than were students with higher socioeconomic status. While this suggests that a stronger emphasis on mathematics vocabulary is warranted, the OECD notes that exposure to terminology alone is not enough; rather students need extensive exposure to problems that ‘stimulate their reasoning abilities and promote conceptual understanding, creativity and problem-solving skills’ (OECD, 2016b, p.3). It is positive to note that some teachers interviewed as part of this review reported observing improvements in mathematics vocabulary among their students following the implementation of Project Maths.

This issue of literacy further raises the question of the extent to which cross-curricular linkages are supporting students’ mathematics learning. Indeed, teachers noted that students can be confused by inconsistencies in wording (e.g., units of measurement) across subjects. As discussed in Chapter 2, increased effort should be made to integrate concepts across subjects such as geography, history and science, and to ensure consistency of terminology across subjects. This could benefit all students in, for example, Space and Shape, where performance on PISA in Ireland is weak, especially among female students.

A final point that needs to be highlighted relates to the view expressed in Focus Groups that many students do not have the skills needed to formulate answers to examination questions that require written explanations. This is of particular concern in the context of the common learning outcomes across syllabus strands that expect students will be able to explain findings, justify conclusions, and communicate mathematics verbally and in written form. Interestingly, based on teachers’ reports in TIMSS 2015, some 13.6% of students are taught by teachers who never ask them to explain their answers, and a further 26.1% are asked to do so only in some classes (Chapter 5). Again, teachers cited a lack of instructional time as obstructing the use of desirable methodologies and the opportunities for students to develop the deep level of understanding needed to answer questions in this way.

Implications

There is a need to clarify expectations in relation to the role of literacy in teaching and learning mathematics, and teachers should be supported in helping students to formulate explanations for answers (i.e., communication and justification). There would be value in emphasising the importance of teaching the vocabulary of mathematics, including the labels for key concepts, as opportunities arise during teaching and learning. Some of the impetus for this can come from dialogic interaction between teachers and students, where students are supported in using mathematical vocabulary in a range of contexts. Greater consistency should be achieved in the use of mathematical terminology, including units of measurement, across curriculum areas.

7.7. Establishing Greater Continuity between Primary and Junior Cycle Mathematics

The question of the adequacy of the Common Introductory Course (CIC) as a framework for bridging the gap between the primary mathematics curriculum and Junior Cycle mathematics arose in the course of this review. The issue emerged primarily from Focus Group interviews with teachers, which revealed that teachers are divided on their thoughts and feelings about the CIC. Comments uncovered both very positive *(‘excellent’)* and very negative *(‘a waste of time’)* opinions among teachers.

The CIC sets forth the minimum mathematics content to be covered with all students in First year (DES/NCCA, 2013). In scope, it revisits the topics covered in Fifth and Sixth classes in primary school, while touching on all five strands in the Junior Certificate mathematics syllabus. Upon completion, teachers may ‘extend or explore to a greater depth’ (p. 33) topics which they feel will benefit the student group in question, based on the progress of the group. It is intended that the CIC will ‘lay the foundation for conceptual understanding which learners can build on subsequently’ (p. 33). The CIC gives teachers discretion to decide on the order in which topics are introduced. Teachers are encouraged to see the course as a whole, making connections between topics as appropriate.

Teachers are also urged to use strategies which encourage the development of students' synthesis and problem-solving skills.

The positive comments from teachers generally concerned the capacity of the CIC to touch on all syllabus strands and the scope for teachers to add in additional topics according to the needs of the class. Teachers further identified the CIC as an opportunity to see where individual students' strengths lie and hence to identify those in need of additional supports. Much of the negativity around the CIC stemmed from the mixed-ability nature of First year classes. Teachers reported spending a lot of time on basic numeracy skills with First year students because, in their experience, many are coming from primary school without the mathematical skills necessary to engage in Junior Cycle mathematics. Teachers also felt that students with greater abilities in mathematics are not adequately challenged by the CIC and can become bored, though the provision of more challenging should offset this. It was also reported that some students find the CIC easy and, as a result, can develop unrealistic expectations about their abilities, and then struggle in Second year. Teachers also felt that the gap between the CIC and Second year Higher level mathematics is too wide, notwithstanding the fact that teachers have the autonomy to increase the attention given to more difficult topics in First year, on a needs basis.

It became obvious from teachers' discussions that there is variation in how the CIC is approached across schools and sometimes across classes within schools that have more than one First year class. Teachers reported sampling, adapting and adding to the CIC in different ways. For example, a teacher described postponing co-ordinate geometry until Second year on the basis that students would not retain what they had learned about this topic in First year. From the perspective of teachers more generally, such variation is problematic, as it can lead to 'unfairness' when decisions are being made about access to Higher level mathematics in Second year, for example.

As set forth in the Junior Certificate syllabus, the CIC specifies the minimum mathematics content that has to be covered by all students, while providing flexibility for teachers to cover topics in more detail. The CIC places the onus on teachers to ensure that First year mathematics provides enough challenge for students at all levels. Teachers are also tasked with ensuring those with the most difficulties in mathematics have the skills needed to engage with Ordinary level mathematics in Second year and that those with the greatest abilities in mathematics are adequately prepared for Higher level mathematics. Given the mixed-ability nature of First year classes, teachers must differentiate for these student groups. However, teachers highlighted barriers such as insufficient instructional time and large classes. In the presence of such barriers, there may be potential for teaching in smaller groups of similar ability within class, a practice that seems to be underutilised in Second year based on TIMSS 2015 data (Chapter 5).

It is worth noting again that the curriculum states that 240 hours are required for mathematics in Junior Cycle, which averages as 80 hours in each year of the Junior Cycle. TIMSS 2015 indicates that an average of 109.7 hours were allocated to mathematics in Second year in 2015; however, data are not available on the average hourly allocation in First year (PISA provides data for Third year). It is possible that mathematics in First year is being restricted in the context of the 'carousel model' of subject sampling that is practiced in some schools. However, it is important to ensure that mathematics is prioritised on First year timetables to ensure that students have the foundations needed for mathematics in subsequent years.

Implications

Given the variety in approaches to First year mathematics described by teachers, it would be beneficial to clarify in greater detail the scope and function of the CIC and the flexibility afforded to teachers to cover material in greater breadth and depth on its completion in First year. Teachers should be supported in planning to better meet the mathematical needs of the students in their classes using strategies such as differentiation, group work and project work, and in striking an appropriate balance between Number and Statistics & Probability, on the one hand, and other content areas, on the other hand.

7.8. Assessment of Mathematics at Junior Cycle

Assessment as it relates to the mathematics performance covers a broad range of issues including the structure and content of tests and other forms of assessment at classroom, school, national and international levels, as well as the outcomes of such assessments. Some of these are considered elsewhere in this report. This section focuses on formative and summative assessment as they relate to Junior Certificate mathematics. The issue of the readability of mathematics questions was addressed in an earlier section.

The finding by Echazarra et al. (2016) (reported in Chapter 5) that instruction in mathematics classes in Ireland (as reported by students in PISA 2012) is characterised by a strong emphasis on formative assessment needs to be interpreted with care. PISA characterises formative assessment as involving teachers telling students what is expected of them when they get a test or assignment, telling them how well they are doing in mathematics classes, and telling them what to do to get better. It does not include the frequency or quality of the observations made by teachers, whether outcome of such observations are documented, whether students are supported in reflecting on and communicating their own learning, and whether students are assigned teacher-made tasks that are designed to assess their reasoning and other aspects of higher-level thinking. These activities are identified in the Junior Cycle Framework (DES, 2015) as key aspects of formative assessment across all subject areas, and need to be built into the assessment component of the new specification for mathematics in Junior Cycle.

The Junior Cycle Framework also proposes the introduction of two Classroom-Based Assessments in all subject areas, including mathematics. Administered by schools, on the basis of guidelines to be provided by the NCCA, and assessed by the students' teachers to generate summative descriptors of performance, such assessments are also viewed as an opportunity for teachers to provide formative feedback to students. While some teachers in our Focus Group interviews did not engage in the discussion around these assessments[36], others suggested a focus on statistics, and the provision of problems with multiple solutions and varying levels of credit, depending on the sophistication of students' solutions. Such problems can be described as having multiple entry points and multiple representations.

Classroom-Based Assessments may also provide an opportunity for students to engage in solving complex problems that cannot be solved in the context of a short examination paper. The process of mathematical modelling (translating between the real-world problems and mathematics in both

[36] The non-engagement of some teachers on topics related to assessment may have been linked to a dispute involving one of the teacher trade unions, which had told members not to co-operate with the implementation and scoring of Classroom-Based Assessments in English.

directions) (e.g., Doerr & English, 2003) could be promoted in the context of solving such problems.[37] These assessments, and the types of instruction they encourage, may lead to more students working in small groups, and undertaking projects that require at least one week to complete (activities described by PISA as compatible with 'student-orientated instruction').

In our Focus Group meetings, teachers were generally concerned about the introduction of a single two-hour examination, as per the Junior Cycle Framework (DES, 2015a). Although it was acknowledged that a two-hour exam might reduce the pressure on students, concern was expressed that students might not be adequately prepared to undertake two longer papers at Leaving Certificate if they not have the experience of two papers at Junior Certificate. Several teachers raised the question of content coverage, and seemed unconvinced that it would be appropriate to sample from course content to a much greater extent that is currently the case. A few felt that the re-introduction of choice should be considered in the context of a shorter paper, to ensure that students weren't unduly disadvantaged in the context of a more limited set of topics, though this, in turn, could result in greater gaps between, for example, Junior Certificate and Leaving Certificate courses.

A number of teachers called on examiners to indicate the number of marks available for each item, in addition to, or instead of, an indication of the time that should be allocated to each item. Teachers of students who typically take Junior Certificate mathematics at Foundation level expressed concerns that a common paper (for Ordinary and Foundation levels) might be too difficult for these students.

Currently, there is a clear shift towards computer-based assessment (CBA) in international assessments, with PISA having already implemented CBA in most participating countries in 2015, and TIMSS scheduled to introduce CBA in its 2019 assessment. In this context, and considering the broader use of technology in a range of real-life contexts, there is a case to be made for introducing CBA as part of assessment at Junior Cycle. This could involve administration of examinations via computer at some point in the future, but there are other possibilities in the shorter term. For example, Classroom-Based Assessments, and/or Assessment Tasks linked to the Junior Certificate examination[38] could be completed or administered using CBA. Computer-based tasks could ask students to solve problems by constructing or evaluating mathematical models. Students could be asked to demonstrate their problem-solving skills by presenting outcomes using technology (spreadsheets, interactive charts, tables etc.). Potential uses of technology for assessment (and, as discussed elsewhere, for teaching and learning), should be considered in the course of syllabus review.

Implications

Curriculum revision in mathematics at Junior Cycle should seek to strengthen the use of formative assessment, and such assessment should be an integral part of Classroom-Based Assessments. The latter should be developed in a way that allows students with varying levels of achievement to access

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[37] Lesh and Doerr (2003) describe such models as products that go beyond short answers, and include conceptual tools for constructing, explaining, predicting and controlling mathematically-significant systems. They add that such models can include quantifying, dimensioning, categorising, algebraizing and systematising relevant objects, relationships, actions, patterns, and regularities.

[38] The Framework for Junior Cycle (DES, 2015) envisages that students will complete two Classroom-Based Assessments in most subjects in Second and Third years, an Assessment Task in Third year, and a final examination. The Classroom-Based Assessment tasks would be marked by students' teachers, and outcomes will be recorded in the form of descriptors on a student's composite Junior Certificate Profile of Achievement, while the Assessment Task would be marked by the State Examinations Commission, and would contribute up to 10% of a student's final grade in mathematics. It is intended that the Classroom-Based Assessments will be set at a common level, and that scoring rubrics, identifying varying quality in students' work, will be provided to teachers by the NCCA.

them and demonstrate their mathematical proficiency. They should involve complex real-life problems that are broader in scope than what students typically encounter in examination contexts, and students should be encouraged to use mathematical modelling as they arrive at and describe their solutions. The Assessment Tasks should be viewed as an opportunity for students to demonstrate aspects of their learning (such as the interpretation of their findings) that may not be possible during a two-hour examination designed to assess a broad range of content.

There should be a clear understanding of how content for the new two-hour examination is selected, and the examination should be accessible to students who currently take separate papers at Ordinary and Foundation levels.

It would be important for teachers and students to view teaching and learning in mathematics as important processes in their own right, rather than as preparation for state examinations. This might reduce the pressure on students and encourage them to draw on a broader range of learning strategies that are consistent with Project Maths.

7.9. Attitudes towards Mathematics among Junior Cycle Students

An important issue arising from this review is the attitudes of students in Ireland towards mathematics. Of particular concern are the substantial proportions of Junior Cycle students who have negative attitudes towards learning mathematics, and the implications for mathematics performance, given the strong associations between attitudes and performance.

Focus Group teachers reported that, in the context of the revised syllabus introduced under Project Maths, students are more interested in and engaged with mathematics than before. In TIMSS 2015, 61.3% of students in Ireland reported that they like mathematics. However, in the same study, just 48.3% reported that they like learning mathematics (Chapter 3). Students who liked learning mathematics scored significantly higher on TIMSS mathematics than those who did not like learning mathematics.

The findings from TIMSS suggest that some students may have an issue with mathematics learning more so than with mathematics itself. Indeed, in the same study, over one-half of Second year students (57.5%) in Ireland reported that they find mathematics boring and over one-third (37.1%) disagreed that they learn many interesting things in mathematics. Additionally, a substantial percentage of students (45.4%) disagreed that their mathematics teacher gives them interesting things to do. In the absence of *recent* comparative data from an earlier (pre-Project Maths) cycle of TIMSS, it is difficult to say what impact if any Project Maths has had on students' liking of mathematics learning. Similar proportions of students in TIMSS 2015 and PISA 2012 reported that they do not look forward to mathematics class (61.8% in TIMSS 2015 and 59.8% in PISA 2012). These data suggest that there continues to be scope for increasing students' enjoyment of mathematics learning.

Teachers reported that students enjoy the learning and teaching methodologies encouraged by the Project Maths Development Team, but, as described earlier, teachers can have difficulty implementing them due to time constraints, as well as constraints imposed by exams. Interestingly, teachers reported that these methodologies are easier to implement with Statistics & Probability (the strand generally favoured by students)[39] and more difficult to implement with Algebra and Geometry (strands that students find most difficult). It appears from this review that there is scope for teaching more of the curriculum *'in the spirit of Project Maths'*, and using methodologies that appeal to students. For

[39] It is worth noting that Statistics & Probability is the strand that teachers would have had least experience with, and hence might not be viewing it through the lens of the old syllabus and more established methodologies.

example, findings from TIMSS 2015 suggest there is potential for wider use of general teaching practices which promote conceptual understanding, such as relating class content to students' daily lives and encouraging classroom discussions among students (Chapter 5).

It is noteworthy that the majority of students in Ireland (77.7%) in TIMSS 2015 are classified as experiencing engaging or very engaging teaching (e.g., they report that their teacher listens to what they have to say, and their teacher does a variety of things to help them learn) in mathematics classes. However, around one-in-five students are classified as experiencing less than engaging teaching in mathematics classes. Analyses of the TIMSS 2015 data showed that students' who report experiencing engaging teaching in mathematics classes also report that they like learning mathematics. While it is possible that students who like learning mathematics are more likely to perceive the mathematics teaching they experience as engaging, it is also possible that experiencing engaging teaching encourages a liking of mathematics learning among students.

The teachers consulted also reported that Junior Cycle students respond well to the real-world practicality and problem solving. However, data from international assessments indicate a considerable percentage of students in Ireland do not like solving mathematical problems: 53.8% of students in TIMSS 2015 indicated that they do not like to 'solve mathematics problems'; and 29.8% of students in PISA 2012 reported that they do not like to 'solve complex problems'. Notably, both studies indicated that, compared to the OECD averages, students in Ireland are given less opportunity for deciding their own problem-solving procedures in class (Chapter 5). It is also worth noting that a large percentage of students in Ireland (42.8%) can be classified as not confident in mathematics and a minority (15.7%) as very confident, based on TIMSS 2015. Students who are very confident or confident in mathematics score significantly higher on TIMSS mathematics than those who are not confident in mathematics. High levels of mathematics anxiety have also been observed among students in Ireland, with 35.0% of students in TIMSS 2015 reporting that mathematics makes them nervous, and 29.7% of students in PISA 2012 reporting that they get very nervous doing mathematics problems. Hence, there is scope for enhancing mathematics confidence among Junior Cycle students and for lowering their mathematics anxiety.

Teachers reported that, at times, students can have unrealistic expectations about what they can achieve in mathematics. They attributed this in some cases to a lack of challenge during First year, and also to the influence of parents who may want their child to take Higher level mathematics, whether or not they have the ability. Parents' concerns can centre on the bonus points for Higher level mathematics at Leaving Certificate, but can also reflect perceived stigma around their child taking mathematics exams at Ordinary or Foundation levels. As described previously, teachers felt the bonus points awarded for Higher level mathematics at Leaving Certificate attract students who might otherwise opt for Ordinary level. Teachers emphasised that students who do not have the ability to engage with Higher level mathematics are at risk of developing negative attitudes towards mathematics. Findings from PISA 2012 also indicate that students may respond differently to failure; female students score higher than male students on mathematics self-responsibility, indicating that they are more likely to blame themselves for failures in mathematics than to blame other factors such as teachers or textbooks. It should be acknowledged that students in Ireland place a high value on mathematics learning, as in both PISA 2012 and TIMSS 2015, Ireland's score on the valuing mathematics scales were significantly higher than the corresponding OECD averages. Interestingly, however, data from TIMSS 2015 also show that many students in Ireland do not want a job that involves using mathematics.

TIMMS 2015 and PISA 2012 both indicate that, compared to female students, male students in Ireland place greater instrumental value on mathematics, but do not differ from female students in their overall liking of mathematics learning. In TIMSS 2015, similar proportions of male (57.5%) and female (57.1%) students in Ireland reported finding mathematics boring, but a greater percentage of male students (66.0%) than female students (59.9%) agreed that they learn interesting things in mathematics. It is worth noting that some teachers consulted felt that the curriculum is somewhat more oriented to male students than female students – firstly because of the kinds of examples used, and secondly, because, in their opinion, female students are more inclined to rote learning. Additionally, PISA 2012 provided evidence suggesting that male students are more open to problem solving than female students. In Focus Groups, teachers described female students as less confident and more anxious about mathematics than male students. This is borne out to some extent in the achievement data from both PISA 2012 and TIMSS 2015 (Chapter 3). The view was also expressed in Focus Group interviews that, compared to male students, female students tend to be more concerned with grades. Indeed, in PISA 2012 a greater percentage of female students (69.4%) than male students (55.0%) reported worrying about getting poor grades in mathematics. These gender differences are of particular interest in the context of a larger than average gender difference on PISA 2015 mathematics in Ireland than in any PISA cycle since 2000, and the lower representation of females among higher achievers in mathematics in PISA and TIMSS (Chapter 2). However, it should also be noted that female students outperform male students in Junior Certificate mathematics, especially at Higher level, though not at the highest grade (A) (Chapter 4).

Findings of this review also revealed another group of students that would benefit from specific interventions to improve their attitudes to mathematics. In TIMSS 2015, students from lower socioeconomic backgrounds scored lower on average than other students on liking learning mathematics, valuing mathematics and mathematics confidence (Chapter 3). Also, data indicated that students in SSP schools under DEIS like learning mathematics less and have lower mathematics confidence than their counterparts in non-SSP schools.

The analyses of TIMSS data in Chapter 3 show that on average Second year students in Ireland have greater intrinsic and instrumental motivation for mathematics learning than students on average across the 16 participating OECD countries. Additionally, these students do not differ from the students in the OECD-16 on average in terms of their mathematics confidence or the extent to which they report experiencing engaging teaching in mathematics classes. In Ireland, students' instrumental value of mathematics is not strongly related to performance in mathematics. However, students' liking of mathematics and students' confidence in mathematics are both positively associated with performance in mathematics. A strong association is also observed between students liking mathematics and students feeling confident in mathematics. Students who report experiencing engaging teaching in mathematics class tend to like mathematics and to feel confident in mathematics.

Implications

Consideration should be given to the findings of the current review that relate to students' attitudes towards mathematics. These indicate a need to continue to promote positive attitudes to mathematics among Junior Cycle students. Interventions which aim to enhance students' experience in mathematics class, to increase their liking of learning mathematics, and to enhance their mathematics confidence will be especially beneficial. Female students and students from lower socioeconomic backgrounds, in particular, will benefit from such interventions. More opportunities could be provided for students to engage in mathematics activities (such as mathematics clubs and competitions) inside and outside the classroom. In the classroom, students will benefit from the increased use of engaging teaching practices and from ensuring that students can participate in activities that interest them. Additionally, the contexts of mathematics problems used in class, in examinations and in mathematics resources should be made relevant to students' lives. The interest and enjoyment factors are also something to consider in the design of Classroom-Based Assessments. The potential of ICTs to enhance students' enjoyment of learning mathematics should be explored and maximised. The claim, by some Focus Group teachers, that the mathematics curriculum is more orientated to male than to female students is worth further investigation.

7.10. Professional Development for Junior Cycle Mathematics Teachers

As described in Chapter 5, a relatively high level of participation in professional development was reported by Junior Cycle mathematics teachers in recent years, and relatively few teachers had not availed of professional development. For example, in TIMSS 2015, 13.5% of Second year students were being taught by teachers who had attended fewer than six formal hours of professional development in the two years prior to TIMSS, compared with an OECD average of 35.7%. As discussed, it is likely that the high level of participation reported is linked to activities associated with the implementation of Project Maths, including those organised by the Project Maths Development Team, and the participation of some teachers in the Professional Diploma in Mathematics for Teaching (see Chapter 1).

TIMSS 2015 provided information on the kinds of development activities engaged in by mathematics teachers (Chapter 5). For example, in the two years prior to testing, the vast majority (over three-quarters) of students were taught by teachers who participated in professional development relating to mathematics content, pedagogy and curriculum. Fewer than half of students were taught by teachers who had participated in professional development related to addressing students' individual needs, or to assessment of mathematics. Around three-quarters of students were taught by teachers who had participated in development activities focused on integrating ICTs into mathematics. Overall, teachers in Ireland reported higher levels of participation in these professional development activities than did teachers on average across the 16 OECD countries in TIMSS.

Data from TIMMS 2015 also indicate that mathematics teachers in Ireland have high levels of confidence in a range of teaching activities such as assessing student comprehension of mathematics, and making mathematics relevant to students (Chapter 5). They also have higher confidence than teachers on average across the OECD-16 for most activities, with the exception is showing students a variety of problem-solving strategies. However, TIMSS also shows that around 20% to 25% of students have teachers who are not fully confident where activities such as developing students' higher-order thinking skills, providing challenging tasks for the highest-achieving students, and adapting teaching to engage students' interest are concerned (Chapter 5). In Focus Group interviews, some teachers reported that they lacked confidence in the use of ICTs for mathematics teaching and learning, though others seemed very confident.

The mathematics teachers consulted as part of this review felt they had benefitted from the development activities in which they had engaged since the implementation of Project Maths, and they highlighted the initial 10 sessions in particular. Teachers reported that they would like more frequent and ongoing professional development around Project Maths and would like the 10 initial sessions to be repeated in some form, though relevant materials are currently available on the website of the Project Maths Development Team.

Findings of this review suggest other future areas of emphasis for professional development for Junior Cycle mathematics teachers. In particular, the findings point towards activities supporting and reinforcing the use of methodologies consistent with Project Maths and towards strengthening teachers' confidence in their application.

A highly important area is professional development focused on the integration of ICTs into mathematics teaching and learning. Although 65.2% of students in TIMSS had teachers that participated in professional development activity in this area (compared to an OECD-16 average of 45.1%), it was clear in Focus Group discussions that some teachers lack confidence in the use of ICTs for mathematics teaching and learning and that others are unable to maximise its potential. A key issue that emerged in this review is that, where ICTs are being used in the mathematics classroom, they tend to be used for demonstration by teachers, rather than for exploration by students. It is clear that there is significant scope for enhancing the use of ICTs for mathematics, and that teachers need additional support in this area. It must also be acknowledged that some teachers are precluded from using ICTs effectively by a lack of infrastructure or intense competition for available infrastructure in their schools.

Teachers indicated that they value development opportunities that involve collective participation by mathematics teachers. However, mathematics teachers in Ireland have relatively little interaction with other teachers in development activities. In particular, it is uncommon for teachers to work together to try out new ideas, and it is especially uncommon for teachers to visit other classrooms to learn more about teaching. Hence, this is an area where improvements could be made. Increasing opportunities for teachers to interact is consistent with the literature on teacher professional development that shows collective participation and active learning as elements of effective teacher professional development (Weir, Kavanagh, Kelleher, & Moran, in preparation). Teachers' concerns about the amount of time that they are expected to allocate to Lesson Study need to be considered.

Implications

Future professional development in mathematics for Junior Cycle teachers should continue to focus on content and pedagogical content areas, particularly where teachers lack confidence and where students display weakness (e.g., differentiating instruction, working in small groups, visualisation and spatial reasoning, mathematical modelling, algebra). In addition, teachers are likely to benefit from activities focused around the curriculum as a whole; notably, reinforcing and developing teachers' capacity to make interconnections across syllabus strands. Further areas that might underpin development activities are strengthening teachers' confidence and reinforcing their capacity to make professional judgements on students' performance and learning needs.

7.11. Future Research

Although the purpose of the current report was to evaluate the impact of Project Maths on the performance of students in Junior Cycle mathematics, it was not possible to draw on comparable(pre- vs. post intervention) data in areas like achievement, attitudes and approaches to teaching. Instead, the study drew on existing international data sets that were not designed specifically to evaluate the impact of Project Maths.

The lack of data on performance might seem surprising, given Ireland's participation in PISA and TIMSS in 2015. However, as noted earlier, the use of PISA 2015 data to draw inferences about performance is difficult because of the transition to computer-based assessment in PISA. Hence, difference between 2015 and earlier cycles, when students completed PISA on paper, may be due to the transition to computer rather than to the effects of Project Maths. PISA is also problematic to the extent that students in Transition and Fifth years are included in the sample, as well as students in Second and Third years. Hence, performance on PISA might be affected by the instructional activities in Transition Year as well as those implemented in the context of Project Maths at Junior Cycle. In the case of TIMSS, a difficulty arises because no recent data are available – the last time Ireland participated in TIMSS was in 2015.

The use of examination results to make inferences about the effects of Project Maths on performance is also problematic in that both questions and marking schemes change from year to year.

Future evaluations of curricula might draw on custom-made tests and interest inventories that can be administered to equivalent samples when a revised syllabus is launched for the first time, and again after it has been in place for a number of years. This would allow for a more systematic appraisal of impact than is possible using data from international studies.

The current study was also restricted in the data it had access to on teaching and learning. Data were available from PISA 2012, when students responded to questionnaire items about the teaching and learning they experienced. Data were also available via the Teacher Questionnaire administered to teachers of students in Second year in TIMSS 2015. While it was possible to benchmark the data from these students against OECD average scores, and combine them with observations made by teachers during the Focus Group interviews, no recent (pre-Project Maths) data on actual teaching and learning in classrooms were available.

Implication

Going forward, it would seem important to generate data based on observations of students engaged in teaching and learning situations, along the lines of the Lyons et al. (2003) study or the TIMSS video studies, in which students were videotaped as they engaged in mathematics lessons, and inferences are made about the instructional processes that are observed. Such data might serve to validate questionnaire data, and, if gathered on more than one occasion, could be used to track changes over time. It would also be useful to examine the effects of Lesson Study and other approaches to professional development in terms of their impact on teaching and learning in classroom settings. Regular participation in international studies should provide more solid trend data than were available in the past.

7.12 Conclusion

Based on currently-available data, it can be concluded that the Project Maths has had a small positive impact on student performance in mathematics, as measured by the PISA and TIMSS studies. It is also clear that Project Maths has had a significant impact on approaches to teaching and learning mathematics in schools and on students' attitudes towards mathematics. Nevertheless, there are clear challenges that must be addressed if the progress achieved to date is to be built on. The development of a new curriculum specification for Junior Cycle mathematics provides a timely opportunity to build on areas of strength in students' mathematical learning, and address areas of weakness. The development of the new curriculum also provides an opportunity to integrate new approaches to assessment that arise from the Junior Cycle Framework.

From a performance perspective, it is clear that Junior Cycle students do well on content areas such as Number in TIMSS and Uncertainty & Data in PISA, and that performance on other areas, such as Space & Shape in PISA, and, to a lesser extent, Algebra and Geometry in TIMSS, needs to be improved. Indeed, it is difficult to see how performance on PISA can improve substantially without addressing weaknesses in Space & Shape. In addition to considering a stronger focus on visualisation and spatial reasoning, there is a need for reconsider the focus on synthetic geometry and proof in the current syllabus, and identify ways in which the teaching of Geometry can be brought into line with other aspects of syllabus. There is a need to ensure continuity between Algebra at Junior Certificate and

Leaving Certificate Ordinary levels, both in in terms of syllabus content and teaching approach. There is also a need to ensure that high-achieving students in Ireland do better, as they have lagged behind their peers in countries that perform at the same overall level as Ireland in studies like PISA and TIMSS.

From a teacher perspective, it is clear that implementation of the mathematics syllabus under Project Maths has led to substantial changes in teaching methodologies, though for some aspects (such as Statistics & Probability, Number, Problem-solving) more than others (Algebra, Geometry & Trigonometry). Lack of instructional time was cited by teachers as a major impediment to the implementation of the syllabus in line with the methodologies espoused by Project Maths. Given that average annual instructional time in Second and Third years is low relative to other OECD countries, there is a need to identify ways in which instructional time can be increased. Greater flexibility and differentiation in presenting the Common Introductory Course could also ensure that the best use is made of available instructional time. Greater integration across content areas could also lead to enhanced learning. While some aspects of mathematics instruction may change as a result of changes to the curriculum (though teachers generally felt that substantial revision was not required at this time), others will emerge as new teaching and assessment approaches are introduced or reinforced. There is evidence from national and international studies that traditional approaches to teaching and learning (memorisation, teacher-directed instruction, an over-emphasis in preparing students to sit examinations) were widely used prior to Project Maths. TIMSS 2015 data, as well as reports by Focus Group teachers, suggest that considerable progress has been made in introducing more meaning-orientated approaches. However, there continues to be scope for development, including a stronger emphasis on student-oriented instruction directed at the needs of students with varying levels of ability. Some of these issues can be addressed through professional development, including stronger collaboration among teachers at school-level. Teacher confidence in areas such as adapting teaching to engage students' interest, providing challenging tasks for higher-achieving students, and developing students' higher-order thinking skills also need to improve. There is a clear need to ensure that both teachers and students capitalise on the affordances of ICTs.

From a student perspective, reports of Focus Group teachers and students' responses in TIMSS 2015 indicate that a majority of students like mathematics and are enthused about learning it. However, just over 40% of Second years in TIMSS view mathematics as one of their favourite subjects, while just over 50% like solving maths problems. Moreover, students in Ireland are broadly similar to students on average across OECD countries in their liking of mathematics, their confidence in their own mathematics, and their liking of learning mathematics, while they are slightly higher in the extent to which they value and recognise the importance of mathematics for their futures. There is considerable scope for improvement on these measures, most of which are positively correlated with mathematics performance. There is also scope to reduce female students' anxiety about mathematics, and some of this may come from increased confidence and greater success in aspects of mathematics on which they have done less well in the past (spatial reasoning, problem solving).

Finally, the introduction of new assessment arrangements, including Classroom-Based Assessments, offer opportunities for students to engage with aspects of mathematics and approaches to problem solving in mathematics that have not been available to them in the past. Such assessments should also offer opportunities for formative feedback. The transition to a single exam paper in mathematics will present a challenge, not least in ensuring that students who, in the past, would have taken the Junior Certificate mathematics exam at Foundation level, can demonstrate their abilities. The transition to computer-based assessment in international studies suggests that consideration needs to be given to the role of ICTs in developing and assessment students' mathematical knowledge in more interactive ways.

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Chapter 5 Appendix

Table A5.1. Correlations between class size and achievement in mathematics, in 16 OECD countries including Ireland, in TIMSS 2015

Country	*r*
Australia	.23
Canada	.13
Chile	.09
England	.50
Hungary	.21
Ireland	.42
Israel	.20
Italy	.16
Japan	.05
Korea	.11
New Zealand	.25
Norway	.08
Slovenia	.05
Sweden	.15
Turkey	.00
United States	-.04
OECD 16 Average	.16

OECD-16 = 16OECD countries participating in PISA 2015.

Table A5.2. Average minutes per week instructional time for mathematics, in Ireland and across 16 OECD countries on average, in TIMSS 2015

	Instructional time for mathematics		
	Mean (minutes per week)	SE	SD
Chile	300.7	(6.85)	77.61
Canada	270.1	(4.21)	60.72
United States	258.1	(5.81)	85.44
Italy	247.0	(4.03)	45.55
Israel	243.4	(2.59)	45.38
New Zealand	223.8	(3.38)	40.24
Australia	213.6	(2.99)	49.21
England	195.4	(5.29)	54.09
Ireland	**192.7**	**(1.25)**	**15.98**
Turkey	191.7	(4.56)	65.57
Hungary	185.6	(3.75)	45.77
Slovenia	180.0	(1.85)	33.22
Korea	177.1	(1.93)	28.86
Sweden	166.2	(2.26)	28.63
Norway	165.4	(3.37)	42.56
Japan	155.6	(1.96)	29.77
OECD 16 Average	210.4	(0.96)	46.79

Table A5.3. Correlations between instructional time and achievement in mathematics, in 16 OECD countries including Ireland, in TIMSS 2015

Country	*r*
Australia	.00
Canada	-.11
Chile	-.07
England	-.13
Hungary	.10
Ireland	-.05
Israel	.29
Italy	-.02
Japan	.14
Korea	.00
New Zealand	.04
Norway	-.06
Slovenia	-.02
Sweden	.05
Turkey	-.02
United States	-.09
OECD 16 Average	.00

OECD-16 = 16 participating OECD countries.

Table A5.4. Percentages of students whose teachers report engaging in various general teaching practices to promote conceptual understanding in 'every or almost every' class, in 16 OECD countries including Ireland, in TIMSS 2015

	%																
	Aus	Can	Chl	Eng	Hun	**Ire**	Isr	Ita	Jap	Kor	NZ	Nor	Slo	Swe	Tur	US	O-16
Relate the lesson to students' daily lives	27.8	42.9	65.6	13.7	43.6	27.9	34.9	51.7	13.0	12.6	14.7	17.7	40.2	14.0	46.6	32.7	31.2
Ask students to explain their answers	58.3	72.0	51.3	84.6	60.0	60.4	72.1	72.5	20.9	30.0	59.7	48.4	50.0	61.1	70.9	74.5	59.2
Ask students to complete challenging exercises that require them to go beyond the instruction	23.6	25.5	30.5	43.1	13.3	17.7	26.3	16.8	1.5	12.5	21.7	10.7	10.5	13.0	14.3	34.1	19.7
Encourage classroom discussions among students	41.5	50.7	20.4	44.9	16.2	26.4	42.3	46.4	6.8	12.0	37.7	20.1	24.7	43.0	15.9	57.0	31.6
Link new content to students' prior knowledge	64.8	72.3	64.7	79.0	83.9	69.8	74.0	78.1	30.7	32.3	61.2	48.2	81.8	46.5	75.9	78.2	65.1
Ask students to decide their own problem solving procedures	19.5	37.6	51.1	21.2	46.5	15.7	44.8	49.8	12.4	25.0	19.0	16.5	26.5	28.1	52.7	36.0	31.4
Encourage students to express their ideas in class	51.5	76.2	68.3	68.4	51.0	51.4	69.5	65.6	n	35.1	55.6	25.4	51.3	47.7	71.2	68.0	57.1

Aus=Australia; Can=Canada; Chl=Chile; Eng=England; Hun=Hungary; Ire=Ireland; Isr=Israel; Ita=Italy; Jap=Japan; Kor=Korea; NZ=New Zealand; Nor=Norway; Slo=Slovenia; Swe-Sweden; Tur=Turkey; US=United States; O-16=OECD 16 (16 participating OECD countries). n = no data.

Table A5.5. Percentages of students whose teachers ask them to do various instructional actvities in mathematics class in 'every or almost every class', 'about half of classes', 'some classes' and 'never', and average performance in mathematics, in Ireland, in TIMSS 2015

	Ireland											
	Every or almost every class			About half of classes			Some classes			Never		
	%	Mean	SE	%	Mean	SE	%	Mean	SE	%	Mean	SE
Listen to me explain new mathematics content	61.6	526.5	(4.07)	29.4	519.6	(5.75)	9.0	507.0	(7.99)	0.0	n	n
Listen to me explain how to solve problems	53.0	519.9	(4.84)	28.8	529.4	(5.69)	17.8	517.3	(6.45)	0.4	558.6	(21.29)
Memorise rules, procedures, and facts	14.2	523.8	(7.68)	21.7	511.4	(8.58)	56.7	525.3	(3.73)	7.5	536.8	(9.60)
Work problems (individually or with peers) with my guidance	50.4	517.8	(3.94)	34.8	535.5	(4.52)	13.4	507.0	(11.99)	1.4	529.3	(27.66)
Work problems together in the whole class with direct guidance from me	39.2	518.0	(4.89)	39.7	530.9	(4.48)	19.5	511.4	(7.83)	1.7	565.8	(22.21)
Work problems (individually or with peers) while I am occupied by other tasks	11.9	511.1	(9.30)	19.2	527.3	(7.56)	34.8	524.4	(4.28)	34.1	522.1	(5.33)
Work on problems for which there is no immediately obvious method of solution	1.8	513.7	(15.4)	18.2	538.0	(7.06)	59.6	524.5	(3.72)	20.5	504.3	(8.23)
Take a written test or quiz	2.6	527.0	(11.66)	9.4	520.6	(11.22)	86.4	522.2	(3.02)	1.6	549.6	(10.57)
Work in mixed ability groups	15.1	514.7	(7.45)	24.6	522.1	(5.10)	43.7	525.6	(4.28)	16.6	523.3	(11.63)
Work in same ability groups	8.7	532.0	(10.90)	16.2	524.0	(8.08)	51.4	525.0	(4.19)	23.8	512.5	(6.93)

n=not applicable.

Table A5.6. Percentages of students whose teachers ask them to do various instructional actvities in mathematics class in 'every or almost every class', 'about half of classes', 'some classes' and 'never' across 16 OECD countries on average, in TIMSS 2015

	OECD 16											
	Every or almost every class			About half of classes			Some classes			Never		
	%	Mean	SE	%	Mean	SE	%	Mean	SE	%	Mean	SE
Listen to me explain new mathematics content	62.2	514.6	(1.12)	25.1	510.6	(2.24)	11.6	510.9	(2.92)	1.0	535.1	(7.67)
Listen to me explain how to solve problems	57.3	513.5	(1.18)	26.6	510.7	(2.13)	15.0	516.5	(2.46)	1.1	527.9	(6.39)
Memorise rules, procedures, and facts	22.0	517.2	(2.12)	29.7	510.0	(1.52)	43.7	514.5	(1.38)	5.0	540.0	(5.43)
Work problems (individually or with peers) with my guidance	54.5	516.3	(1.24)	32.7	511.7	(1.64)	12.3	507.2	(3.07)	0.5	524.7	(12.93)
Work problems together in the whole class with direct guidance from me	40.0	512.6	(1.44)	34.1	513.4	(1.56)	24.5	514.9	(2.04)	1.4	528.7	(10.73)
Work problems (individually or with peers) while I am occupied by other tasks	13.6	514.8	(2.82)	15.4	513.0	(2.52)	32.3	510.8	(1.82)	38.7	514.3	(1.51)
Work on problems for which obvious method of solution there is no immediately obvious method of solution	6.2	516.8	(5.21)	21.8	518.3	(1.98)	59.7	515.0	(1.15)	12.4	502.4	(2.77)
Take a written test or quiz	8.0	513.7	(3.82)	14.3	506.8	(3.84)	76.7	515.4	(1.01)	1.1	513.7	(8.69)
Work in mixed ability groups	13.6	512.7	(3.72)	21.7	506.9	(2.08)	52.6	517.0	(1.30)	12.1	516.4	(2.54)
Work in same ability groups	9.1	514.8	(3.91)	18.3	506.6	(2.32)	52.1	513.7	(1.32)	20.4	514.6	(2.17)

OECD 16 = 16 participating OECD countries.

Table A5.7. Percentages of students whose teachers ask them to do various instructional actvities in mathematics class in 'every or almost every class', in 16 OECD countries including Ireland, in TIMSS 2015

	%																
	Aus	Can	Chl	Eng	Hun	**Ire**	Isr	Ita	Jap	Kor	NZ	Nor	Slo	Swe	Tur	US	O-16
Listen to me explain new mathematics content	75.1	53.7	85.3	66.0	37.4	61.6	80.32	72.7	52.7	62.9	56.7	40.8	57.4	45.8	90.6	57.3	62.2
Listen to me explain how to solve problems	64.4	43.8	73.4	57.8	39.6	52.9	78.8	63.1	57.2	67.9	45.6	36.0	53.5	42.2	87.5	52.8	57.3
Memorise rules, procedures, and facts	27.1	10.1	26.9	18.3	16.3	14.2	27.2	43.5	37.6	27.6	9.8	6.9	44.0	11.1	19.7	11.0	22.0
Work problems (individually or with peers) with my guidance	70.0	58.2	54.8	61.9	51.2	50.4	56.7	53.9	60.5	39.5	60.5	62.4	40.2	46.1	40.5	64.2	54.5
Work problems together in the whole class with direct guidance from me	46.9	40.1	51.0	38.4	37.2	39.1	48.8	46.7	47.8	47.5	29.9	29.1	26.4	15.1	38.4	57.1	40.0
Work problems (individually or with peers) while I am occupied by other tasks	29.1	13.3	10.9	21.5	4.1	11.9	2.7	6.8	4.1	23.2	17.0	3.9	7.4	4.0	16.0	21.3	13.6
Work on problems for which there is no immediately obvious method of solution	7.5	5.8	10.3	5.3	2.5	1.8	10.8	16.1	4.2	11.1	2.7	3.0	3.7	0.8	5.5	7.7	6.2
Take a written test or quiz	5.6	2.6	11.5	1.9	6.2	2.6	11.1	8.3	14.6	7.4	5.5	0.7	3.2	1.4	34.7	10.1	8.0
Work in mixed ability groups	16.9	18.7	13.4	8.2	2.4	15.1	10.2	8.4	30.2	12.0	17.1	4.8	5.2	15.2	13.0	27.0	13.6
Work in same ability groups	9.7	5.0	8.3	30.7	3.6	8.7	7.4	3.8	8.0	19.6	6.6	5.5	5.1	2.9	13.2	7.8	9.1

Aus=Australia; Can=Canada; Chl=Chile; Eng=England; Hun=Hungary; Ire=Ireland; Isr=Israel; Ita=Italy; Jap=Japan; Kor=Korea; NZ=New Zealand; Nor=Norway; Slo=Slovenia; Swe-Sweden; Tur=Turkey; US=United States; O-16=OECD 16 (16 participating OECD countries).

Table A5.8. Percentages of students whose mathematics teachers report various interactions with other teachers 'often' or 'very often', in 16 OECD countries including Ireland, in TIMSS 2015

	%																
	Aus	Can	Chl	Eng	Hun	**Ire**	Isr	Ita	Jap	Kor	NZ	Nor	Slo	Swe	Tur	US	O-16
Discuss how to teach a particular topic	76.5	53.9	63.1	58.7	65.1	56.2	78.0	60.1	37.7	52.4	70.1	59.0	76.3	66.0	57.8	63.4	62.1
Collaborate in planning and preparing instructional materials	67.6	55.1	45.8	47.0	53.6	50.9	77.6	51.0	26.9	62.6	55.8	49.9	53.3	55.7	55.5	60.9	54.3
Share what I have learned about my teaching experiences	69.1	51.2	54.6	55.5	73.9	53.8	76.5	47.5	4.9	62.2	61.5	58.8	78.2	69.8	77.7	60.1	62.0
Visit another classroom to learn more about teaching	16.5	8.1	8.8	25.3	18.7	5.0	22.8	6.4	22.6	40.0	17.2	9.0	7.8	7.9	40.0	14.6	16.9
Work together to try out new ideas	40.4	40.4	24.2	36.4	40.3	27.5	67.8	26.3	28.7	50.6	41.1	34.6	46.0	38.4	52.9	46.5	40.1
Work as a group on implementing the curriculum	70.2	41.2	39.0	56.6	39.7	63.0	68.5	32.3	22.5	71.9	62.0	47.3	54.6	42.2	47.7	57.1	51.0
Work with teachers from other grades to ensure continuity of learning	46.0	28.6	23.0	43.7	59.5	42.8	70.8	28.0	25.1	45.8	53.8	21.8	34.3	26.9	39.3	28.0	38.6

Aus=Australia; Can=Canada; Chl=Chile; Eng=England; Hun=Hungary; Ire=Ireland; Isr=Israel; Ita=Italy; Jap=Japan; Kor=Korea; NZ=New Zealand; Nor=Norway; Slo=Slovenia; Swe-Sweden; Tur=Turkey; US=United States; O-16=OECD 16 (16 participating OECD countries).

Table A5.9. Percentages of students whose mathematics teachers report aspects of their (teacher) confidence is 'very high', 'high', 'medium' and 'low', and average achievement in mathematics, TIMSS 2015 - Ireland

	Very high			High			Medium			Low		
	%	Mean	SE	%	Mean	SE	%	Mean	SE	%	Mean	SE
Inspiring students to learn mathematics	32.7	537.0	(4.19)	47.2	523.7	(4.94)	19.7	496.8	(7.29)	0.5	478.4	(19.07)
Showing students a variety of problem-solving strategies	31.2	537.8	(4.44)	50.2	524.3	(3.98)	16.8	492.2	(9.49)	0.8	497.7	(24.46)
Providing challenging tasks for the highest achieving students	21.7	539.3	(6.30)	53.6	527.8	(3.94)	22.5	498.3	(8.04)	0.8	478.3	(24.70)
Adapting my teaching to engage students' interest	28.2	519.8	(5.87)	53.9	525.0	(3.91)	17.0	518.2	(7.83)	0.9	464.6	(10.53)
Helping students appreciate the value of learning mathematics	27.1	526.5	(5.53)	53.4	530.4	(3.30)	17.8	494.0	(8.89)	1.7	522.6	(25.39)
Assessing student comprehension of mathematics	38.0	534.8	(4.97)	53.9	519.3	(3.40)	8.1	487.0	(15.75)	0.0	n	n
Improving the understanding of struggling students	29.2	526.7	(7.41)	52.4	522.8	(4.16)	17.3	515.3	(5.96)	0.6	525.1	(46.05)
Making mathematics relevant to students	28.6	526.8	(5.91)	54.0	523.1	(4.18)	17.0	513.0	(6.29)	0.4	567.4	(6.90)
Developing students' higher-order thinking skills	20.1	540.8	(6.26)	54.4	533.3	(3.31)	23.0	492.2	(6.25)	0.9	429.1	(34.98)

Table A5.10. Percentages of students whose teachers report aspects of their (teacher) confidence is 'very high' or 'high', TIMSS 2015, OECD-16 Countries

	%																
	Aus	Can	Chl	Eng	Hun	**Ire**	Isr	Ita	Jap	Kor	NZ	Nor	Slo	Swe	Tur	US	O-16
Inspiring students to learn mathematics	75.4	80.8	89.3	81.9	76.2	79.8	86.4	86.0	31.7	74.6	72.5	87.3	92.3	76.5	84.6	74.9	78.1
Showing students a variety of problem-solving strategies	82.4	92.4	97.3	82.7	87.6	81.3	87.6	90.1	49.3	85.3	78.1	85.2	93.5	89.4	89.4	88.2	85.0
Providing challenging tasks for the highest achieving students	74.1	73.6	86.7	87.4	73.8	75.3	73.5	66.4	41.5	73.9	71.8	85.2	92.7	78.4	60.8	75.4	74.4
Adapting my teaching to engage students' interest	76.9	84.7	93.6	74.9	79.3	82.0	92.3	85.6	53.6	75.6	72.2	65.9	90.6	66.5	81.8	72.2	78.0
Helping students appreciate the value of learning mathematics	78.7	81.4	95.3	76.5	81.6	80.5	87.9	84.8	44.1	83.8	75.3	72.3	86.6	73.6	85.0	74.1	78.8
Assessing student comprehension of mathematics	86.5	93.1	94.5	87.2	86.4	91.9	91.9	87.2	48.3	85.7	80.8	85.8	89.0	91.2	89.3	87.9	86.1
Improving the understanding of struggling students	73.2	81.4	76.3	80.0	75.3	82.6	87.1	74.7	54.3	83.0	69.4	60.8	81.4	71.8	94.0	74.9	76.2
Making mathematics relevant to students	70.4	81.9	95.4	61.4	74.2	82.6	85.1	73.1	31.7	65.5	67.8	67.5	88.1	68.1	86.7	71.2	73.2
Developing students' higher-order thinking skills	63.7	69.5	88.8	65.3	74.5	74.5	81.1	56.3	25.8	62.5	60.3	70.4	86.6	65.2	77.1	71.2	68.3

Aus=Australia; Can=Canada; Chl=Chile; Eng=England; Hun=Hungary; Ire=Ireland; Isr=Israel; Ita=Italy; Jap=Japan; Kor=Korea; NZ=New Zealand; Nor=Norway; Slo=Slovenia; Swe-Sweden; Tur=Turkey; US=United States; O-16=OECD 16 (16 participating OECD countries).

Table A5.11. Calculator use in mathematics class and average achievement, in 16 OECD countries including Ireland, in TIMSS 2015

Country	Calculators available for students to use in mathematics classes								
	Yes, unrestricted	Yes, restricted	No	Yes, unrestricted		Yes, restricted		No	
	%	%	%	Mean	SE	Mean	SE	Mean	SE
Australia	49.5	49.9	0.66	514.6	(3.86)	504.2	(3.81)	514.3	(6.37)
Canada	57.2	42.2	0.59	532.2	(2.88)	527.7	(3.77)	527.3	(19.18)
Chile	5.2	73.0	18.8	413.0	(16.46)	422.7	(4.45)	463.0	(10.22)
Hungary	13.0	73.6	13.4	539.5	(15.45)	511.6	(4.12)	504.7	(12.93)
Ireland	**72.0**	**26.7**	**1.3**	**520.3**	**(3.37)**	**529.0**	**(5.95)**	**514.1**	**(37.50)**
Israel	76.5	20.3	3.2	516.8	(4.78)	494.5	(15.16)	519.6	(36.86)
Italy	16.9	64.8	18.2	488.7	(8.06)	490.6	(3.30)	509.5	(6.20)
Japan	6.6	58.7	34.7	587.2	(8.15)	581.3	(3.13)	595.8	(4.18)
Korea	3.0	35.1	61.9	579.5	(13.18)	598.5	(5.10)	611.0	(3.45)
New Zealand	42.2	47.7	10.1	498.5	(6.19)	487.9	(6.42)	502.8	(14.35)
Norway	32.9	66.2	0.9	513.5	(3.35)	513.4	(2.79)	461.0	(4.81)
Slovenia	0.5	71.5	28.0	556.7	(15.75)	516.2	(2.71)	516.4	(4.13)
Sweden	31.4	67.8	0.87	502.4	(4.88)	499.8	(3.64)	492.4	(2.47)
Turkey	15.6	32.5	52.0	439.2	(8.99)	463.7	(8.31)	459.3	(6.94)
United States	42.7	52.2	5.2	521.0	(5.26)	518.9	(5.30)	489.1	(13.14)
England	10.0	89.5	0.5	544.8	(17.86)	514.8	(5.13)	406.1	(36.85)
OECD 16 Average	29.7	54.7	15.6	516.7	(2.52)	510.9	(1.49)	505.4	(4.57)

OECD 16 = 16 participating OECD countries.

Table A5.12. Standads Errors for Percentages of students whose teachers report various issues as limiting how they teach in class ‘some’ or ‘a lot’, Ireland

Issue	A lot	Some	Not at all
Students lacking prerequisite knowledge or skills	1.62	2.79	2.76
Students suffering from lack of basic nutrition	1.32	2.30	1.32
Students suffering from not enough sleep	1.18	3.10	3.01
Disruptive students	1.86	2.88	3.24
Disinterested students	1.86	3.05	2.60
Students with physical disabilities	0.31	1.30	1.40
Students with mental, emotional, or psychological impairments	1.21	2.74	3.01

Table A5.13. Percentages of students whose teachers report various issues as limiting how they teach in class 'some' or 'a lot', in 16 OECD countries including Ireland, in TIMSS 2015

	%																
	Aus	Can	Chl	Eng	Hun	**Ire**	Isr	Ita	Jap	Kor	NZ	Nor	Slo	Swe	Tur	US	O-16
Students lacking prerequisite knowledge or skills	85.7	88.0	96.4	84.9	85.0	90.9	82.6	90.6	67.1	72.8	88.9	90.5	77.9	84.0	97.7	94.6	86.1
Students suffering from lack of basic nutrition	30.5	36.8	49.5	26.1	22.9	21.0	35.0	22.9	1.3	32.3	37.3	43.5	5.8	22.3	64.3	29.7	30.1
Students suffering from not enough sleep	66.9	72.6	86.7	60.5	65.1	65.7	63.5	55.1	43.7	69.5	70.0	67.0	42.2	60.7	82.5	73.3	65.3
Disruptive students	69.7	69.7	87.9	68.6	54.0	46.2	71.2	77.6	19.9	90.8	76.7	57.5	52.6	63.7	91.8	76.4	67.1
Uninterested students	79.2	83.2	92.0	71.8	79.0	77.9	76.6	96.6	55.5	87.3	81.4	77.8	64.2	78.5	97.5	88.6	80.5
Students with physical disabilities	10.9	8.2	16.3	2.9	2.4	7.2	20.7	16.5	2.5	19.5	8.9	1.2	3.0	6.4	7.8	14.0	9.3
Students with mental, emotional, or psychological impairments	59.1	54.9	67.5	43.6	52.4	46.7	75.2	61.4	17.9	70.0	46.4	41.0	42.2	52.5	43.5	57.6	52.0

Aus=Australia; Can=Canada; Chl=Chile; Eng=England; Hun=Hungary; Ire=Ireland; Isr=Israel; Ita=Italy; Jap=Japan; Kor=Korea; NZ=New Zealand; Nor=Norway; Slo=Slovenia; Swe-Sweden; Tur=Turkey; US=United States; O-16=OECD 16 (16 participating OECD countries).